Paper: Relativistic Kinematic Effects in the Interaction Time of Whistler-Mode Chorus Waves and Electrons in the Outer Radiation Belt

Response to Review - #1

General

This draft presents calculations regarding the interaction time for electrons with chorus waves, and the resultant effect on pitch angle transport ("delta alpha"). The main proposed contribution is that the calculation is presented as being consistent with special relativity. Impicitly, the reader is to assume that this is being performed for the first time in the literature.

I believe that the basic principle of the work presented in this draft is sound, and potentially important. Although I do have a major concern regarding the assumption of an inertial satellite frame of reference

Even if that major concern is cleared up, I believe that further major revisions are necessary before publication can be recommended.

These revisions mostly centre around

- Calculations – I have some serious questions regarding some of the calculations. Or clarifications that need to be made regarding the validity of the results
- Data – I do not see what significant value the presented Van Allen Probes data adds, for the manuscript in the current form. More work needs to be done, I would suggest.
- Context – we need much more justification and references to literature. And discussion.

The authors appreciate all suggestions given by Reviewers 1 and 2, which were essential to improve this article. A careful revision was carried out in the manuscript taking into account all the comments and suggestions

Major comments

Calculations

1. All of the calculations rely on the comparisons of inertial frames of reference. Does a satellite have an inertial frame?

Yes, the satellite can be considered as an inertial frame, as explained below.

Not only is the satellite orbiting the Earth (and therefore undergoing acceleration), but it is also rotating around its own axis (and as a result so are the devices that measure BuBvBw etc). Please clarify. We need to have absolute clarity on this. Please can the authors give a thorough justification to explain why this is a reasonable approach. How are the inertial frames constructed and justified, given the above mentioned 2 sources of acceleration?

We add the discussion below in lines 149-157:
To justify the use of an inertial frame associated with the satellite, consider, for instance, the maximum acceleration achieved by the satellite at the perigee (data from satellite orbit can be found e.g. at Mauk et al., 2013) is 8.2m/s^2. The interaction time between the electron and the wave is of the order of 10^{-3} s. Since the orbital period of the satellite is 537.1 min, its acceleration is nearly constant during the interaction. Therefore, the change in the speed of the satellite in its orbit during one interaction time is around 8.2 x 10^{-3} m/s which is 6 orders of magnitude smaller than the speed of the satellite at the perigee, which is 9.8 km/s. Similarly, the spin period of the satellite is 11 s (Breneman et al., 2022), which leads to a change in angle of about 1.8 arcmin through the interaction time relative to one wave cycle. Thus, for the purposes of the present article, it is reasonable to consider the satellite as an inertial reference frame during the interaction time relative to one wave cycle. Moreover, it is a standard approach in the literature to consider the satellite as an inertial reference frame.

Breneman, A.W., Wygant, J.R., Tian, S. et al. The Van Allen Probes Electric Field and Waves Instrument: Science Results, Measurements, and Access to Data. Space Sci Rev 218, 69 (2022). https://doi.org/10.1007/s11214-022-00934-y)

Mauk, B.H., Fox, N.J., Kanekal, S.G. et al. Science Objectives and Rationale for the Radiation Belt Storm Probes Mission. Space Sci Rev 179, 3–27 (2013). https://doi.org/10.1007/s11214-012-9908-y

Relatedly, if there is a justification for this, we sould strongly suggest that the authors make a diagram that illustrates what S, S', and the different relativistic considerations are regarding the wave and the satellite, in order to help the reader understand what is going on

Resp.: The diagram was included in the manuscript as Figure 3

2. e.g. going from equations 14 to 20, and in particular 19 to 20
All of this analysis assumes that

- the forces on the particle are constant during the timescale 0
- It also assumes that T is determined by some version of “T=X/V” (e.g equation 10 from Lakhina et al 2010), or a relativistic version equation 13 in this paper.

Both of these are quite restrictive approximations, although we accept that they are relatively standard methods that are used in the literature e.g. Kennel&Petschek 1966, Lakhina et al 2010 as cited by the authors. The approximations are as follows.

- The presented equations are a big approximation to the real answer, which requires the solution of coupled first order odes for gyrophase, pitch angle, energy, position (e.g. see exact solutions up to second order in Bwave/B0 in Allanson et al 2022 Front. Astron. Space Sci. 8:805699. doi:10.3389/fspas.2021.805699)
- the interaction time could be shorter than this if the particle is scattered away very quickly. Likewise it could be longer if the particle is trapped (e.g. see Bortnik et al 2008 GEOPHYSICAL RESEARCH LETTERS, VOL. 35, L21102, doi:10.1029/2008GL035500, 2008)

Therefore it is important to state these approximations, and to say that the answers could be very different if (i) and (ii) play an important role, e.g. for higher amplitude/nonlinear waves. Conversely, the results in this paper are probably only true for a quasilinear interaction (e.g. Kennel and Engelmann 1966 The Physics of Fluids 9, 2377 (1966).

Resp.: We thank the referee for this suggestion, and we include the following discussions and references throughout the manuscript, mainly in lines 230-236 and 336-347:

The manuscript aims to improve the calculation of the interaction time in the quasilinear wave-particle interaction regime. Thus, we use first-order solutions such as those done by Lakhina and Tsurutani (2010). The weak turbulence in plasma and nonlinear events occurrence rate is around 10 to 15% considering the average occurrence of whistler-mode chorus waves (Zhang et al., 2018). Thus, non-linear events have a solution of the wave-particle interaction equations based on, at least, second-order terms in wave amplitude, e.g., Allanson et al., 2022; Artemyev et al., 2023; Omura, 2021; Osmane et al., 2016; Bortnik et al., 2008).

There are several difficulties in calculating the trapping time in non-linear interactions (e.g., Omura, 2021; Omura et al., 2013; 2008) using in situ measurements (Hsieh et al., 2022). Though, the first-order approach can give approximate solutions to nonlinear regimes, and Equation (13) can be used to estimate the interaction time from in-situ measurements (similar to the application done by Hsieh et al. (2022)) since it is important to determine the energy gain of electrons undergoing a wave-particle interaction (Hsieh et al., 2022). However, the interaction time is known to be shorter than the trapping time (Hsieh et al., 2022; Bortnik et al., 2008)

Anton V. Artemyev, Jay M. Albert, Anatoli I. Neishtadt, Didier Mourenas; The effect of wave frequency drift on the electron nonlinear resonant interaction with whistler-mode waves. Physics of Plasmas 1 January 2023; 30 (1): 012901. https://doi.org/10.1063/5.0131297

Omura, Y., Katoh, Y., and Summers, D. (2008), Theory and simulation of the generation of whistler-mode chorus, J. Geophys. Res., 113, A04223, doi:10.1029/2007JA012622.

Osmane, A., Wilson, L. B., Blum, L., Pulkkinen, T. I. (2016), On The Connection Between Microbursts And Nonlinear Electronic Structures In Planetary Radiation Belts. The Astrophysical Journal, ApJ 816 51, https://dx.doi.org/10.3847/0004-637X/816/2/51

Allanson, O., Thomas, E., Clare, W., Thomas, N. (2022), Weak Turbulence and Quasilinear Diffusion for Relativistic Wave-Particle Interactions Via a Markov Approach. Front. Astron. Space Sci. 8:805699. doi:10.3389/fspas.2021.805699

Bortnik, J., Thorne, R. M., and Inan, U. S. (2008), Nonlinear interaction of energetic electrons with large amplitude chorus, Geophys. Res. Lett., 35, L21102, doi:10.1029/2008GL035500.

Zhang, J., Thorne, R., Artemyev, A., Mourenas, D., Angelopoulos, V., Bortnik, J., Kletzing, C. A., Kurth, W. S., & Hospodarsky, G. B. (2018). Properties of Intense Field-Aligned Lower-Band Chorus Waves: Implications for Nonlinear Wave-Particle Interactions. Journal of Geophysical Research: Space Physics, 123(7), 5379-5393. https://doi.org/10.1029/2018JA025390

3. Equation (3) in the manuscript is incorrect. E.g. see
Equation (1) in Lakhina et al 2010
Or equation (13) in Artemyev et al Space Sci Rev (2016) 200:261–355 DOI 10.1007/s11214-016-0252-5
or equation 1 in Shprits et al Journal ofAtmosphericandSolar-TerrestrialPhysics70(2008)1694–1713
It should only be k_parallel and v_parallel in the doppler shift. The velocity component of interest is
v_parallel = v*cos\alpha

Resp.: We thank the referee for this point and we clarify this point as follows. In the manuscript, we write Equation (3) as a general equation for the Doppler shift in the wave frequency, because of that, the scalar product is explicit. We explain in lines 115-116 that we are considering the wave vector parallel to the ambient magnetic field for the wave's direction being co-propagating and counter-propagating to the energetic electron. In this case, for parallel (antiparallel) propagation, we use the equatorial electron pitch angle \alpha (180-\alpha) will be used in equation (3) as done by Lakhina et al. (2020), Artemyev et al. (2016), and Camporeale (2015).

To generalize the angle in the following equation, i.e. in Eq.(4), we call \delta the angle between the wave vector and the electron velocity vector. This correction was included in equation (4) and lines 118-119. Also, Figure 2 was corrected, and the discussion was added in lines 120-121.

Therefore there are a range of speeds (|v|) and pitch angles (\alpha) that are resonant with any given wave frequency \omega. E.g.
see equations 151 and 152 from Omura Earth, Planets and Space (2021) 73:95.

Resp.: Yes, we agree, and this is shown by the n (harmonic number) in Eq. (4) in the submitted manuscript. To clarify this point, we add a sentence in lines 130-133 and 253-254.

Therefore equation 4 not quite correct. This could have serious implications for the rest of the draft?

Resp: Equation (4) (line 119) was corrected. The angle between the electron velocity vector and the wave normal angle is named \delta; both vectors' orientations are defined as related to the ambient magnetic field in a tridimensional coordinate system. In the manuscript, we solve Eq. (4) for \delta equals \alpha and \pi - \alpha for co-propagating and counter-propagating waves, respectively.

The point is that there are many different values of energy that can resonate with a given wave, and these are a function of pitch-angle, e.g. see an example for the n=-1 resonance in Camporeale (2015), Resonant and nonresonant whistlers-particle interaction in the radiation belts, Geophys. Res. Lett., 42, 3114–3121, doi:10.1002/2015GL063874.

Resp.: We agree with the referee that there are many different energies resonating with a given wave frequency since we have the possibility to match several harmonics with the same wave; besides, for parallel propagating whistler waves, the angle between the wave vector and electron speed vector coincides with the electron's pitch angle, as shown by Camporeale, E. 2015. This information and reference were added in the manuscript in lines 130-133.

Data

4. Case studies 1, 2 and 3 all have magnetic field wave amplitudes >= 1nT (see table 1). With B0 roughly 100nT. That gives Bw/B0 ~ 1/100. These are very intense waves and almost certainly fall into the nonlinear regime (e.g. phase trapping and phase bunching etc) e.g. see

Zhang et al JGR, 2018, 123, 5379–5393, https://doi.org/10.1029/2018JA025390
and
Zhang et al GRL, 2019, 46, 7182–7190. https://doi.org/10.1029/2019GL083833
As per the above comment (2), I question whether interaction time method described in the paper is applicable. Or to put it another way, I would need to be convinced – please!

Resp: We thank the referee for this point, and we clarify that the interaction time calculated in this manuscript accounts for the first-order cyclotron effect only (as in lines 62, 113, 228, 234). However, some authors have used the first-order solution for the interaction time as an approximation to non-linear problems because of the difficulties in calculating trapping time, e.g. Hsieh et al., 2017. Thus, we included this discussion in lines (334-346) and the citation of Hsieh et al. (2020) and (2017), Omura (2021), Omura et al. (2013), (2008) to show an example in the literature of the interaction time in nonlinear cases. In spite of that, we considered only linear interactions throughout the paper. To be consistent, we changed the case studies presented in Table 1 and we applied the derived equations to the linear wave-particle cases. Also, Figures 6, 7, D1, and E1 were updated.
Hsieh, Y.-K., and Omura, Y. (2017), Nonlinear dynamics of electrons interacting with oblique whistler-mode chorus in the magnetosphere, J. Geophys. Res. Space Physics, 122, 675– 694, doi:10.1002/2016JA023255.

5. implications of point 3 regarding resonant particles. There is not one single resonant energy. There is a single resonant v_parallel for a given value of n, but this corresponds to many energies and pitch-angles. Therefore we do not understand figure 2 as it is currently presented, or any results or discussion that make use of the results from equation 5, and discussions about the given value of resonant energy

Resp.: Figure (2) is an example of Eq. (5) for a given ambient plasma parameters, wave normal angle, equatorial electron pitch angle, and one harmonic number. Besides, Figure (2) was corrected for the angle between the wave vector and electron velocity vector. This figure has the purpose of giving an example of how the resonant electron kinetic energy varies as a function of the wave frequency, for waves propagating parallel and anti-parallel to the ambient magnetic field. Also, it compares the resonant kinetic energy considering a relativistic and non-relativistic equation. Accordingly, if the harmonic number n is changed, all the curves would be shifted to the higher wave frequency side. We inserted a sentence about this point in lines 131-133.

6. Lines 208 – 267: There is a lengthy description of some Van Allen Probes data here. But none of it really adds any genuine value to the manuscript in our

opinion. From line 268 onwards there is a discussion of some parameters that are used and calculated for the interaction time. But that information is not really linked scientifically to the discussion of the data in lines 208-267, except in a very superficial way. What are the implications of one on the other? What is the importance of the results and how does it related to the data? What new inferences can we make because of the data in the table etc? What might this show us in the Van Allen Probes data etc?

Resp.:

We apologize if we were not clear. Section 4 was thoroughly revised and split into two parts in order to organize the dataset in a subsection (starting in line 200).

In the manuscript, this section aims to discuss the applicability of the model described before, mainly the scientific application of Equations (13) and (21), to four case studies observed during the Van Allen Probes data. The in situ measurements are used first to identify the periods of the high-energy electron flux dropouts and enhancements in the outer radiation belt concomitant with whistler-mode chorus wave activities for each case study shown in Table 1. It suggests that the resonance between the chorus waves and particles may have locally contributed to this electron flux variability. Then, they are used as inputs in the equations derived in the previous sections. To clarify the importance of the discussions about the PSD in this article, we decided to write a new paragraph before line 208 and rewrote the discussions about Figures 7, C1, and F1, as described below:

Figures 7, C1, and F1 show the time evolution of the radial PSD profiles at inbound/outbound regions of Van Allen Probe B, which allows the identification of the local relativistic electron loss and/or local low-energy acceleration in the outer radiation belt. The pitch angle diffusion mechanism driven by whistler mode chorus waves can cause this kind of local electron flux loss/acceleration, in which the local (L*), order of magnitude, and energy level are identified at the same period analyzed in previous sections.

Thus, we discuss the relevance of the complete relativistic description to estimate the interaction time and the average diffusion coefficient for two main different ambient conditions described by the electron plasma frequency ratio, which is shown in low-density (high group velocity, e.g. in case 1) and medium-density (low group velocity, e.g. case 3) ambient plasmasphere. The analyses allow us to show the average pitch angle diffusion coefficient may be significantly underestimated if the relativistic description is not considered. For events with the lower group velocity, the diffusion coefficient ratio (between relativistic and non-relativistic calculation) is around 1.4. As a consequence, accurately calculating the interaction time with full consideration of Special

Relativity can enhance the modeling of the electron flux in Earth's outer radiation belt. This approach improves the estimation of wave-particle interaction time and pitch angle diffusion coefficient

Reference:

Tsyganenko, N. A., & Sitnov, M. I. (2005). Modeling the dynamics of the inner magnetosphere during strong geomagnetic storms. Journal of Geophysical Research, 110, A03208. https://doi.org/10.1029/2004JA010798

Context/References/Discussion

7. ~line 50: The authors claim that the relativistic approach is often simplified or misused, but they provide no references or discussion to support this claim. Please provide references and much more discussion! This is the foundation of the motivation for the paper. We suggest that there needs to be a much more thorough justification and motivation in the introduction of this draft

Resp.: We include the discussion and references related to the motivation and justification of the paper in lines 46 - 68:

In the magnetosphere, the kinematics description of the wave-particle interaction for relativistic electrons usually considers the relativistic Doppler shift in the resonance condition (e.g., Thorne et al., 2005; Summers et al., 1998) and the relativistic motion equation (e.g., Omura, 2021). Often, the resonant kinetic energy of the electrons results from the resonance condition and the motion equation, together with the wave group velocity (e.g., Omura, 2021; Hsieh et al., 2020; Summers et al., 2012; Glauer et al., 2005; Lyons et al., 1972). The wave-particle interaction time (T_r) is a crucial parameter in estimating time-dependent processes as the energy and pitch angle diffusion coefficients (Walker, 1993; Lakhina et al., 2010; Tsurutani et al., 2013; Hsieh et al., 2020; 2022). However, the relativistic kinematics description mentioned above is incomplete to calculate this parameter. In this paper, we add to the latter approach a complete relativistic description of the problem: the relativistic velocity addition (between the electron and the wave) and the implications of the different reference frames in the estimates of the change in pitch angle and the diffusion coefficient.

We calculate the parameters for four case studies to give a quantitative comparison between the complete relativistic description and a non-relativistic approach (used here as an approximation to calculate the interaction parameters). The interaction time is calculated using the test particle equations (Tsurutani, 1974; Lakhina et al., 2010; Horne et al., 2003b; Bortnik et al., 2008) along with the special relativity theory applied to whistler-mode chorus waves propagating in cold plasma magnetosphere (where group velocity is $\sim 0.3c$ to

$0.5 c$) and energetic electrons (with energy ~ 0.1 to 2 MeV). We consider that the resonance occurs in the electron's reference frame. At the same time, the result of such interaction and their parameters are measured in the local inertial reference frame of the satellite.

We considered parallel propagating whistler-mode chorus waves linearly interacting with relativistic electrons to derive first the group velocity equation, then the resonant relativistic kinetic energy, and finally the interaction time. Thus, we calculate the change pitch angle and the diffusion coefficient rates. We use the Van Allen Probes measurement of wave parameters, ambient magnetic field, density, electron fluxes, and equatorial pitch angle to apply the interaction time equation. A complete calculation of these parameters can improve relativistic outer radiation belt electron flux variation models.

8. Line 69: Appleton/Hartree equation: please provide a citation and a justification for why this is the correct relation to use

Resp.: We add the following justification and reference in lines 81-83: whistler mode chorus waves which occur in frequencies higher than the ion cyclotron frequency, besides the wave-particle interaction outside the plasmasphere, the dispersion relation for this case is obtained from the solution of the Appleton-Hartree equation (Bittencourt, 2004).

9. Equation (20) How does the presented formula for $\Delta\alpha$ compare to those presented in e.g. papers by Lakhina et al 2010, Tsurutani & Lakhina 1997, Kennel&Petschek66 and Allanson et al 2022? What differences has it made to include relativity, and what differences are observed because the integration was performed approximately and not exactly?

Resp.: The following discussion was included in lines 230-244 and 332-335:

The change in pitch angle for non-relativistic electrons due to wave-particle interaction in a quasi-linear regime is given by equations (3.6) in Kennel and Petschek (1966), and (11) in Tsurutani and Lakhina (1997). Lakhina et al. (2010), derived equation (11) by considering the relativistic resonant condition and the non-relativistic equation of motion. Allanson et al. (2022) show the exact equation for pitch angle scattering and second-order equations for weak turbulence and nonlinear regimes.

In this manuscript, the change in pitch angle is given by equation (20), which is consistent with equations (3.6) in Kennel and Petschek (1966); (11) in Tsurutani and Lakhina (1997) and equation (11) in Lakhina et al. (2010). In the limit that \gamma ~ 1, for non-relativistic electrons, Tr equals to \delta t (in Kennel and Petschek, 1966 and Tsurutani and Lakhina, 1997) or to \tau in (Lakhina et al., 2010). Also, Equation (20) in this manuscript is similar to Equation (S3) in Allanson et al. (2021).

The main difference we observe using a more complete relativity description is that the interaction time is often longer than that calculated with a non-relativistic description, as shown in Table 1. The consequence of underestimating the interaction time is that diffusion coefficients are also underestimated and may deteriorate modeling results.

10. Lines 280 onwards: There is no meaningful discussion section. Section 5 has some conclusions. But the authors have not contextualised their work in detail with respect to previous results in the literature. This is an important missing piece of the draft and we would suggest needs to be fixed. Furthermore, now that we have these new estimates, how could they be used in practice by scientists in the future?

Resp.: We thoroughly revised Section 5 (Conclusion) to present the main results and contextualize them.

Technical
11. line 5 ("wave velocity"): is this group or phase velocity?

Resp.: We add the wave group velocity in line 5

12. Line 15 ("interaction time can be ~30% lower for quiet periods"): we are not sure that it is appropriate to characterise things in this way. The authors have performed a small subset of case studies. Also, the details of the interaction time are fundamentally related to the microphysics of the wave-particle interaction, and not really whether it is an active or quiet period. Perhaps it is best not to phrase things in this way.

Resp.: We agree with the referee, and we remove this sentence

13. Line 99: The Shprits 2008 paper cited is a paper about radial diffusion. You should replace this with a local diffusion paper by Shprits in the same series in JASTP

Resp.: The reference cited in the paper was correct to the following: Yuri Y. Shprits, Dmitriy A. Subbotin, Nigel P. Meredith, Scot R. Elkington, Review of modeling of losses and sources of relativistic electrons in the outer radiation belt

II: Local acceleration and loss, Journal of Atmospheric and Solar-Terrestrial Physics, Volume 70, Issue 14, 2008, Pages 1694-1713, ISSN 1364-6826, https://doi.org/10.1016/j.jastp.2008.06.014.

14. Line 134: "well-known formula of the addition of velocities". What is this formula? Please provide the formula and a citation

Resp.: Equation (7) in the manuscript is the general equation for the relativistic addition of velocities between vg and vgc. In line 166, we add the citation of Jackson, J. D. Classical Electrodynamics, whose citation was included in line 378

15. Equation 18: the authors use \Delta on the LHS but "d" on the RHS for infinitesimal changes. They should either use one or the other, and be consistent on both sides of the equation.
Resp.: We made the suggested change in equation (18), line 222.

Response to Referee #2

The authors appreciate all suggestions given by Reviewers 1 and 2, which were essential to improve this article. A careful revision was carried out in the manuscript taking into account all the comments and suggestions

The authors argue that taking into account special relativity effects would alter the wave-particle interactions between chorus and energetic electrons in the radiation belts. Unfortunately their results (most notably shown in Table 1) demonstrate the opposite, that is that relativistic effects are most likely inconsequential. And here is why. Their calculation assumes that electrons interacting with chorus waves will experience weak scattering, and thus nonlinear effects such as phase-trapping (Bortnik et al. 2008 GRL) or physical trapping (Artemyev et al. 2013 PoP, Osmane et al. ApJ 2016) can be neglected.

Resp.: We clarify in the manuscript that the derived equations described in this paper are applicable to a quasi-linear regime, and thus we choose case studies in order to limit the application to the linear wave-particle interaction cases, as shown in Table 1.

Thus, by assuming that the waves are indeed sufficiently small in amplitude (clearly not the case for some of their events since delta B/B_0 >=1%) and that we are in a quasi-linear regime, the transit time they are calculating with or without relativistic effects are comparable, with some minor differences of 0.1-0.01 ms. What would actually justify their thesis would be to show that the pitch-angle diffusion coefficient they can compute from Delta_alpha (Equation 20) would lead to notable differences in the particle scattering. But they do not provide such an analysis and the change in the pitch-angle shown in Table 1 (assuming again we are within a quasi-linear regime despite the large amplitudes of the waves) are essentially identical for both relativistic and nonlinear relativistic effects with differences of the order 1/10 or 1/100.

Resp.: We thank the referee for this point. We chose specific periods in the four case studies to consider only linear processes, as we are interested in showing the relevance of a more complete relativistic description in linear wave-particle interaction. We also include in the manuscript (Section 4, subsection 4.1) the calculation of the average pitch angle diffusion coefficient for one subelement, following a similar approach as done by Lakhina et al. (2010). The new results are included in Table 1.
We calculate the ratio between the diffusion coefficient computed from a complete relativistic description and a non-relativistic one. The same was done for the interaction time. We include the discussion about these comparisons in the manuscript in subsection 4.2. For the four case studies, results show that the

relativistic interaction time is longer than the non-relativistic one. The quantitative evaluation of this ratio (calculated from in situ parameters from cases 1-4) shows that for case 1, the interaction time can be 3 times longer.
We found that the average diffusion coefficient ratio equals 5.4, for case 1, which was the most significant difference in our case studies. The results are now in the corrected version of Table 1.

If we are interested with the nonlinear regime, the transit time they compute only make sense when compared with some nonlinear timescales, such as the trapping time. But since their transit time are essentially identitical I don't see how relativistic effect could turn a linear interaction into a nonlinear one either. I am therefore not certain how the paper could be salvaged since the relativistic effect they study seems to have no impact for both linear and nonlinear wave-particle interactions. If the editors are willing to accept a null result, the authors could perhaps compute the diffusion coefficients with relativistic effects included, and if the difference is marginal publish it as such.

Resp.: We agree that the relativistic effects can not change the linear wave-particle interaction into a non-linear nature. Thus, to be consistent with linear regimes, we clarify this point throughout the manuscript and choose only linear events for the case studies presented in Table 1.
In addition, we include a paragraph in the Conclusion (lines 336-347) to discuss the applicability of the equations derived in the manuscript. The authors consider that the results from the average pitch angle diffusion coefficient calculated for the case studies are relevant, which elucidate how much the coefficients can be underestimated by taking a non-relativistic physical description of the related events. Thus, an inaccurate wave-particle interaction model, applied to calculate the pitch angle diffusion coefficients, could deteriorate outer radiation belt electron flux models. Because of these results, the authors consider the approach described in this manuscript adds an important contribution to the literature on wave-particle interaction due to whistler mode chorus waves in the outer radiation belt.