Reply to the referee comments

Manuscript ID: ANGEO-2023-2
Scalar-potential mapping of the steady-state magnetosheath model

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I thank the both referees for their careful reading and thoughtful comments. Reply to each comment is given here.

Referee 1

1. *This manuscript describes a methodology for mapping magnetosheath locations relative to specific empirically-based models for the bow magnetosheath location with boundaries described by confocal paraboloids. Analytic solutions for plasma streamlines (and potential) and mapped to a space bounded by more realistic boundaries. The authors have made significant efforts to provide more details and description of the techniques used, along with their advantages and to this referee's comments, concerns, and questions. There remain several minor issues with English usage, and some recommended edits, then this manuscript will be considered by this referee to be publishable in Annales Geophysicae.*

   Line 1 (Abstract): 'applications in studying' → 'applications for studying'

   Reply (ref.01.01):

   - Done. Page 1, line 1.

2. *Line 2 (Abstract): 'the planetary magnetospheres In particular,' → 'planetary magnetospheres. In particular,'*

   Reply (ref.01.02):

   - Done. Page 1, line 1.

3. *Lines 7-8 (Abstract): 'a set of the shell variable and the connector variable' → 'a set of variables (shell variable and connector variable)'*

   Reply (ref.01.03):

   - Done. Page 1, lines 7–8.

4. *Line 9 (Abstract): 'zenith angle in the magnetosheath' → 'zenith angle within the magnetosheath relative to the magnetopause'*

   Reply (ref.01.04):

   - "the zenith angle within the magnetosheath with respect to the direction to the Sun" on page 1, line 9.

5. *Line 13 (Abstract): 'by using the analytic expression' → ', using analytic expressions'*

   Reply (ref.01.05):

   - Done. Page 1, lines 13–14.

6. *Line 14 (Abstract): 'is applicable' → 'and is applicable'*

   Reply (ref.01.06):

   - Done. Page 1, line 14.

7. *Line 17: treated as ignored' → 'ignored'*

   Reply (ref.01.07):

   - Done. Page 1, line 17.

8. *Line 25: 'on the assumption' → 'using the assumption'*

   Reply (ref.01.08):

   - Done. Page 1, line 25.

9. *Line 28: 'solution; the' → 'solution. The' (it's better to break up this very long sentence)*

   Reply (ref.01.09):

   - Done. Page 2, line 28.

10. *Line 33: 'an non-parabolic shape of empirical magnetosheath' → 'a non-parabolic empirical magnetosheath shape'*

    Reply (ref.01.10):

    - Done. Page 2, lines 33.

11. *Line 70: directions → direction*

    Reply (ref.01.11):

    - Done. Page 3, line 72.

12. *Lines 110-111: 'makes a difference to the radial mapping in using' → 'differs from the radial mapping by using'*

    Reply (ref.01.12):

    - Done. Page 5, lines 111–112.

13. *Line 126: 'makes a difference' → 'differs'*

    Reply (ref.01.13):

- Done. Page 5, line 135.

14. *Line 136: 'This is achieved on' → 'This is based on'*

    Reply (ref.01.13):

    - Done. Page 6, line 145.

15. *Lines 137-138: 'is the same when normalized to the magnetosheath thickness (...)' → 'when normalized to the magnetosheath thickness (...) remains constant'*

    Reply (ref.01.15):

    - "when scaled to the magnetosheath thickness (defined as the distance from the magnetopause to the bow shock along the magnetopause-normal direction) remains constant."

16. *Line 146: 'of the nearest magnetopause' → 'associated with the minimum distance to the magnetopause'*

    Reply (ref.01.16):

    - Done. Page 7, line 155.

17. *Fig.4 caption: 'zenith angle of the nearest magnetopause' → 'zenith angle along the direction nearest to the magnetopause'*

    Reply (ref.01.17):

    - Done. Page 7, figure 4 caption, line 1.

18. *Line 182: 'direction such as' → 'direction as'*

    Reply (ref.01.18):

    - Done. Page 9, line 191.

19. *Line 193: 'Having the nearest magnetopause at a distance' → 'Using the minimum distance to the magnetopause'*

    Reply (ref.01.19):

    - Done. Page 9, line 202.

20. *Line 221: 'simplify given as' → 'simply given by:'*

    Reply (ref.01.20):

    - Done. Page 11, line 233.

21. *Line 226: 'Equations (28)' → 'Equation (28)'*

    Reply (ref.01.21):

    - Done. Page 11, line 235.

22. *Line 226-227: 'set separately to avoid the numerical digergence problem.' → 'set separately.'*

    Reply (ref.01.22):

    - Done. Page 11, line 235.

23. *Line 284: 'in the Cartesian' → 'in Cartesian coordinates'*

    Reply (ref.01.22):

    - Done. Page 14, line 293.

24. *Line 316: 'problems on this.' → 'problems with this approach.'*

    Reply (ref.01.22):

    - Done. Page 18, line 362.

25. *Lines 338-339: 'Additional references to analytic models of the magnetosheath magnetic field (using expansions in Legendre polynomials) that make use of flexible magnetopause and bow shock boundary models (e.g., Romashets and Vandas, JGR, [2019])' This sentence is essentially repeated a few lines above, and should be removed*

    Reply (ref.01.25):

    - Deleted.

Referee 2

1. *General comments*

   The changes made by the authors improved the impression of the manuscript. Specifically, more focus is put on achieving a reasonable grid in the flank region rather than the ability to generalize the method, which is in line with the outcome of the study. The added figures improved the clarity, especially Fig. 1-3. The updated section headers are also appreciated as they highlight the difference between previous works and the present study. To represent a significant scientific advancement, the manuscript would need to be extended by benchmarking with real spacecraft data and comparing it to the performance of previous methods (e.g. Soucek and Escoubet (2012)). Having said that, I will continue with more specific comments regarding the content of the current manuscript. There are still concerns regarding the generality of the mapping procedure and the presentation of the method.'

   Reply (ref.02.01):

   - Thank you very much for the comments. While I understand the wish to include a benchmarking test using the real spacecraft data, I disagree with the suggestion that such a test needs to be included in this manuscript. Of course, the referee is right in his/her claim, but the goal of the manuscript is to introduce the algorithm for magnetosheath modeling with discussions about merits and demerits from the viewpoint of the algorithm construction. The current manuscript already has a sufficient amount of information and materials. The extension to spacecraft data should be left for a future study, otherwise the manuscript would be significantly long and hard to read. But I happily worked on the further revision based on the specific and technical comments as below.

2. *Specific comments*

The generalizability of the mapping procedure remains a bit unclear. The phrase "arbitrary shape" has been changed to "non-parabolic shape", but does it also need to be axisymmetric? The discussion reads:

line 326: "Our method has the possibility to be extended to three-dimensional, non-axisymmetric modeling by the use of magnetopause normal mapping. It is possible to obtain the steady-state magnetosheath potential in a more general sense without referring to the KF94 solution. [...] Various numerical solvers are known for solving the Laplace equation such as the Jacobi method, the Gauss-Seidel method, and the successive over-relaxation (SOR) method. These Laplace solvers are numerically more expensive than the mapping method, but the computation in 3-D is feasible with the contemporary computational resources."

Here, it seems like the Laplace equation needs to be numerically solved for a 3D non-axisymmetric magnetosheath. But I thought your method was to use the analytic expressions from the KF solution and map them onto a magnetosheath with new boundaries. Is this not possible in the non-axisymmetric case? If so, this is quite a crude restriction which should be noted (perhaps in the introduction and/or around line 155). Staying on the topic of the generality of the method, the following sentence is a bit strange:

line 170: "We use a specific exponent for the Shue model (with an alpha exponent of 0.5) in an effort to show that the analytic model is 'simple'. The solar wind conditions for which this exponent is applicable is not often encountered"

This is a direct response to a previous referee comment. The impression is that you are only showing that the model is simple in a special case which is rarely encountered. With this result, you cannot claim that the general method is 'simple'. Thus, this sentence weakens your argument that the method is simple and/or computationally inexpensive. To improve credibility, would it be possible to give the results with a general alpha exponent?

Reply (ref.02.02):

- I see that the text in the revision 1 (line 326 and 170) is confusing. I added a section 5 "Discussion" in the revision 2 (page 16, line 311), which shows an application of the method to different exponents of the Shue model, the approach of generalizing the method to the three-dimensional magnetosheath, and discussion on the different approaches.

3. *The methods section still seems unnecessarily lengthy (compared to the scientific contribution of the study), since the same set of equations are repeated twice with only some changes in the notation. However, if the authors after thorough consideration regard all details as necessary, it can be included as-is.*

   Reply (ref.02.03):

   - I find the manuscript concise enough, i.e., it is as compact as possible and the readers can still reproduce the results from the information shown in the manuscript.

4. *I have a number of suggestions regarding the figures which might give them a more solid impression:*

   The figure titles are inconsistent – for example, in Fig. 11 the titles describe which functions are plotted and in Fig. 2 the titles refer to the grid and mapping method. The point of this study is that Fig. 11 (left panel) is different from Fig. 2 (right panel), so the 'structure' of the figures should be similar and Fig. 2 should be clearly referred to when discussing Fig. 11.

   Instead of referring to the figure panels as left/right, why not introduce subfigures (e.g. Fig 2a)? I also suggest to add colorbars so that absolute numbers can be compared between the results of the different methods. In addition, the captions could probably be more informative.

   It would be nice to have figures that should be compared with eachother side by side (e.g. Fig. 2 (left panel) vs Fig. 2 (right panel) vs Fig. 11 (left panel)), but I understand that this might not be reconcileable with the order in which they are referred to in the text. However, as stated above, there can be more references to the figures (e.g. Fig. 2 vs Fig. 11) when making comparisons.

   Maybe combine Fig 1 and 3 to facilitate the comparison (keep all plots but make 2x2 subfigures).

   Reply (ref.02.04):

   - Figure 2 caption (page 6): "Note that the same function is plotted here for different mapping methods. Contours represent the velocity potential normalized to the solar wind, $\Phi^{(\mathrm{vel})}/U_x$, which is in units of the planetary radii. The color range is chosen for the visual demonstration purpose from $0.2R_\mathrm{E}$ to $90R_\mathrm{E}$ (left panel) and from $2R_\mathrm{E}$ to $200R_\mathrm{E}$ (right panel)."
   - Figure 11 caption (page 15): "Note that two different functions are plotted here for the same grid and mapping method. The color range is chosen for the visual demonstration from $5.5R_\mathrm{E}$ to $314R_\mathrm{E}$ (left panel) and from $0R_\mathrm{E}$ to $15R_\mathrm{E}$ (right panel)."
   - Figure 12 caption (page 15): "The color range of the magnetic potential is from $-35R_\mathrm{E}$ to $348R_\mathrm{E}$."

5. *On line 123, the reader might ask: You say that Soucek et al (2012) were able to avoid the problem, so why is your orthogonality needed?*

Reply (ref.02.05):

- We added a paragraph explaining the drawback of the Soucek method (page 5, lines 126–133):

  "Although the method introduced by Genot et al (2011) and later adapted by Soucek et al. (2012) is computational less expensive than global magnetosheath simulations, the density and velocity fields from the bow shock to a given point in the magnetosheath still needs to be computed along the streamline in an incremental way. Moreover, the Rankine-Hugoniot relations need to be solved before calculating iteratively the streamline, the flow velocity vector, to track the plasma density flow velocity along the streamline. Naturally, the uncertainty in this calculations depends on the step size (larger uncertainties for larger step sizes) and the errors accumulate along the streamline. The method introduced in this work overcomes this issue by constructing a magnetopause-normal grid system such that computational efforts are improved (no need to solve the Rankine-Hugoniot relations and the error does not accumulate in the flank region)."

6. *Technical corrections*

   Text

   - *u and v are introduced on line 50 but defined/explained on line 72-73. Consider defining them where they are introduced.*
   - *The stream function should be mentioned closer to Eq (9).*
   - *Line 108-110 and line 116-118 are almost the same sentence, a bit repetitive.*
   - *line 323-324: references are in the wrong format (parentheses).*

   Equations

   - *Eq (39): Parentheses in the denominator that should not be there.*
   - *Eq (41): Are $e_{mp,x}$ and $e_{mp,y}$ the x and y components of $e_m p^{(k)}$? If so, they should have the superscript (k).*

Reply (ref.02.06):

- Definitions of the shell and connector variables. Done. Page 2, lines 50–51.

- Stream function. Done. Page 3, lines 69–70.
- Soucek and Escoubet (2012). I leave as is. Both are necessary.
- Parentheses for reference. Done. Page 18, lines 369–370.
- Parenthesis in Equation (39) corrected (page 12).
- Superscript (k) in Eqs. (41) and (42) added (page 12).