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Reflection Properties of Gravito-MHD Waves in an Inhomogeneous Horizontal Magnetic Field

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Abstract In this article we derive the dispersion equation for gravito-MHD waves in an isothermal, gravitationally stratified plasma with a horizontal inhomogeneous magnetic field. Sound and Alfvén speeds are constant. Under these conditions, it is possible to derive analytically the equations for gravito-MHD waves. The large values of the viscous and magnetic Reynolds numbers in the solar atmosphere imply that the dissipative terms in the MHD equations are negligible, except in layers around the positions where the frequency of the MHD wave equals the local Alfvén or slow frequency. Outside these layers the MHD waves are accurately described by the equations of ideal MHD.
We consider waves which propagate energy upward in the atmosphere. For the plane boundary, z = 0, between two isothermal plasma regions with horizontal but different magnetic fields, we discuss the boundary conditions and derive the equations for the reflection and transmission coefficients.
In the simpler case of a gravitationally stratified plasma without magnetic field, these coefficients describe the reflection and transmission properties of gravito-acoustic waves.

Keywords: Sun:photosphere, Sun:corona, magnetohydrodynamics, waves

1. Introduction

The problem of wave propagation in magnetized stellar atmospheres remains unsolved despite continuous efforts over the last half century. The main reason is the complicated mathematical description of the physical processes that influence wave propagation in realistic magneto-atmospheres. The motivation for studying gravito-MHD waves is to shed light on their role in the dynamics and formation of the solar corona as they are believed to be associated with the long-standing problem of the solar wind acceleration and coronal plasma heating. These waves transport energy, and when part of this energy is dissipated, they can heat plasmas. An important aspect of the gravito-MHD wave

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propagation is the reflection and transmission of such waves in the "open" magnetized atmospheres of late-type stars. The considerable interest in this aspect of the propagation problem stems from the conjecture that Alfvén wave reflection may play a crucial role in the acceleration of stellar winds (see Moore *et al.*, 1991; 1992; Rosner *et al.*, 1991). The EUV imaging telescopes on the *Solar and Heliospheric Observatory* (SOHO) and the *Transition Region And Coronal Explorer* (TRACE) spacecraft made it possible to directly observe the MHD waves in the corona. Compressive longitudinal waves were detected in polar plumes (DeForest and Gurman, 1998; Nakariakov *et al.*, 1999). These waves were interpreted as slow magnetoacoustic waves (Ofman, Nakariakov, and DeForest, 2000; Ofman, Nakariakov, and Sehgal, 2000). Thompson *et al.* (1999), reported observations of the so-called coronal Moreton waves (flare waves). Most probably, these waves are magnetoacoustic waves because they are compressive and they propagate essentially isotropically in the low β (β is the ratio of the kinetic (thermal) to the magnetic plasma pressure) coronal plasma. Theoretical analysis of MHD waves in the corona shows that a key element of the coronal wave theory is the interaction of the waves with coronal structures (Roberts, 1991; Roberts and Ulmschneider, 1997), which affect the MHD waves in a number of different ways: they can guide the waves due to refraction and reflection, resonantly absorb the wave energy when the wave phase speed coincides with local Alfvén or cusp speeds, make the waves dispersive, lead to linear coupling of the waves and enhance the efficiency of their nonlinear coupling. A number of solar models were put forward to investigate the features of gravito-MHD waves in the solar atmosphere. Some authors studied gravito-MHD waves in a stratified isothermal atmosphere with a vertical magnetic field and obtained analitical solutions (Ferraro and Plumpton, 1958; Zhugzhda, 1979; Zhugzhda and Dzhailov, 1982, 1984a, 1984b). Numerical results are obtained for compressible MHD perturbations in a spherically symetric isothermal static stellar atmosphere embedded with a radial magnetic field (Leroy and Schwartz, 1982; Lou, 1996; Rosenthal *et al.*, 2002). Beneath the solar surface the magnetic field can be described by confined, toroidal thin flux tubes. When these flux tubes break through the photosphere, it is observed that the magnetic field lines incline in most cases from the vertical direction. They fan out and create a local magnetic canopy, *i.e.* structures with horizontal magnetic field, through the chromosphere. Lites *et al.* (1996), presented observations of quiet regions near the center of the solar disk using the *Advanced Stokes Polarimeter*. These observations reveal a component of the solar magnetic field heretofore unobserved: isolated, small-scale predominantly horizontal magnetic flux structures in the solar photosphere. They find that such magnetic fields are weak, significantly less than 1000 G. A horizontal (canopy) magnetic field with uniform Alfvén speed was used for the chromosphere of a simple planar solar-like model by Campbell and Roberts (1989). A simple planar three-layer model, including a non magnetic interior and a constant β chromosphere and corona was analyzed by Pintér and Goossens (1999).

In this work we consider the driven wave reflection (and transmission) in the two-layer model of an isothermal, gravitationally stratified plasma embedded in an inhomogeneous horizontal magnetic field with constant Alfvén speed. We

think that this simple analytical model can be an effective tool to describe (although roughly) the behavior of gravito-MHD waves in a solar atmosphere. Of course, the assumption of a steady unidirectional horizontal magnetic field is obviously a crude representation of the three-dimensional magnetic structures of the real solar chromosphere and corona. Magnetic fluxes are continuously emerging through the solar surface and expanding into the atmosphere. Consequently, the orientation of the magnetic field changes temporarly and spatially.
This paper is organized as follows. In Section 2 we used the ideal MHD equations to mathematically describe our problem. We derive the dispersion equation for gravito-MHD waves and present it in the dimensional and dimensionless form. In Section 3 we derive and discuss the boundary conditions for unperturbed relevant physical quantities in the basic state and for perturbations as well. We explain the presence of singularities by the resonant absorption phenomena. Reflection and transmission coefficients are derived in Section 4. Finally, Section 5 contains the conclusion.

2. MHD Equations

We start from the standard set of MHD equations describing the dynamics of a fully ionized plasma having the properties of a perfect gas given by the:
continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

momentum equation

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \rho \boldsymbol{g} + \frac{1}{\mu_0} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B},$$

magnetic induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}), \tag{1}$$

Gauss law

$$\nabla \cdot \boldsymbol{B} = 0,$$

and an adiabatic law for a perfect gas

$$\frac{\partial p}{\partial t} + \boldsymbol{v} \cdot \nabla p = \frac{\gamma p}{\rho} \left(\frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla \rho \right).$$

2.1. The Basic State

In this paper we consider an unbounded isothermal magnetized plasma with a constant Alfvén speed. The applied magnetic field is horizontal and the gravitational acceleration is constant.

The initial basic state is defined by the model itself which implies a stationary, static (*i.e.* the hydrostatic equilibrium), gravitationally stratified isothermal plasma with an embedded horizontal magnetic field. The equilibrium quantities are assumed to depend only on the z coordinate. Thus, in Cartesian coordinates, we have:

$$\boldsymbol{g} = -g\boldsymbol{e}_z, \quad g = const,$$

$$\boldsymbol{B}_0 = B_0(z)\boldsymbol{e}_x, \tag{2}$$

$$\rho_0 = \rho_0(z), \quad p_0 = p_0(z).$$

This basic state is assumed to satisfy the perfect-gas law:

$$p_0 = \rho_0 R_M T_0,$$

with $R_M = k/m$, where k is Boltzman constant and m is the mean mass of plasma particles. The unperturbed plasma is initially in hydrostatic equilibrium and assumed to be stepwise isothermal $T_0 = const$, *i.e.* with constant temperature in each of the two regions separated by the boundary $z = 0$.
A fully ionized plasma in magnetohydrostatic equilibrium satisfies

$$\frac{d}{dz}\left(p_0(z) + \frac{B_0^2(z)}{2\mu_0}\right) + \rho_0(z)g = 0 \tag{3}$$

or

$$\frac{d}{dz}\left(\frac{v_s^2}{\gamma} + \frac{v_A^2}{2}\right) + \left(\frac{v_s^2}{\gamma} + \frac{v_A^2}{2}\right)\frac{d}{dz}\ln\rho_0(z) + g = 0. \tag{4}$$

Here, $\gamma = \frac{c_p}{c_v} = \frac{5}{3}$ is the ratio of the specific heats, $v_s^2 = \gamma\frac{p_0(z)}{\rho_0(z)} = \gamma R_M T_0$ and $v_A^2 = \frac{B_0^2(z)}{\mu_0\rho_0(z)}$ are the squares of the sound and Alfvén speeds respectively. In this model both v_s and v_A are assumed to be constant. Therefore, Equation (4) for a magnetohydrostatic equilibrium can be rewritten as:

$$\frac{d}{dz}\ln\rho_0(z) + \frac{1}{H} = 0, \tag{5}$$

with a density scale length

$$H = \frac{v_s^2}{\gamma g} + \frac{v_A^2}{2g} = \frac{1+\beta}{\beta}H_0,$$

where $H_0 = \frac{v_s^2}{\gamma g} = const$ is the isothermal density scale length in a non-magnetized atmosphere and $\beta = \frac{p_0(z)}{p_{0m}(z)} = \frac{2v_s^2}{\gamma v_A^2} = const$.
The solutions for the density and magnetic field profiles that follow from Equation (5) and the Alfvén speed definition are obtained as:

$$\rho_0(z) = \rho_0(0)e^{-z/H}, \quad B_0(z) = B_0(0)e^{-z/2H}. \tag{6}$$

We assume that the plasma density, the kinetic plasma pressure $p_0(z)$ and the magnetic plasma pressure $p_{0m}(z) = B_0^2/(2\mu_0)$ decrease exponentially in the vertical direction with the same constant H, while the magnetic field strength $B_0(z)$ decreases with $2H$:

$$B_0(z) = B_0(0)e^{-z/2H}. \tag{7}$$

Here, $B_0(0) = v_A\sqrt{\mu_0\rho_0(0)}$.

2.2. Linearized MHD Equations

The plasma dynamics of small amplitude waves is described by a standard set of non-linear MHD equations for an ideal plasma (Equations (1)), which are perturbed by taking any unknown physical quantity $f(x,y,z,t)$ as

$$f(x,y,z,t) = f_0(z) + \delta f(x,y,z,t), \tag{8}$$

where the perturbations $\delta f(x,y,z,t)$ have the form:

$$\delta f(x,y,z,t) = f_1(z)e^{(-i\omega t + ik_x x + ik_y y)},$$

while unperturbed quantities $f_0(z)$ satisfy the magnetohydrostatic balance Equation (5) of the basic state.
Equations (1) linearized following Equation (8) can be reduced to a system of two coupled ordinary differential equations (Pintér, Čadež, and Roberts, 1999):

$$\frac{d\xi_{1z}}{dz} = \frac{C_1}{D}\xi_{1z} - \frac{C_2}{D}P_1, \quad \frac{dP_1}{dz} - g\frac{d\rho_0(z)}{dz}\xi_{1z} = C_3\xi_{1z} - C_4P_1, \tag{9}$$

where $\xi_{1z} = iv_{1z}/\omega$ is the z-component (vertical component) of the fluid displacement and $P_1 = p_1 + p_{1m}$ is the total pressure perturbation made of the perturbed kinetic plasma pressure p_1 and the perturbed magnetic pressure $p_{1m} = \boldsymbol{B}_0 \cdot \boldsymbol{B}_1/\mu_0$, where $\boldsymbol{B}_1$ is the perturbed magnetic field. These equations govern the linear motions of a one-dimensional inhomogeneous magnetic plasma in a gravitational field. The coefficients in Equations (9) are:

$$D(z) = \rho_0(z)(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2),$$

$$C_1(z) = \rho_0(z)g\omega^2(\omega^2 - \omega_A^2),$$

$$C_2(z) = \omega^4 - k_p^2(v_s^2 + v_A^2)(\omega^2 - \omega_c^2), \tag{10}$$

$$C_3(z) = \rho_0(z)(\omega^2 - \omega_A^2) + \frac{\rho_0(z)g^2(\omega^2 - \omega_A^2)}{(v_s^2 + v_A^2)(\omega^2 - \omega_c^2)},$$

$$C_4(z) = \frac{g\omega^2}{(v_s^2 + v_A^2)(\omega^2 - \omega_c^2)}.$$

The density distribution is given by Equation (6) and the characteristic frequencies are:

$$\omega_A^2 = k_x^2 v_A^2, \quad \omega_s^2 = k_x^2 v_s^2, \quad \omega_c^2 = k_x^2 v_c^2,$$

where

$$v_c^2 = v_s^2 v_A^2/(v_s^2 + v_A^2)$$

is the cusp speed. The Alfvén and slow or cusp frequency play a fundamental role in the phenomena of resonant absorption as will be shown in Section 3.

2.3. Dispersion Equation

Taking into account Equation (6), the Equations (9) and (10) allow the following solutions for the fluid displacement ξ_{1z} and the total pressure perturbation P_1[1]:

$$\xi_{1z}(z) = \xi_{1z}(0) e^{z/2H} e^{ik_z z}, \tag{11}$$

$$P_1(z) = P_1(0) e^{-z/2H} e^{ik_z z}. \tag{12}$$

Finally, Equations (9) with solutions in Equations (11) and (12) yield the dispersion equation for gravito-MHD waves:

$$k_z^2 = \frac{\omega^2(\omega^2 - \omega_A^2) - \frac{g}{H}\omega_A^2}{(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2)}\omega^2 - \frac{1}{4H^2}$$

$$- \frac{(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2) + g^2(\omega^2 - \omega_A^2) - \frac{g}{H}(v_s^2 + v_A^2)(\omega^2 - \omega_c^2)}{(v_s^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_c^2)} k_p^2, \tag{13}$$

where $k_p^2 = k_x^2 + k_y^2$ is the horizontal component of the wave vector $\boldsymbol{k}$. We can rewrite the dispersion equation (Equation (13)) in the dimensionless form

[1] Equations (11) and (12) show that the amplitudes of the fluid displacement ξ_{1z} and the total pressure perturbation P_1 are exponential functions of z. However, the phase averaged energy density $\langle\epsilon\rangle$ of these waves remains z invariant. Namely, $\langle\epsilon\rangle$ is the sum of the corresponding thermal, kinetic and magnetic mean energy densities

$$\langle\epsilon\rangle = \langle\epsilon_T\rangle + \langle\epsilon_K\rangle + \langle\epsilon_M\rangle,$$

i.e.

$$\langle\epsilon\rangle = \frac{v_s^2}{2\rho_0(z)}\rho_1\rho_1^* + \frac{\rho_0(z)}{2}\boldsymbol{v}_1 \cdot \boldsymbol{v}_1{}^* + \frac{1}{2\mu_0}\boldsymbol{B}_1 \cdot \boldsymbol{B}_1{}^*,$$

where $*$ denotes the complex conjugate quantities, and ρ_1 and $\boldsymbol{v}_1 = (v_{1x}, v_{1y}, v_{1z})$ are the perturbed density and plasma velocity respectively. From the definitions of the fluid displacement $\xi_{1z} = iv_{1z}/\omega$ and the total pressure perturbation $P_1 = p_1 + \boldsymbol{B}_0 \cdot \boldsymbol{B}_1$, one can easily write the following proportionalities: $|\boldsymbol{v}_1| \sim \xi_{1z}$, for the perturbed plasma velocity, $\rho_1 \sim P_1$ for the perturbed plasma density, and $|\boldsymbol{B}_1| \sim P_1/B_0$ for the perturbed magnetic field. According to Equations (6), (11) and (12)

$$\boldsymbol{v}_1 \cdot \boldsymbol{v}_1{}^* \equiv |\boldsymbol{v}_1|^2 \sim exp(z/H), \quad \rho_1\rho_1^* \equiv |\rho_1|^2 \sim exp(-z/H), \quad \boldsymbol{B}_1 \cdot \boldsymbol{B}_1{}^* \equiv |\boldsymbol{B}_1|^2 \sim const,$$

which finally gives that the averaged wave density $\langle\epsilon\rangle$ is z invariant.

by using dimensionless physical quantities: $K_p = k_pH$, $K_z = k_zH$, where K_p and K_z are the dimensionless horizontal and vertical wave numbers scaled to $1/H$, while $\Omega = \omega H/v_s$, $\Omega_A = \omega_A H/v_s$, $\Omega_s = \omega_s H/v_s$ and $\Omega_c = \omega_c H/v_s$ are the dimensionless frequencies scaled to v_s/H. The dimensionless dispersion equation for gravito-MHD waves is:

$$K_z^2 = \frac{\Omega^2(\Omega^2 - \Omega_A^2) - \frac{(1+\beta)}{\gamma\beta}\Omega_A^2}{\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_A^2)(\Omega^2 - \Omega_c^2)}\Omega^2 - \frac{1}{4}$$

$$- \frac{\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_A^2)(\Omega^2 - \Omega_c^2) + \frac{(1+\beta)^2}{\gamma^2\beta^2}(\Omega^2 - \Omega_A^2) - \frac{(1+\beta)}{\gamma\beta}\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_c^2)}{\left(1 + \frac{2}{\gamma\beta}\right)(\Omega^2 - \Omega_A^2)(\Omega^2 - \Omega_c^2)} K_p^2. \tag{14}$$

Inspection of this equation shows that it is cubic in Ω^2, meaning that there exist three distinct MHD modes of propagating waves in the considered model of a stratified atmosphere in a magnetic field. Our analysis will be restricted to the propagation of pure modes only. Equation (14) for K_z^2 has two singularities: $\Omega = \Omega_A$ and $\Omega = \Omega_c$. In their neighborhood the condition $K_z^2 > 0$ for wave propagation is surely satisfied. Gravitational effects promote no new modes but they modify the dispersion properties of the modes existing in homogeneous magnetized plasmas. The restoring force acting upon perturbations is a combination of gravitational and magnetic forces whose proportional efficiency depends on the wave parameters and on the magnetic field strength. The dispersion properties depend qualitatively on the parameter β, measuring the relative contributions of hydrodynamic and magnetic effects. In the case without magnetic field, when $1/\beta = 0$ (or $H = H_0$) and $\Omega_A = \Omega_c = 0$, Equation (14) becomes:

$$K_z^2 = \Omega^2 - \frac{1}{4} - \frac{K_p^2}{\Omega^2}\left(\Omega^2 - \frac{(\gamma - 1)}{\gamma^2}\right),$$

or in the terms of the dimensionless characteristic frequencies *i.e.* acoustic cut-off frequency $\Omega_{co} = \frac{\omega_{co}H_0}{v_s} = \frac{1}{2}$ and Brunt-Väis*ä*l*ää* frequency $\Omega_{BV} = \frac{\omega_{BV}H_0}{v_s} = \sqrt{\frac{(\gamma-1)}{\gamma^2}}$:

$$K_z^2 = \Omega^2 - \Omega_{co}^2 - \frac{K_p^2}{\Omega^2}(\Omega^2 - \Omega_{BV}^2). \tag{15}$$

This equation describes gravito-acoustic waves in an isothermal stratified atmosphere (Mihalas, 1984) as in the case of the quiet Sun.

3. Boundary Conditions

3.1. Boundary Condition for the Unperturbed Basic State

Let us consider a basic state composed of two half-spaces with constant sound speeds v_s and Alfvén speeds v_A, separated by a horizontal boundary plane $z = 0$.

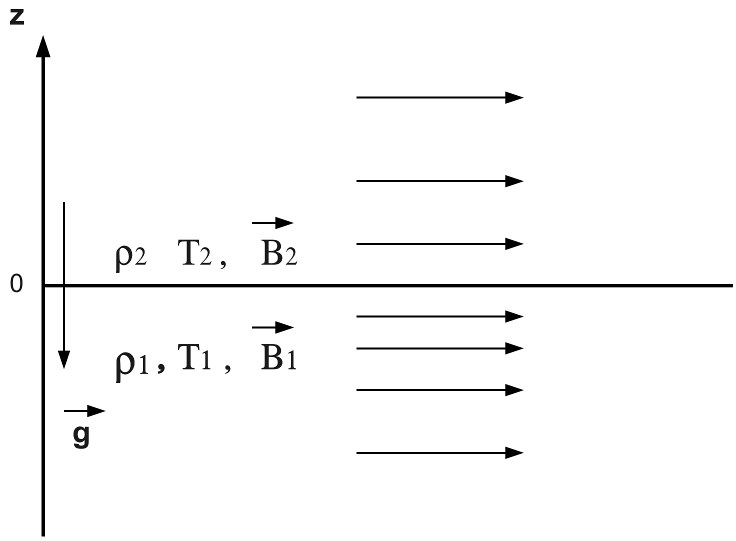

Figure 1. Sketch of the two-region model.

The magnetic field is horizontal, inhomogeneous in the z-direction and aligned along the x-axis throughout the whole space.
The two regions are characterized by the corresponding Alfvén and sound speeds: v_{A1}, v_{s1} and v_{A2}, v_{s2} which are assumed constant, and plasma densities ρ_{01} and ρ_{02} adjacent to the lower and upper side of the boundary *i.e.* at $z = 0 \pm \zeta$, with $\zeta \rightarrow 0$. Thus we have for the unperturbed density profile $\rho_0(z)$ the following expression:

$$\rho_0(z) = \rho_{01} e^{-z/H_1}, \quad z < 0, \quad (region1)$$

$$(16)$$

$$\rho_0(z) = \rho_{02} e^{-z/H_2}, \quad z > 0, \quad (region2),$$

where $H(n) = v_{sn}^2/\gamma g + v_{An}^2/2g$, $n = 1, 2$. There is a density jump (also there is a jump of all other equilibrium quantities: temperature, gas pressure and magnetic induction) across $z = 0$, Figure 1. The boundary condition that has to be applied at $z = 0$ in the basic state is the continuity of the total unperturbed pressure $p_{tot} = p_0 + p_{m0}$ (kinetic plus magnetic) at $z = 0$:

$$\rho_{01}\left[\frac{v_{s1}^2}{\gamma} + \frac{v_{A1}^2}{2}\right] = \rho_{02}\left[\frac{v_{s2}^2}{\gamma} + \frac{v_{A2}^2}{2}\right]. \quad (17)$$

We can rewrite this equation in terms of the parameter β for regions 1 and 2, as:

$$\frac{\rho_{02}}{\rho_{01}} = \frac{s\beta_2(1+\beta_1)}{\beta_1(1+\beta_2)}, \tag{18}$$

where $\beta_1 = 2v_{s1}^2/\gamma v_{A1}^2$, $\beta_2 = 2v_{s2}^2/\gamma v_{A2}^2$ and $s = v_{s1}^2/v_{s2}^2$. In the case when the region 2 (*i.e.* $z > 0$) above the interface $z = 0$ is taken to be isothermal and permeated by a non-uniform horizontal magnetic field $\boldsymbol{B}(z) = B_0(z)\boldsymbol{e}_x$ and the region 1 (*i.e.* $z < 0$) below the interface $z = 0$ is also isothermal but with different temperature and field free (*i.e.* $\beta_1 \gg 1$), Equation (18) has the form:

$$\frac{\rho_{02}}{\rho_{01}} \approx \frac{s\beta_2}{(1+\beta_2)}. \tag{19}$$

The equilibrium considered here can be unstable when the plasma in the upper region is heavier than that in the lower region, *i.e.* $\rho_{02}/\rho_{01} > 1$. This instability, discussed by Yu (1965) and Thomas (1983) among others, is known as Rayleigh-Taylor instability.

3.2. Boundary Conditions for Perturbations

Equations (9) and (10) possess singularities at $\omega^2 = \omega_A^2$ and $\omega^2 = \omega_c^2$. The presence of these singularities is an indication of a number of interesting effects connected with the phenomena of resonant absorption and phase mixing. Since all plasmas in nature are, to a higher or lower degree, inhomogeneous and since waves can be excited easily in plasmas, resonant absorption and phase mixing frequently occur. These phenomena have been studied in the context of wave damping and heating for controlled thermonuclear fusion experiments (Chen and Hasegawa, 1974; Balet, Appert, and Vaclavik, 1982, also Vaclavik and Appert, 1991; Poedts *et al.*, 1992), and for solar and astrophysical plasmas (Kuperus, Ionson, and Spicer, 1971; Poedts, Goossens, and Kerner, 1990; Goossens, 1991; Beliën, Martens, and Keppens, 1999), whereas resonant mode conversion is also studied in magnetospheric physics (Zhu and Kivelson, 1988; Kivelson and Russell, 1995; Mann and Wright, 1995, among others). To explain the mechanism of resonant absorption (or dissipation) we will consider an "extended" two-region model with a thin layer (let its thickness be 2ζ, $\zeta \ll 1$) between two semi-infinite plasma regions that occupies the half space $z < 0$ and the half space $z > 0$. In this layer the local equilibrium quantities: temperature T_0, density ρ_0, gas pressure p_0 and magnetic induction B_0 vary continuously in the z-direction. Therefore, the local speeds v_A and v_s, *i.e.* local frequencies ω_A and ω_c are also functions of z. In $z = 0 - \zeta$ their values are $\omega_A(z) = \omega_A(1)$ and $\omega_c(z) = \omega_c(1)$, while for $z = 0 + \zeta$ they are $\omega_A(z) = \omega_A(2)$ and $\omega_c(z) = \omega_c(2)$. The thin transitional layer between regions 1 and 2 introduces an Alfvén continuum $[\omega_A(1), \omega_A(2)]$ and slow continuum $[\omega_c(1), \omega_c(2)]$ in the waves frequency spectrum. Let us suppose that there is a MHD wave impiging at the boundary between the two plasma regions with a frequency ω in the continuum range. If the frequency ω of this MHD wave at a given point, say at $z = z_r$ $(0-\zeta < z_r < 0+\zeta)$, matches the local Alfvén or slow frequency, then the MHD wave is in resonance with the local Alfvén or slow wave in the point z_r. In our case of ideal MHD this would result in infinite amplitudes of the perturbations leading to the large gradients. When the gradients of the perturbations become large, we cannot assume any longer the plasma is ideal, *i.e.* dissipative effects (*e.g.* resistivity, viscosity, magnetic diffusivity) have to be considered at least within the vicinity of such resonant locations leading to energy dissipation. In the vicinity of this resonant points the dissipative MHD equations are required for a physically meaningful description of the MHD waves while ideal MHD gives an accurate description of the wave dynamics elsewhere. Such dissipation, *i.e.* energy absorption of the incoming MHD wave will result in heating of the plasma converting the energy of the MHD wave into localized thermal heating.

When ω^2 is in the continuum range, Chen and Hasegawa (1974) avoided the logarithmic singularity at $z = z_r$ where $\omega = \omega_A(z_r)$ (or $\omega = \omega_c(z_r)$) in ideal MHD by considering a complex $\omega = \Re\omega + i\Im\omega$. The role of the "artificial" damping, represented by the imaginary part of the frequency ω, is to mimic real dissipation. It could be concluded that if we want to remove the sigularities from the mathematical analysis, dissipation has to be included (see Pintér and Goossens, 1999; Pintér, Erdélyi, and New, 2001; Pintér, Erdélyi, and Goossens, 2007, also Goossens, Erdélyi, and Ruderman, 2011). Dissipations causes "real" damping of the waves and their frequencies become complex $\omega = \Re\omega + i\Im\omega$, with the small $|\Im\omega| \ll \Re\omega$ negative imaginary part which comes from dissipations and is responsible for the damping of the wave. The condition for Alfvén resonance is $\Re\omega = \omega_A(z_r)$ and the condition for slow resonance is $\Re\omega = \omega_c(z_r)$.

It can be noticed that Equations (9) together with Equation (10) do not possess singularities in the unmagnetized stratified plasma as will be shown in Section 4.2. Therefore, there is no possibility for resonant absorption to occur. In this case we are dealing with driven gravito-acoustic waves. We can conclude that singularities in Equations (9) and (10), and associated with them resonant absorption effects, occur in the magnetized plasma with or without gravitational stratification. Another interesting fact is that there are no singularities if the wavenumber $\boldsymbol{k}$ of the perturbations have not got a component in the magnetic field direction. In our model this means that $k_x = 0$.

If we approximate the foregoing thin layer whose thickness is 2ζ with a plane boundary $z = 0$ between two different plasma regions (this is possible for $\zeta \to 0$), then we avoid Alfvén and slow continuum and consequently the resonant absorption. Then, integrating Equations (9) across the boundary $z = 0$ between two arbitrary close points $z = \pm\zeta$, with $\zeta \to 0$, we obtain the following two boundary conditions for the perturbations[2]:

$$\xi_{1z}(\zeta) = \xi_{1z}(-\zeta). \tag{20}$$

[2]The second equation in Equations (9) could be rewritten in the form:

$$\frac{d}{dz}(P_1 - g\rho_0(z)\xi_{1z}) = C_3\xi_{1z} - C_4P_1 - g\rho_0(z)\frac{d\xi_{1z}}{dz}.$$

Integration of this equation in the interval $z = \pm\zeta$ yield boundary conditions for pressure perturbation.

and

$$P_1(\zeta) - g\rho_{02}\xi_{1z}(\zeta) = P_1(-\zeta) - g\rho_{01}\xi_{1z}(-\zeta). \tag{21}$$

Here, $P_1 = \frac{C_{1n}}{C_{2n}}\xi_{1z} - \frac{D_n}{C_{2n}}\frac{d\xi_{1z}}{dz}$, as follows from the first equation in Equations (9) for the regions $n = 1, 2$. The physical significance of the boundary conditions in Equations (20) and (21) are continuity of both the vertical fluid displacement ξ_{1z} and the pressure perturbation $P_1 - g\rho_0(z)\xi_{1z}$ at the boundary $z = 0$ (Pintér, Čadež, and Goossens, 1998; Pintér, Čadež, and Roberts, 1999). In addition to these two conditions we require that the total (kinetic plus magnetic) energy density of the perturbations diminish to zero as $|z|$ tends to infinity. These three conditions will be applied to the solutions for the fluid displacement and the pressure perturbation.

3.3. Gravito-MHD Waves on the Boundary

It is known that a harmonic wave, which propagates through regions 1 and 2, does not change its dimensionless frequency, in the further text $\Omega = \omega H_1/v_{s1}$, and the horizontal dimensionless wavevector component $K_p = k_p H_1$, parallel to the boundary $z = 0$, (Landau and Lifshitz, 1987). However, the normal dimensionless wavevector component K_z has a discontinuity at the boundary $z = 0$, where it changes from K_{z1} to K_{z2} according to the dispersion Equation (14). Here, $K_{z1} = k_{z1}H_1$ and $K_{z2} = k_{z2}H_1$. We assume that a wave propagates from the lower region 1 upward towards the boundary $z = 0$ and that the waves continuing past it are absorbed with no reflection in the upper region 2. In this case, the solution for the fluid displacement ξ_{1z} in the lower region, according to Equation (11) written in dimensionless form, is a superposition of the incident wave with a unit amplitude and a reflected wave with amplitude A_r:

$$\xi_{1z} = e^{[iK_{z1}+\frac{1}{2}]\frac{z}{H_1}} + A_r e^{[-iK_{z1}+\frac{1}{2}]\frac{z}{H_1}}, z < 0 \tag{22}$$

while in the upper region there is only a transmitted wave with amplitude A_t:

$$\xi_{1z} = A_t e^{[iK_{z2}+\frac{s}{2}]\frac{z}{H_1}}, z > 0. \tag{23}$$

The solution for P_1, could be rewritten in the form:

$$P_{1n} = g\Pi_{1n}e^{-z/H_n}\xi_{1z} - \Pi_{2n}e^{-z/H_n}\frac{d\xi_{1z}}{dz}, \tag{24}$$

where:

$$\Pi_{11} = \frac{\rho_{01}\Omega^2(\Omega^2 - \Omega_{A1}^2)}{\Omega^4 - K_p^2\left(1 + \frac{2}{\gamma\beta_1}\right)(\Omega^2 - \Omega_{c1}^2)} \tag{25}$$

$$\Pi_{12} = \frac{\rho_{02}\Omega^2\left(\Omega^2 - \frac{\Omega_{A1}^2}{a}\right)}{\Omega^4 - \frac{K_p^2}{s}\left(1 + \frac{2s}{a\gamma\beta_1}\right)\left(\Omega^2 - \frac{2+\gamma\beta_1}{2s+a\gamma\beta_1}\Omega_{c1}^2\right)}, \tag{26}$$

$$\Pi_{21} = \frac{v_{s1}^2 \rho_{01} \left(1 + \frac{2}{\gamma\beta_1}\right) (\Omega^2 - \Omega_{A1}^2)(\Omega^2 - \Omega_{c1}^2)}{\Omega^4 - K_p^2 \left(1 + \frac{2}{\gamma\beta_1}\right) (\Omega^2 - \Omega_{c1}^2)} \tag{27}$$

and

$$\Pi_{22} = \frac{\rho_{02} \frac{v_{s1}^2}{s} \left(1 + \frac{2s}{a\gamma\beta_1}\right) \left(\Omega^2 - \frac{\Omega_{A1}^2}{a}\right) \left(\Omega^2 - \frac{2+\gamma\beta_1}{2s+a\gamma\beta_1}\Omega_{c1}^2\right)}{\Omega^4 - \frac{K_p^2}{s} \left(1 + \frac{2s}{a\gamma\beta_1}\right) \left(\Omega^2 - \frac{2+\gamma\beta_1}{2s+a\gamma\beta_1}\Omega_{c1}^2\right)} \tag{28}$$

with:

$$a = v_{A1}^2/v_{A2}^2, \quad v_{s1}^2 + v_{A1}^2 = v_{s1}^2 \left(1 + \frac{2}{\gamma\beta_1}\right), \quad v_{s2}^2 + v_{A2}^2 = \frac{v_{s1}^2}{s} \left(1 + \frac{2s}{a\gamma\beta_1}\right)$$

$$\Omega_{A2}^2 = \omega_{A2}^2 H_1^2 / v_{s1}^2 = \Omega_{A1}^2 / a$$

and

$$\Omega_{c2}^2 = \omega_{c2}^2 H_1^2 / v_{s1}^2 = (2 + \gamma\beta_1)\Omega_{c1}^2 / (2s + a\gamma\beta_1).$$

For brevity, in what follows we will use labels: $\beta_1 = \beta$, $\Omega_{A1}^2 = \Omega_A^2$ and $\Omega_{c1}^2 = \Omega_c^2$, meaning that all the results will be presented using the relevant physical quantities in region 1.
The solutions for P_1, according to Equation (12), could be presented as:

$$P_1 = G_{(1,1)} e^{[iK_{z1} - \frac{1}{2H_1}]z} + A_r G_{(1,2)} e^{-[iK_{z1} + \frac{1}{2H_1}]z}, z < 0 \tag{29}$$

$$P_1 = A_t G_{(2,1)} e^{[iK_{z2} - \frac{s}{2}]\frac{z}{H_1}}, z > 0, \tag{30}$$

Here,

$$G_{(1,1)} = g\Pi_{11} - \left[iK_{z1} + \frac{1}{2}\right] \frac{\Pi_{21}}{H_1}$$

$$= \frac{g\rho_{01}(\Omega^2 - \Omega_A^2)}{\Omega^4 - K_p^2 \left(1 + \frac{2}{\gamma\beta}\right) (\Omega^2 - \Omega_c^2)}$$

$$\times \left[\Omega^2 - \frac{(2 + \gamma\beta)(\Omega^2 - \Omega_c^2)}{2(1 + \beta)} - \frac{iK_{z1}(2 + \gamma\beta)(\Omega^2 - \Omega_c^2)}{1 + \beta}\right]$$

$$G_{(1,2)} = g\Pi_{11} + \left[iK_{z1} - \frac{1}{2}\right] \frac{\Pi_{21}}{H_1}$$

$$= \frac{g\rho_{01}(\Omega^2 - \Omega_A^2)}{\Omega^4 - K_p^2 \left(1 + \frac{2}{\gamma\beta}\right) (\Omega^2 - \Omega_c^2)} \tag{31}$$

$$\times \left[\Omega^2 - \frac{(2+\gamma\beta)(\Omega^2-\Omega_c^2)}{2(1+\beta)} + \frac{iK_{z1}(2+\gamma\beta)(\Omega^2-\Omega_c^2)}{1+\beta}\right]$$

$$G_{(2,1)} = g\Pi_{12} - \left[iK_{z2} + \frac{H_1}{2H_2}\right]\frac{\Pi_{22}}{H_1}$$

$$= \frac{g\rho_{02}\left(\Omega^2 - \frac{\Omega_A^2}{a}\right)}{\Omega^4 - \frac{K_p^2(a\gamma\beta+2s)}{sa\gamma\beta}\left(\Omega^2 - \frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)}$$

$$\times \left[\Omega^2 - \frac{(a\gamma\beta+2s)\left(\Omega^2 - \frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)}{2(s+a\beta)} - \frac{iK_{z2}(a\gamma\beta+2s)\left(\Omega^2 - \frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)}{as(1+\beta)}\right].$$

4. Reflection and Transmission Coefficients for Gravito-MHD Waves

4.1. Magnetized Case

Boundary conditions in Equations (20) and (21) applied to Equations (22), (23), (29), (30), with Equation (31) at $z = 0$, yield the following set of two algebraic equations for complex amplitudes A_r and A_t:

$$A_t = 1 + A_r \tag{32}$$

$$[G_{(2,1)} - g\rho_{02}]A_t - [G_{(1,2)} - g\rho_{01}]A_r = G_{(1,1)} - g\rho_{01}, \tag{33}$$

whose solutions are:

$$A_r = \frac{G_{(1,1)} - G_{(2,1)} + g[\rho_{02} - \rho_{01}]}{G_{(2,1)} - G_{(1,2)} - g[\rho_{02} - \rho_{01}]}, \tag{34}$$

$$A_t = \frac{G_{(1,1)} - G_{(1,2)}}{G_{(2,1)} - G_{(1,2)} - g[\rho_{02} - \rho_{01}]}. \tag{35}$$

These are general equations for the reflection and transmission amplitudes of the gravito-MHD waves. The fact that the reflection and transmission amplitudes A_r and A_t turn out to be complex in general indicates that the reflected and transmitted waves are shifted in phase with respect to the incident one. Note that in the case of a single region we have $A_r = 0$ and $A_t = 1$, according to Equation (32), as should be expected. Namely, the plane $z = 0$ now does not separate two different regions and no wave reflection occurs at $z = 0$. The incident wave is fully transmitted in region 2.

In this paper our goal is to derive the equation for the reflection coefficient of gravito-MHD waves, which is defined as the square of the absolute value of the reflection amplitude A_r. Using dimensionless parameters and Equation (34), we can get a very complex expression for the reflection amplitude A_r. For better visibility of this expression we will write the real and the imaginary part of the numerator separately

$$\Re(N_{A_r}) = \frac{g\rho_{01}(\Omega^2-\Omega_A^2)}{\Omega^4 - K_p^2\left(1+\frac{2}{\gamma\beta}\right)(\Omega^2-\Omega_c^2)}\left[\Omega^2 - \frac{(2+\gamma\beta)(\Omega^2-\Omega_c^2)}{2(1+\beta)}\right]$$

$$-\frac{g\rho_{01}\left(\Omega^2-\frac{\Omega_A^2}{a}\right)as(1+\beta)}{\left[\Omega^4 - \frac{K_p^2(a\gamma\beta+2s)}{sa\gamma\beta}\left(\Omega^2-\frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)\right](s+a\beta)}$$

$$\times\left[\Omega^2 - \frac{(2s+a\gamma\beta)}{2(s+a\beta)}\left(\Omega^2 - \frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)\right]$$

$$+ g\rho_{01}\left[\frac{as(1+\beta)}{s+a\beta}-1\right] \tag{36}$$

and

$$\Im(N_{A_r}) = \frac{g\rho_{01}K_{z2}\left(\Omega^2-\frac{\Omega_A^2}{a}\right)(2s+a\gamma\beta)}{\left[\Omega^4 - \frac{K_p^2(a\gamma\beta+2s)}{sa\gamma\beta}\left(\Omega^2-\frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)\right](s+a\beta)}\left(\Omega^2 - \frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)$$

$$-\frac{g\rho_{01}K_{z1}(2+\gamma\beta)(\Omega^2-\Omega_A^2)(\Omega^2-\Omega_c^2)}{(1+\beta)\left[\Omega^4 - K_p^2\left(1+\frac{2}{\gamma\beta}\right)(\Omega^2-\Omega_c^2)\right]}, \tag{37}$$

meaning that the complex nominator is:

$$N_{A_r} = \Re(N_{A_r}) + i\Im(N_{A_r}). \tag{38}$$

In obtaining Equations (36) and (37) we have used Equation (18), which, together with the introduced dimensionless parameters a and s, gives the relation:

$$\frac{\rho_{02}}{\rho_{01}} = \frac{as(1+\beta)}{s+a\beta}.$$

In the same way we can write for the real and imaginary part of the denominator

$$\Re(D_{A_r}) = \frac{g\rho_{01}\left(\Omega^2-\frac{\Omega_A^2}{a}\right)as(1+\beta)}{\left[\Omega^4 - \frac{K_p^2(a\gamma\beta+2s)}{sa\gamma\beta}\left(\Omega^2-\frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)\right](s+a\beta)}$$

$$\times\left[\Omega^2 - \frac{(2s+a\gamma\beta)}{2(s+a\beta)}\left(\Omega^2 - \frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)\right]$$

$$-\frac{g\rho_{01}(\Omega^2-\Omega_A^2)}{\Omega^4 - K_p^2\left(1+\frac{2}{\gamma\beta}\right)(\Omega^2-\Omega_c^2)}\left[\Omega^2 - \frac{(2+\gamma\beta)(\Omega^2-\Omega_c^2)}{2(1+\beta)}\right]$$

$$- g\rho_{01}\left[\frac{as(1+\beta)}{s+a\beta}-1\right] \tag{39}$$

and

$$\Im(D_{A_r}) = -\frac{g\rho_{01}K_{z2}\left(\Omega^2-\frac{\Omega_A^2}{a}\right)(2s+a\gamma\beta)}{\left[\Omega^4-\frac{K_p^2(a\gamma\beta+2s)}{sa\gamma\beta}\left(\Omega^2-\frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)\right](s+a\beta)}\left(\Omega^2-\frac{(2+\gamma\beta)\Omega_c^2}{2s+a\gamma\beta}\right)$$

$$-\frac{g\rho_{01}K_{z1}(2+\gamma\beta)(\Omega^2-\Omega_A^2)(\Omega^2-\Omega_c^2)}{(1+\beta)\left[\Omega^4-K_p^2\left(1+\frac{2}{\gamma\beta}\right)(\Omega^2-\Omega_c^2)\right]}. \tag{40}$$

The complex denominator is given by equation:

$$D_{A_r} = \Re(D_{A_r}) + i\Im(D_{A_r}). \tag{41}$$

Finally, for the complex reflection amplitude A_r we get:

$$A_r = \frac{N_{A_r}}{D_{A_r}} = \frac{\Re(N_{A_r}) + i\Im(N_{A_r})}{\Re(D_{A_r}) + i\Im(D_{A_r})}, \tag{42}$$

or with separated real and imaginary part:

$$A_r = \frac{\Re(N_{A_r})\Re(D_{A_r}) + \Im(N_{A_r})\Im(D_{A_r}) + i[\Re(D_{A_r})\Im(N_{A_r}) - \Re(N_{A_r})\Im(D_{A_r})]}{\Re^2(D_{A_r}) + \Im^2(D_{A_r})}. \tag{43}$$

Similarly we get the transmission amplitude A_t using Equation (35). Here we have a simpler situation because the numerator in the expression for the transmission amplitude is a pure imaginary and given by:

$$\Im(N_{A_t}) = \frac{-2g\rho_{01}K_{z1}(2+\gamma\beta)(\Omega^2-\Omega_A^2)(\Omega^2-\Omega_c^2)}{(1+\beta)\left[\Omega^4-K_p^2\left(1+\frac{2}{\gamma\beta}\right)(\Omega^2-\Omega_c^2)\right]}, \tag{44}$$

i.e.

$$N_{A_t} = i\Im(N_{A_t}). \tag{45}$$

The denominator is the same as for the reflection amplitude (see Equations (34),(35)), and we can write:

$$A_t = \frac{N_{A_t}}{D_{A_t}} = \frac{N_{A_t}}{D_{A_r}} = \frac{i\Im(N_{A_t})}{\Re(D_{A_r}) + i\Im(D_{A_r})} = \frac{\Im(N_{A_t})\Im(D_{A_r}) + i\Re(D_{A_r})\Im(N_{A_t})}{\Re^2(D_{A_r}) + \Im^2(D_{A_r})}. \tag{46}$$

The reflection coefficient is defined as a square of the absolute value of the reflection amplitude A_r, *i.e.* $R = |A_r|^2$, and for gravito-MHD waves its form is:

$$R = \frac{[\Re(N_{A_r})\Re(D_{A_r}) + \Im(N_{A_r})\Im(D_{A_r})]^2 + [\Re(D_{A_r})\Im(N_{A_r}) - \Re(N_{A_r})\Im(D_{A_r})]^2}{[\Re^2(D_{A_r}) + \Im^2(D_{A_r})]^2}. \tag{47}$$

The reflection coefficient is equal to unity if these conditions are fulfilled:

$$\Re(D_{A_r})\Im(N_{A_r}) = \Re(N_{A_r})\Im(D_{A_r}) \tag{48}$$

together with

$$\Re(D_{A_r}) = \Re(N_{A_r}) \tag{49}$$

and

$$\Im(D_{A_r}) = \Im(N_{A_r}). \tag{50}$$

For $K_{z2} = 0$, when gravito-MHD waves in region 2 are evanescent, Equation (50) is satisfied, but the Equation (49) is not. Because of that, another condition is needed. We find that it is given by the relations between wave frequencies- $\Omega^2 = \Omega_A^2$ or $\Omega^2 = \Omega_c^2$. It is not difficult to show that for the waves with freqencies equal to the Alfvén or slow frequency, expressions $\Re(D_{A_r})$ and $\Re(N_{A_r})$ differ only in sign and $\Im(D_{A_r}) = \Im(N_{A_r}) = 0$. Replacing these expressions in Equation (47) we can easily get $R = 1$. This result describes the total internal reflection of the incoming gravito-MHD waves on the plane boundary $z = 0$ between two plasma regions with different temperatures and horizontal magnetic field values. The minus sign between $\Re(D_{A_r})$ and $\Re(N_{A_r})$ shows that the incident and reflected waves are phase shifted for 180^0.
The transmission coefficient, as a square of the absolute value of the transmission amplitude A_t, *i.e.* $T = |A_t|^2$, for gravito-MHD waves is:

$$T = \frac{[\Im(N_{A_t})\Im(D_{A_r})]^2 + [\Re(D_{A_r})\Im(N_{A_t})]^2}{[\Re^2(D_{A_r}) + \Im^2(D_{A_r})]^2} = \frac{\Im^2(N_{A_t})}{\Re^2(D_{A_r}) + \Im^2(D_{A_r})}. \tag{51}$$

For the frequencies $\Omega^2 = \Omega_A^2$ or $\Omega^2 = \Omega_c^2$, when $\Im(N_{A_t}) = 0$, we have $T = 0$, as we expected. A gravito-MHD wave does not transfer its energy in region 2. Theoretically, the reflection coefficient could be equal to zero if $\Re(N_{A_r}) = \Im(N_{A_r}) = 0$. Inspecting Equations (36) and (37) we could not find the gravito-MHD wave frequency for which the condition $\Re(N_{A_r}) = \Im(N_{A_r}) = 0$ is satisfied. Because of that we are prone to conclude that in the case of gravito-MHD waves, the reflection coefficient R can not be zero meaning that all the wave energy can not be transmitted in region 2.

4.2. Non-magnetized Case

From the magnetized case, which is more general, we will deduce equations for reflection and transmission coefficients of the non-magnetized gravitationally stratified plasma.
In the absence of magnetic field, *i.e.* when $v_{A1}, v_{A2} = 0$, or $1/\beta = 0$ and $H_n = H_{0n} = v_{sn}^2/\gamma g$, where $n = 1, 2$ for two different regions, the coefficients in Equation (10) become:

$$D(z) = \rho_0(z) v_s^2 \omega^4,$$

$$C_1(z) = \rho_0(z) g \omega^4,$$

$$C_2(z) = \omega^4 - k_p^2 v_s^2 \omega^2,$$

$$C_3(z) = \rho_0(z)\omega^2 + \frac{\rho_0(z) g^2}{v_s^2},$$

$$C_4(z) = \frac{g}{v_s^2}.$$

Equation (9) has no singularities in the non-magnetized case and this situation is quite good descripted by ideal MHD equations.
Equation (17) for the equality of the unperturbed pressures becomes:

$$\frac{\rho_{02}}{\rho_{01}} = s. \tag{52}$$

The boundary condition in Equation (20) for the continuity of the vertical fluid displacement ξ_{1z} remains unchanged, while Equation (21) for the continuity of the pressure perturbations has a form:

$$p_1(\zeta) - g\rho_{02}\xi_{1z}(\zeta) = p_1(-\zeta) - g\rho_{01}\xi_{1z}(-\zeta). \tag{53}$$

Equations (22) and (23) retain the same form as in the magnetized case, but now dimensionless vertical wavenumbers K_{z1} and K_{z2} satisfy the dispersion equation (Equation (15)). The solution for pressure perturbations p_1 is:

$$p_1 = g\Pi_{1n} e^{-z/H_n} \xi_{1z} - \Pi_{2n} e^{-z/H_n} \frac{d\xi_{1z}}{dz}, \tag{54}$$

with Equations (25)-(28) in the form:

$$\Pi_{11} = \frac{\rho_{01}\Omega^2}{\Omega^2 - K_p^2}, \tag{55}$$

$$\Pi_{12} = \frac{s\rho_{02}\Omega^2}{s\Omega^2 - K_p^2}, \tag{56}$$

$$\Pi_{21} = \frac{\rho_{01} v_{s1}^2 \Omega^2}{\Omega^2 - K_p^2}, \tag{57}$$

$$\Pi_{22} = \frac{s\rho_{02} v_{s2}^2 \Omega^2}{s\Omega^2 - K_p^2} = \frac{\rho_{02} v_{s1}^2 \Omega^2}{s\Omega^2 - K_p^2}, \tag{58}$$

or

$$p_1 = G_{(1,1)} e^{[iK_{z1} - \frac{1}{2H_1}]z} + A_r G_{(1,2)} e^{-[iK_{z1} + \frac{1}{2H_1}]z}, z < 0 \tag{59}$$

$$p_1 = A_t G_{(2,1)} e^{[iK_{z2} - \frac{s}{2}]\frac{z}{H_1}}, z > 0, \tag{60}$$

Here,

$$G_{(1,1)} = \frac{g\rho_{01}\Omega^2}{\Omega^2 - K_p^2}\left[1 - \frac{\gamma}{2} - i\gamma K_{z1}\right],$$

$$G_{(1,2)} = \frac{g\rho_{01}\Omega^2}{\Omega^2 - K_p^2}\left[1 - \frac{\gamma}{2} + i\gamma K_{z1}\right], \tag{61}$$

$$G_{(2,1)} = \frac{gs\rho_{02}\Omega^2}{s\Omega^2 - K_p^2}\left[1 - \frac{\gamma}{2} - \frac{i\gamma K_{z2}}{s}\right].$$

Following the same procedures as in the magnetized case, and applying boundary conditions to adequate the equations, we obtain expressions for the reflection and transmission amplitudes of gravito-acoustic waves:

$$A_r = \frac{\Re(N_{A_r})\Re(D_{A_r}) + \Im(N_{A_r})\Im(D_{A_r}) + i[\Re(D_{A_r})\Im(N_{A_r}) - \Re(N_{A_r})\Im(D_{A_r})]}{\Re^2(D_{A_r}) + \Im^2(D_{A_r})}, \tag{62}$$

$$A_t = \frac{i\Im(N_{A_t})}{\Re(D_{A_r}) + i\Im(D_{A_r})} = \frac{\Im(N_{A_t})\Im(D_{A_r}) + i\Re(D_{A_r})\Im(N_{A_t})}{\Re^2(D_{A_r}) + \Im^2(D_{A_r})}. \tag{63}$$

with

$$\Re(N_{A_r}) = \Omega^2\left(1 - \frac{\gamma}{2}\right)\left[\frac{1}{\Omega^2 - K_p^2} - \frac{s^2}{s\Omega^2 - K_p^2}\right] + s - 1, \tag{64}$$

$$\Im(N_{A_r}) = \gamma\Omega^2 K_{z1}\left[\frac{K_{z2}}{K_{z1}} \cdot \frac{s}{s\Omega^2 - K_p^2} - \frac{1}{\Omega^2 - K_p^2}\right], \tag{65}$$

$$\Re(D_{A_r}) = \Omega^2\left(1 - \frac{\gamma}{2}\right)\left[\frac{s^2}{s\Omega^2 - K_p^2} - \frac{1}{\Omega^2 - K_p^2}\right] - (s-1), \tag{66}$$

$$\Im(D_{A_r}) = -\gamma\Omega^2 K_{z1}\left[\frac{K_{z2}}{K_{z1}} \cdot \frac{s}{s\Omega^2 - K_p^2} + \frac{1}{\Omega^2 - K_p^2}\right], \tag{67}$$

and

$$\Im(N_{A_t}) = \frac{-2g\rho_{01}\gamma\Omega^2 K_{z1}}{\Omega^2 - K_p^2} \tag{68}$$

It is easy to calculate the reflection coefficient for gravito-acoustic waves as $R = |A_r|^2$ and the transmission coefficient for gravito-acoustic waves as $T = |A_t|^2$. Following the same procedure as in the magnetized case and using the above equations we can conclude that the reflection coefficient could be equal to unity if $K_{z2} = 0$. Notice that Equation (50) is satisfied now and, contrary to the magnetized case, there is no additional conditions for wave frequencies to be satisfied. Namely, from Equations (64) and (66) it is obvious that $\Re(N_{A_r}) = -\Re(D_{A_r})$. As a conclusion we can say that the reflection coefficient for gravito-acoustic waves is equal to unity if $K_{z2} = 0$. The condition for the reflection coefficient to be zero is $\Re(N_{A_r}) = \Im(N_{A_r}) = 0$. Here, as in the magnetized case, we could not analytically find the frequency for which this condition is satisfied. We conclude that the reflection coefficient for gravito-acoustic waves can not be zero. Marmolino *et al.* (1993), discussed reflection and transmission coefficient values for the two succession layers suitable to represent the photospheric and the lower chromospheric stratification.

The results of this paper are presented in Figures 2 and 3. The standard quiet-Sun model is chosen for the waves on the photosphere-chromosphere boundary. For the photospheric temperature $T_1 = 6000K$ and chromospheric temperature $T_2 = 10000K$, the parameter s is: $s = v_{s1}^2/v_{s2}^2 = T_1/T_2 = 0.6$.

Figure 2 presents the reflection coefficient for acoustic waves modified by gravity on the photosphere-chromosphere boundary as a function of frequency Ω. For the horizontal phase velocity $V_h = 1/\sqrt{s} \approx 1.29$, when $K_{z2} = 0$, (Jovanović, 2013), the reflection coefficient for these waves is equal to unity. For $V_h > 1/\sqrt{s}$, when K_{z2} has real and positive values, modified acoustic waves could propagate through chromosphere[3]. The characteristic point $V_h = \sqrt{(s+1)/s} \approx 1.633$ is chosen because in this point the reflection coefficient for the pure acoustic wave is zero, $R = 0$.[4] For the modified acoustic waves on the photosphere-chromosphere boundary the reflection coefficient is always greater than zero. Gravitational influence introduces a cutoff frequency $\Omega_{co} = 0.5$ below which modified acoustic waves can not propagate. Therefore, allowed frequencies for these waves are $\Omega > \Omega_{co}$. Gravitational influence is the most pronounced in the frequency range $\Omega_{co} < \Omega < 1$. For the high frequencies the reflection coefficient for these waves is almost the same as in the pure acoustic case. Note that the reflection coefficient for the modified acoustic waves decreases with increasing frequency Ω for a given horizontal phase velocity values.

Figure 3 shows the reflection coefficient for gravity waves on the photosphere-chromosphere boundary as a function of frequency Ω. It is found that these waves could propagate through chromosphere if horizontal phase velocities are $V_h < \Omega_{BV}/\Omega_{co} \approx 0.97$. If $V_h \approx 0.97$, then the reflection coefficient is equal to unity.[5] The frequency $\Omega = \sqrt{s}\Omega_{BV} \approx 0.378$ is the cutoff frequency above which gravity waves can not propagate through the chromosphere. Note that the reflection coefficient for gravity waves increases with increasing frequency for given V_h values. The reflection coefficient for gravity waves on the photosphere-chromosphere boundary is always greater than zero.

[3]For the horizontal phase velocities $V_h < 1/\sqrt{s}$ the modified acoustic waves are evanescent.

[4]For the pure acoustic case the reflection coefficient is $R = \frac{\left(s\sqrt{V_h^2-1}-\sqrt{sV_h^2-1}\right)^2}{\left(s\sqrt{V_h^2-1}+\sqrt{sV_h^2-1}\right)^2}$. It is easy to see that for $V_h = \sqrt{(s+1)/s}$, there is $R = 0$.

[5]For the horizontal phase velocities $V_h > \Omega_{BV}/\Omega_{co} \approx 0.97$ gravity waves are evanescent.

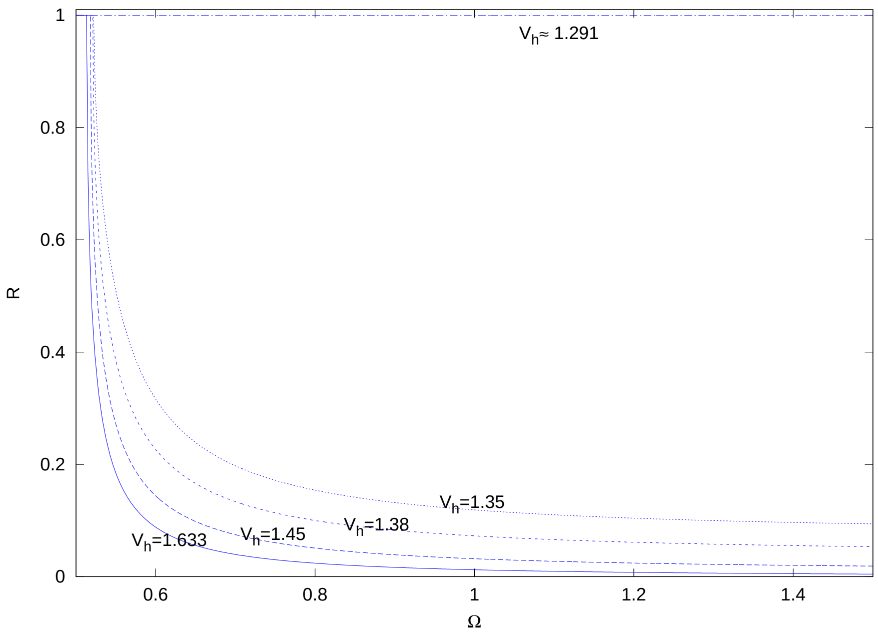

Figure 2. Reflection coefficient, R, for acoustic waves (modified by gravity) on the solar photosphere-chromosphere plane boundary $z = 0$, when $s = 0.6$. It is a function of the frequency Ω. Horizontal phase velocities are $V_h > 1/\sqrt{s} = 1.291$. If $V_h = 1.291$, the reflection coefficient for these waves on the photosphere-chromosphere boundary is $R = 1$. The frequency $\Omega_{co} = 0.5$ is the cutoff frequency below which modified acoustic waves can not propagate.

5. Conclusion

Using a simple model of two semi-bounded isothermal stratified magnetized plasmas, whose plane boundary is $z = 0$, we derived dispersion Equations (13) and (14) and also equations for the reflection and transmission coefficients for gravito-MHD waves (Equations (47) and (51)). For practical reasons we presented our results using physical quantities characteristic for region 1 as Ω_A, Ω_c, β. We did it in a manner that relevant physical quantities in region 2 are expressed as a function of appropriate quantities from region 1 using dimensionless parameters $s = v_{s1}^2/v_{s2}^2$, $a = v_{A1}^2/v_{A2}^2$ and parameter β. Equations (47) and (51) have a very complex form. We discussed the conditions when the reflection coefficient can be equal to unity. We could not analytically find conditions for $R = 0$ (and consequently $T = 1$) and we are prone to assume that the reflection coefficient can not be zero in the applied model. This means that there is no chance that the whole wave energy is transmitted in region 2. Figures 2 and 3 for the reflection coefficient of gravito-acoustic waves on the photosphere-chromosphere boundary in the quiet-Sun model, confirm this assumption. We expect that these equations can be a useful tool to calculate the amount of reflected and, more significantly, the amount of transmitted wave energies for example in the solar or other piece-wise astrophysical plasmas with constant temperature profile. Energy transmitted in region 2 (for example in the

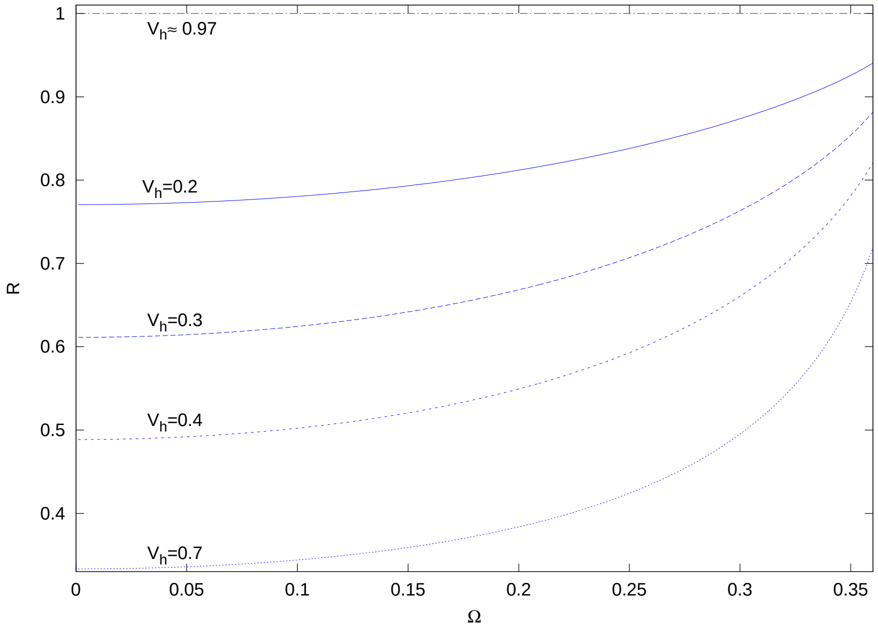

Figure 3. Reflection coefficient, R, for gravity waves on the solar photosphere-chromosphere plane boundary $z = 0$, when $s = 0.6$, as a function of frequency Ω. These waves could propagate through solar chromosphere if horizontal phase velocities are $V_h < \Omega_{BV}/\Omega_{co} \approx 0.975$. For $V_h > \Omega_{BV}/\Omega_{co} \approx 0.975$, gravity waves are evanescent. If $V_h = 0.975$, then the reflection coefficient for gravity waves is equal to unity. The frequency $\Omega = \sqrt{s}\Omega_{BV} \approx 0.37$ is the cutoff frequency above which gravity waves can not propagate.

solar corona) through various dissipative mechanisms can lead to the heating of the plasma. Another heating mechanism, discussed in Section 3.2 is resonant absorption when the frequencies of the incoming gravito-MHD wave equals the local Alfvén or local slow wave frequency. In this case ideal MHD equations must be replaced by dissipative MHD equations (for example visco-resistive MHD) which include dissipative effects like: resistivity, viscosity, magnetic diffusivity. When these dissipative phenomena are taken into account, energy absorption of the incoming gravito-MHD wave will heat the plasma. Finally, we expect that this mathematical treatment of gravito-MHD waves can be the basis for further numerical analysis.

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