Second Revision

Paper: Relativistic Kinematic Effects in the Interaction Time of Whistler-Mode Chorus Waves and Electrons in the Outer Radiation Belt

Response to Reviewer - #1

> The authors have answered comments and corrected the manuscript accordingly. I can recommend for publication.

We thank to the reviewer for this carefull work in this manuscript

Response to Review - #2

The authors have done a good job incorporating additional literature and answering several of the reviewers' feedback, especially concerning the validity of the inertial frame of reference and by adding the effect on the diffusion coefficient through the Lakhina et al method. The authors have convinced me that incorporating relativistic effects to account for the transit time of interaction is potentially worthy of additional studies. However, the use of the Lakhina et al. method (which is heuristic at best, and has often been used in the literature because it is less cumbersome than the Kennel and Engelman or Hamiltonian methodologies), to account for a higher order effect (length and time contraction), is not a good enough choice. The \Delta t in Equation (19) is entirely arbitrary and all the changes highlighted by the authors between relativistic and non-relativistic effects stem from it. The numerous definition of Delta t given in the literature are certainly plausible but nonetheless ad hoc and would require a more carefully theoretical (e.g. through Hamiltonian methods) or/and numerical analysis. I will therefore recommend publication with the following additional caveat added to the paper: when compared with the Lakhina et al. methodology for pitch-angle scattering, we find that relativistic effects result in larger pitch-angle diffusion. Our results indicate that more accurate descriptions of pitch-angle scattering by whistler waves (e.g. through the Kennel and Engelman method or through Hamiltonian methods) can also potentially be significantly affected by the addition of relativistic effects.

R. We thank to referee for this recommendation. We added it at the Conclusion in lines 345-348

Additional comments:

L160 The definition of the guiding-centre trajectory in the paper seems incorrect to me. What the authors are using is the exact particle's motion in a frame they do not define. What is known as the guiding-centre trajectory is the one accounting for various particle's drifts perpendicular to the mean field plus the parallel motion (à la Northrop and Teller).

R. In this manuscript the guiding center is defined as the center of a circular orbit of the electron around the magnetic field line, according to Baumjohann and Treumann, (1997). We added this information and the reference in lines 145-146.

Baumjohann and Treumann, (1997) Basic Space Plasma Physics, Imperial College Press, 1ed, ISBN 1-86094-079-X

And the guiding-centre velocity is frame specific (for instance, sometimes it's defined in the E cross B frame of reference). What the authors define as a guiding centre drift is the resonant velocity (Equation 4) projected along the mean field. However, it is not clear in which frame this velocity of Equation (4) is defined to start with. Which makes me wonder why the authors did not more simply compute the resonance in the frame of the wave (which has the advantage of having no electric field for parallel propagation) and then transform in the frame of the satellite with the relativistic effect accounted for.

R. The Eq. 4 is in the frame of the satellite because this is the frame where the measurements are taken, including the velocity of the electron's guiding center. This information was added in line 120.

L240 the last sentence of the paragraph in Equation 21 is unclear. Moreover, in the pitch-angle diffusion coefficient of Equation 21 the average is taken over some random-phases or for a collection of particles (ensemble-average). The author seem to average over multiple interaction times by assuming some power-law distribution in the wave-packet element duration tau. Therefore Equation 21 is a diffusion equation for a collection of particle, and the definition of the last sentence of L240 is for a particle with a given pitch-angle and energy that encounters a large number of wave-packets. It's not clear to me if these two definitions are consistent with one another but the former one (à la Kennel and Engelman) is the only one that makes sense to me when applied to a kinetic equation for a collection of particles.

R. We apologize that this point is not clear in the manuscript.
We consider the wave-particle interaction occurs at the equator. Then, the change in the electron's pitch angle derived in Eq. (20) considers the interaction with one chorus wave subelement with a constant time duration (tau). However, Santolik et al., 2004 showed that the whistler-mode chorus wave time duration can follow a power law distribution. Thus, in Eq. (21), we use a time average in pitch angle calculation to account for a subelement that has a power law time (such as done before by Lakhina et al., 2010). This emphasizes the relevance of the interaction time which is the main topic of this manuscript. As a consequence of such construction, a limitation of this approach is that other averaged effects, such as spectrum fluctuation (Kennel and Petschek, 1966) or random phase (Li et al., 2015), bounce-orbit (Lyons et al., 1972; Glauert and Horne, 2005), and ensemble contributions (Tao et al., 2011, 2012) affecting the pitch angle diffusion coefficient

have to be considered separated. Finally, Table 1 compares the pitch angle diffusion coefficient resulting from different interaction times for relativistic and non-relativistic approaches used to describe the interaction between electron and wave (with a subelement time duration given by a power law distribution) interaction. This discussion was inserted in the manuscript in lines 225, 238-242, also the following references were included.

Glauert, S. A., and Horne, R. B. (2005), Calculation of pitch angle and energy diffusion coefficients with the PADIE code, J. Geophys. Res., 110, A04206, doi:10.1029/2004JA010851.

Li, X., Tao, X., Lu, Q., and Dai, L. (2015), Bounce resonance diffusion coefficients for spatially confined waves, Geophys. Res. Lett., 42, 9591–9599, doi:10.1002/2015GL066324.

Tao, X., Bortnik, J., Albert, J. M., Liu, K., and Thorne, R. M. (2011), Comparison of quasilinear diffusion coefficients for parallel propagating whistler mode waves with test particle simulations, Geophys. Res. Lett., 38, L06105, doi:10.1029/2011GL046787.

Tao, X., Bortnik, J., Albert, J. M., and Thorne, R. M. (2012), Comparison of bounce-averaged quasi-linear diffusion coefficients for parallel propagating whistler mode waves with test particle simulations, J. Geophys. Res., 117, A10205, doi:10.1029/2012JA017931.