Reply to referee comments

Manuscript ID: ANGEO-2023-2
Potential mapping method for the steady-state magnetosheath model

Y. Narita

I thank the both referees for their careful reading and thoughtful comments. Reply to each comment is given here.

Referee 1

1. *This manuscript describes a methodology for mapping magnetosheath locations relative to specific empirically-based models for the bow shock and magnetopause into an equivalent magnetosheath location with boundaries described by confocal paraboloids. Analytic solutions for plasma streamlines (and potential) and magnetic fields (and magnetic potential) can then be conformally mapped to a space bounded by more realistic boundaries.*

   Reply (ref.01.01):

   - No, not exactly. We are transforming the magnetosheath scalar potential not by a conformal mapping but by a non-conformal (and non-orthogonal) mapping in this manuscript. It is of course ideal if the harmonic functions (given as the solution of Laplace equation) were transformed into an arbitrarily-bounded magnetosheath using the conformal mapping. After extensive theoretical research (both analytically and numerically), it became clear that the conformal mapping of magnetosheath cannot be constructed uniquely. The reason for this is that the magnetosheath is not properly bounded for solving the Laplace equation. The magnetosheath is bounded only in the radial direction from the planet (or normal to the magnetopause) by the bow shock and the magnetopause. there is no boundary along the streamline, and the conformal mapping (Cauchy-Riemann condition or orthogonality condition) is no longer unique. Nevertheless, the algorithm we develop in the manuscirpt is a useful approach, because one can utilize the analytic solutions and the algorithm can relatively easily be implemented to various boundary shapes (though we chose only one example), which is of great help for future planetary research (missions and simulations). We highlight the problem with the conformal mapping in section 1 (page 2, lines 32–39) and section 5 (page 16, lines 314–320).

2. *In general, this article does not represent a significant advancement. It reads more like an Appendix of a larger study, with the Appendix detailing a technique to map locations between confocal, parabolic boundaries and empirically-based boundaries. While this technique is similar to previous efforts as described by Soucek and Escoubet [2012], Trattner et al., JGR [2015], and others, there is no effort here to demonstrate that this particular mapping technique better matches observations than previous techniques.*

   Reply (ref.01.02):

   - W accept the critique that the manuscript reads more like an appendix of thesis. This impression comes from the slight mismatch between the manuscript goal (tool or algorithm development) and the journal scope (such as scientific message). We aim to develop a numerical grid scheme for space science applicaitons. Numerical grid schemes are one of the favored discussion topics in fluid dynamics, computational physics, and informatics, but not so widely acknowledged in space science journals. See, for example, a grid generation using the conformal mapping by

Lin and Chandler-Wilde, J. Hydroinformatics, 2, 255–267, 2000 https://doi.org/10.2166/hydro.2000.0023. We nevertheless choose AnGeo for the dissemination of our study because the space science community should benefit the most from our algorithm development.

- The drawback with the radial mapping by Soucek and Escoubet (2012) is that the quality of mapping (distortion effect due to non-orthogonal grids) becomes quickly degraded in the flank to distant-tail region. Our mesh is robust against the distortion effect in the tail region. This point is elaborated in section 3 (pages 4–6) in the revised manuscript.

3. *Some of the references to empirical models of boundary shapes/sizes are inconsistent with the description provided here, or are examined under extremely specific solar wind conditions, or do not properly represent the knowledge of the physical boundaries far down the flank. Specifically,*

   (a) *The Farris et al., JGR [1991] empirical bow shock model is not a paraboloid model. It is an ellipsoid model (eccentricity of 0.81), describing the bow shock on the dayside. It is not a proper representation of the far flank bow shock.*

   (b) *The Cairns et al., JGR [1995] paraboloid bow shock model also does not properly represent the far flank bow shock. The distant bow shock shape approaches that of a hyperboloid.*

   (c) *The authors have selected a very specific exponent for the Shue et al., JGR [1997] model (alpha = 0.5) in an effort to show that the analytic model is 'simple'. The solar wind conditions for which this exponent is applicable (from the Shue et al. mode) is not often encountered (IMF Bz ¿ +8 nT, with specific values of solar wind dynamic pressure in order that alpha = 0.5).*

Reply (ref.01.03):

- True and agreed. We added the referee's comments (page 8, lines 160–164 and lines 170–172).

4. *Additional references to analytic models of the magnetosheath magnetic field (using expansions in Legendre polynomials) that make use of flexible magnetopause and bow shock boundary models (e.g., Romashets and Vandas, JGR, [2019]) should be provided and discussed.*

Reply (ref.01.04):

- Oh, thank you very much for introducing us this excellent paper! Yes, we cited the paper (page 16, lines 333–335).

5. *Although the claim is made in the manuscript that this technique can be applied to arbitrary boundary model shapes, it is not demonstrated that under general*

circumstances, the equations can be written in a closed form.

Reply (ref.01.05):

- Critique is well taken. We changed “arbitrary” into “non-parabolic” as we applied only one example (page 15, line 310).

6. *The technique described relies on determining the (straight line, or minimum) distance from a given point within the magnetosheath to the magnetopause. This is along the normal direction from the magnetopause. However, Lines 109–110 state that the task is to find the shell variable ‘v’ and the connector variable ‘u’ in the empirical magnetosheath. However, while the connector variable ‘u’ of the empirical magnetosheath is normal to the magnetopause surface, it is not a straight line through the magnetosheath? and doesn’t represent the minimum distance from the given point to the magnetopause. In other words, the distance from the magnetopause extends over a (narrow) range of connector variable ‘u’ values.*

Reply (ref.01.06):

- The confusion comes from the difference between the grids we used in the mapping (shown in figure 3, page 6, in the revised manuscript) and the u-v contours we showed in the original manuscript. We span the magnetopause-normal grids both in the KF model and in the empirical model, and here we see straight lines extending to the magnetopause.

7. *The rationale for the methodology described is confusing. For most implementations, the solar wind parameters are known, and the corresponding parameters at a given point within the magnetosheath are desired. However, the methodology here is to start with known parameters at a given place within the magnetosheath (relative to empirical models), conformal map to a location relative to the KF paraboloid boundaries, calculate the ‘u’ and ‘v’ values and determine the B-field, streamline, and potentials. The solar wind drivers appear to be missing. It appears that part of this strategy is based on the Toepfer et al. [2022] motivation; but a clear description for the order of steps for this technique is missing.*

Reply (ref.01.07):

- Yes, in the case of space plasma missions orbiting the Earth; But no, in the case of planetary mission (in which even the availability of plasma data is still chanllenging due to the mass, power budget, and telemetry budget). The advantage with the manuscript is that one can compute the magnetosheath potential for various scenarios of upstream conditions without extensive numerical efforts, assuming that the boundaries (bow shock and magnetopause) are well parametrized to the solar wind condition.

8. *Several of the equations presented are incorrect. For example, Eq.5 is infinite everywhere, due to the denominator. How are Eqs. 20 and 23 are used to derive Eq.24? Why do the units not match for the terms within Eq.39? How do Eqs.31-32 lead to Eq.33 when ymp=0?*

   Reply (ref.01.08):

   - Equation (5). Corrected (page 4, Eq. 13). Thank you!
   - It is straightforward to derive Eq. (24) from Eq. (23), but Equation (24) offers an alternative approach to find the minimum distance to the brute-force method, but the manuscript can read even withtout this equation. Equation (24) was deleted in the revised manuscript.
   - Equation (39). “2” should read “x” (page 11, Eq. 36). Thank you!
   - Equations (31) becomes singular in the subsolar direction and Eq. (33) needs to be set separately to avoid the numerical digergence problem (page 10, lines 224–227).

Referee 2

1. *General comments*

   This manuscript proposes a method for generalizing the mapping of flow lines and magnetic field in the magnetosheath. The proposed method aims to be computationally inexpensive and generalizable, which is desirable for many applications including statistical studies. However, a number of critical issues need to be addressed if the manuscript is to be considered for publication. The results presented are not general enough to be of use for actual applications; thus, the manuscript represents no (or very minor) scientific advancement. The novelty of the study needs to be explained, in particular by a more focused comparison with previous works. Furthermore, the method should be presented more clearly; it is currently difficult to tell if the proposed method is incorrect or if the presentation is unclear and contains too many mistakes (typos and inconsistencies). Please find below my detailed comments and suggestions for improvement.

   Reply (ref.02.01):

   - Thank you for suggestion that the novelty should be clear in the manuscript. We compare the grid pattern between the radial mapping by Soucek and Escoubet (2012) (figure 1, page 5) and our magnetopause-normal mapping (figure 3, page 6). The radial mapping has the problem of artificial converging flow pattern in the flank region (see figure 2 right panel, page 5). This is due to the strong grid non-orthogonality. Our method can properly transform the scalar-potential and there is no artificial converging flow pattern (figure 11, page 15).

2. *Specific comments*

   Concerning the whole manuscript

   Coordinate system: The authors introduce a new, non-parabolic coordinate system (u,v,ϕ) in which the potentials for the velocity and magnetic field are to be expressed. However, it is not mentioned whether the new coordinate system is orthogonal; in fact, from Fig. 3 it appears that the grid in the right panel is not. The parabolic coordinate system (used in KF94) is orthogonal by construction, and the gradient and Laplace equation in this system are defined and given explicitly (Eq. (8)-(9) in the KF94 paper). In this manuscript, however, the potentials are obtained from the shell and connector variables v and u in the new coordinate system (according to line 274-285 and 289). The authors do not define the gradient and curl in this system and thus the evaluation of Eq. (1) is not defined in the manuscript.

   Reply (ref.02.02):

   - The coordinates with u and v are orghotonal around the magnetopause, and the grid non-orthogonality is suppressed in our method (section 3.3, page 5 and figure 3, page 6;). To make it fully orthogonal (curvilinear and locally orthogonal grids) one needs to find a conformal mapping. Mathematical studies conclude that there is no unique conformal mapping to the magnetosheath problem because the magnetosheath is bounded only by the magnetopause and the bow shock in the radial direction but not along the streamlines.

- The azimuthal coordinate ϕ is still orthogonal to the u and v coordinates (page 5, lines 111–112).
- The computation of the u and v variables and their gradient and curl is performed in the Cartesian so that the connection represented by the Christoffel symbol vanish in the computation. Computation in the Cartesian domain is also beneficial to the practical application because spacecraft trajectories are often represented in the Cartesian coordinates (page 14, lines 283–286).

3. *Generalization of the method: In section 3, the mapping algorithm is reduced to an axisymmetric geometry and the calculations are made specifically for the Farris (1991) bow shock and Shue (1997) models. Yet, it is stated in sections 1 and 4 that the method is easy to generalize to an arbitrary magnetopause shape. Would it be possible to express the derivations in more general terms? I suggest giving expressions on a form which allows for non-axisymmetric geometries and aberrated GSE coordinate systems (see for example the asymmetric magnetosheath thickness and aberrated x axis in https://doi.org/10.1002/jgra.50465), or otherwise indicate to the reader which modifications and transformations are necessary to generalize the method. Are there restrictions on the choice of magnetopause model? Does the method require an analytic expression for the magnetopause?*

   Reply (ref.02.03):

   - Non-axisymmetric case. It is possible to obtain the steady-state magnetosheath potential in a more general sense without referring to the KF94 solution. For example, for a non-axisymmetric geometry of magnetosheath (e.g., Dimmock and Nykyri, 2013), one needs to sove the Laplace equation for a given set of boundaries (bow shock and magnetopause). Various numerical solvers are known for solving the Laplace equation such as the Jacobi method, the Gauss-Seidel method, and the successive over-relaxation (SOR) method. These Laplace solvers are numerically more expensive than the mapping method, but the computation in 3-D is feasible with the contemporary computational resources (page 16, lines 327–332).
   - Magnewtopause model. There is no restriction regarding the choice of the magnetopause model. The magnetopause-normal direction needs to be computed either analytically using the gradient of the magnetopause function as ∇f_{mp}, or numerically for a user-defined magnetopause shape (page 16, lines 335–337).

4. *Figures 1 and 2 show examples of results of the mapping, but not in which way the proposed method is better than (or even different from) the method*

by Soucek (2012). They are also introduced very early in the manuscript, before the potential functions or the mapping procedure have been described, and do not contain much relevant information. I suggest replacing them with figures showing the steps of the mapping procedure and/or a comparison with the Soucek (2012) method.

Reply (ref.02.04):

- Figure 1 and figure 3 compare directly the quality of grid pattern between the method by Soucek and Escoubet (2012) and ours. The artificially introduced converging flow pattern in the radial mapping method is shown in figure 2 (page 6).
- Agreed. Stepwise figures are shown in the revision (figures 5–9).

5. *Title*

The phrase "Potential mapping method" can be interpreted as "Possible mapping method". Perhaps the title could be rephrased to avoid misunderstanding.

Reply (ref.02.05):

- Agreed. We change the title into "Scalar-potential mapping of the steady-state magnetosheath model" to avoid confusion (title field, page 1).

6. *Abstract*

The abstract lacks a motivation for the study (a short background and a science question to be answered). It is very technical, especially the second sentence, and difficult to follow before reading the manuscript.

Reply (ref.02.06):

- Agreed. The abstract was rewritten by including the short background (page 1, lines 1–3) and the science question (page 1, lines 3–4).

7. *Section 1*

This section needs to be more concise and explain the novelty of the proposed method. In particular, the advantages compared to Soucek an Escoubet (2012) need to be clearly explained. Furthermore, since this study is very similar to the present work, it would be informative if the manuscript contained a comparison between the two methods in cases where the difference between them is the most important, together with an explanation as to why the proposed method is advantageous.

Reply (ref.02.07):

- Agreed. We have cut the introduction text and added paragraph addressing the question (page 2, lines 40–44).

8. *The introduction should be restructured. For example, lines 36-38 "While the radial mapping [...]") say the same thing as lines 52-53 ("While the radial mapping [...]"). The "gap" that this study fills is also stated twice (line 25-26 and line 48-49).*

   Reply (ref.02.08):

   - Agreed. We shortened the introduction (section 1, pages 1–2), and restructured the manuscript with the review of KF94 model (section 2, pages 2–4) and discussion about different mapping methods (section 3, pages 4–6).

9. *Line 52-53: "While the radial mapping is nearly boundary-fitted on the dayside, the orthogonality of mapping degrades in the flank to tail region.": this sentence is essential as it mentions the difference between the proposed method and the previous one. However, the term "orthogonality of mapping" should be better explained and perhaps illustrated in a figure. Currently, it is not clear what "orthogonality" refers to.*

   Reply (ref.02.09):

   - Agreed. The orthogonality (or non-orthogonality) of grids is graphically displayed in figure 1 (page 5) and figure 3 (page 6).

10. *Section 2*

    The title should be more informative, for example indicating that the section reviews theory from previous works.

    Reply (ref.02.10):

    - New section headers are:
      - Sec. 1 – Introduction
      - Sec. 2 – Revisiting the magnetosheath scalar potential
        - 2.1 Parabolic coordinates
        - 2.2 Velocity potential
        - 2.3 Stream function
        - 2.4 Magnetic scalar potential
      - Sec. 3 – Mapping method comparison
        - 3.1 Mapping problem
        - 3.2 Radial direction as reference
        - 3.3 Magnetopause-normal direction as reference
      -
        - 4.1 Overview of the procedure
        - 4.2 Setup
        - 4.3 Step 1: Measuring the distance to magnetopause
        - 4.4 Step 2: Computing the thickness of empirical magnetosheath

      * 4.5 Step 3: Computing the magnetosheath thickness in the KF system
      * 4.6 Step 4: Mapping the position vector onto the KF system
      * 4.7 Step 5: Evaluating the shell and connector variables
      * 4.8 Step 6: Computing the potentials and stream function
  - Sec. 4 – Magnetopause-normal mapping
  - Sec. 5 – Concluding remark

11. *This section reviews previous results from KF94. It should begin with an introductory sentence explaining what the section contains. It was not entirely clear when the review of previous work ends and the new work starts. Also, the level of detail in the section seems a bit unnecessary. Would the authors perhaps consider referring to the works by Kobel and Flückiger (1994) and Guicking et al. (2012), instead of writing out all the equations in the main text? Alternatively, detailed equations could be placed in an appendix to improve the flow in the text.*

    Reply (ref.02.11):

    - We added a short paragraph about the section organization in the paper at the end of section 1 (page 1, lines 45–47).
    - It is better for the benefit to the readers if the manuscript contains all the necessary information (equations) in a coherent way, rather than simply citing the original referencces. The reason for this is that the coordinate system used by Kobel and Flückiger (1994) has the co-focal point as the origin. In our method, the bow shock and the magnetopause do not share the same focal point.

12. *Section 3*

    Section 3.1: This section contains confusing terminology and probably some typos. These include:

    - *Line 130: "magnetosheath-to-magnetopause distance": the magnetosheath is a region bounded by the magnetopause, so this phrase does not make sense. Instead of magnetosheath, consider writing "position vector r" (since αemp is the distance from r to the magnetopause).*
    - *"magnetosheath-normal direction": → "magnetopause-normal direction" (line 136 and Figure 4 caption).*
    - *"magnetopause thickness" → "magnetosheath thickness" (line 137, Figure 4 caption, line 250, possibly more places in the text).*

    Reply (ref.02.12):

    - Corrected into "computing the distance to the magnetopause" (page 7, line 140)

- Corrected into “the distance from the planet to the bow shock $r_{\rm bs}$, the distance from the planet to the magnetopause $r_{\rm mp}$, the relative distance to the magnetopause $\alpha_{\rm emp}$” (page 7, lines 146–147).
- Corrected into “the magnetosheath thickness” (page 7, line 147–148).

13. *Line 138-139: are these the only input parameters, or should the magnetopause and bow shock shapes also be regarded as input?*

    Reply (ref.02.13):

    - Yes, assuming that the shape of bow shock and that of magnetopause are already known. We changed the sentence into “The position vector $\boldsymbol{r}$, the bow shock stand-off distance $R_{\rm bs}$, the bow shock shape, the magnetopause stand-off distance $R_{\rm mp}$, and the magnetopause shape are assumed to be known in our mapping” (page 7, lines 148–149).

14. *Also, the reason for defining a unit vector orthogonal to the magnetopause was not clear when reading the manuscript for the first (or second) time. The motivation for this choice should be emphasized in this section and better explained in the introduction.*

    Reply (ref.02.14):

    - The need for the magnetopause-normal direction is elaboted in section 3 (pages4–6). The grids need to be as orthogonal as possible particularly around the magneopause.

15. *Section 3.2-3.5: see above comments about generalization of the method. Equations (18)-(20) and (26)-(28) are specific to the Shue and Farris models; it should be clarified that they do not describe a generalized method.*

    Reply (ref.02.15):

    - Equations (18)–(20) in the original manuscript are packed into a subsection “Setup” (section 4.2, pages 7–8). An explanation was added at the end of section 4.2 that one needs to compute the radial distance from the planet to the bow shock or magnetopause as a function of the zenith angle when using different shapes (page 8, lines 177–179).
    - Steps 1 and 2 in the original manuscript are compressed into section 4.2 in the revised manuscript. The procedure begins with Step 3 in the original manuscript, and is introduced as Step 1 in the revision (page 8, section 4.3).

16. *Section 3.6: The derivations are very similar to what has already been done in section 3.2-3.5. For the sake of getting a better flow in the text, perhaps it would be possible to reduce the number of equations, or make an appendix with the details?*

Reply (ref.02.16):

- We thoroughly checked the logical flow in the text. We moved Eq. (17) in the original manuscript onto Eq. (47) in appexndix in the revised manuscript. We corrected Eqs. (13) and (36) in the revised manuscript. We deleted Eqs. (24) and (25) in the original manuscript. All the other equations are necessary and should appear as is.

17. *Section 3.8: Line 270-272: What is meant by the sentence "The mesh pattern [...]" Why is this important?*

Reply (ref.02.17):

- The sentence was deleted. It becomes important when discussing the conformal mapping of magnetosheath coordinates, but we conclude that it is beyond the scope of the manuscript.

18. *Section 4*

This section needs to be reworked when the above comments have been taken into account.

Reply (ref.02.18):

- We went through the conclusion section after revising the main text part. We changed "arbitrary shape" into "non-parabolic shape" (page 15, line 310) and also added more discussion on page 16, lines 327–337.

19. *Line 292: "wider range" – compared to what? (This also appears in the abstract.)*

Reply (ref.02.19):

- Corrected into "wide range" (page 1, line 12 and page 15, line 308).

20. *Line 294: "The method is applicable to an arbitrary shape of magnetosheath domain": in its current state, the method is specific to the Shue and Farris models, and thus this sentence is too strong (see above comments about generalization of the method).*

Reply (ref.02.20):

- Changed into "non-parabolic shape" (page 15, line 310).

21. *Line 302-304: Here the authors mention non-orthogonality – is the coordinate system (u,v,ϕ) orthogonal in the magnetosheath?*

Reply (ref.02.21):

- No, not orthogonal between u and v any more due to the non-uniqueness or non-existence of conformal mapping in the magnetosheath. But our method restores the orthogonality near the magnetopause.

22. *Line 309-311: "In reality, non-axisymmetric [...]" – this should be expanded on or incorporated in other parts of the manuscript.*

Reply (ref.02.22):

- Done (page 16, lines 327–332).

23. *Technical corrections*

- *Line 4: "solution of Laplace equation" → "solution of the Laplace equation"*
- *Line 51: "magnetopause. But" → "magnetopause, but"*
- *Line 53: "orthogonality of mapping" → "orthogonality of the mapping"*
- *Line 63: "flow velocity is U is" → "flow velocity U is?´*
- *Line 164: "[...], the cylindrical distance" → "[...], and the cylindrical distance"*
- *Line 207: "such that relative distance" → "such that the relative distance"*
- *Line 277: "streamline" → "streamlines"*
- *Line 278: "nose of magnetopause" → "nose of the magnetopause"*
- *Line 293: "solar wind condition" → "solar wind conditions"*

Reply (ref.02.23):

- "solutions" (page 1, line 8).
- "magnetopause. But" was deleted.
- "orthogonality of mapping" was deleted.
- "flow velocity $\boldsymbol{U}$ is" (page 3, line 68).
- "and the cylindrical distance" (page 8, line 177).
- "such that the relative distance" (page 10, line 212–213).
- "streamlines" (page 14, line 291).
- "nose of the magneopause" (page 14, line 292).
- "solar wind conditions" (page 15, line 309).