Review of “A technique for volumetric incoherent scatter radar analysis” by Stamm et al.

This manuscript describes an inverse technique for volumetric velocity reconstructions with direct relevance to the future EISCAT_3D radar. The problem in question is fundamentally ill-posed, namely solving for six unknown vector components (3 components of electric field and 3 components of neutral wind) given only measurements of 3 variables (the three components of ion velocity). As an ill-posed problem, the solution is highly reliant on a priori assumptions, and the final solution is only reasonable if all the a priori assumptions are reasonable. The justifications given for the a priori assumptions in this manuscript are inadequate and require additional examination and explanation, as described below. Furthermore, the formulation of the inverse problem contains conceptual flaws.

Major Comments

1. **The manuscript does not handle the F-region parallel ion velocities correctly.** The momentum equation in Eq. 3 is a vector equation, and it is approximately valid for the two perpendicular components. Nonetheless, this equation is not valid for the parallel component at F-region altitudes. Using that fact that $v_{\|} \times \mathbf{B} = 0$ and assuming that $E_{\|}$ is small, the parallel component of Eq. 3 reduces to $v_{\|} = u_{\|}$, implying that the parallel ion velocities are always equal to the parallel neutral velocities. This is generally not true in the F-region. A proper treatment of ion parallel velocity in the F-region requires the inclusion of gravity, ion pressure gradients, and ambipolar electric fields. The ion inertia terms can also become important during times of rapidly varying ion upflow.

   In principle, EISCAT_3D measurements could be used to volumetrically reconstruct all three components of the F-region ion velocities, including the spatial variations of the ion upflow velocities. The algorithm presented in this manuscript, however, would fail to do that. This manuscript is not solving for $v_{\|}$, but instead solving for $E_{\|}$ and $u_{\|}$ assuming the two quantities are related to $v_{\|}$ through an invalid parallel momentum equation.

   Figure 8 shows low uncertainties in the vertical neutral wind estimates extending all the way up to 200 km altitude. This is unreasonable since the ion velocities that the radar measures become collisionally decoupled from the neutral velocities at high altitudes, meaning the radar data cannot actually be giving meaningful information on neutral velocities at those altitudes. This unreasonable result is a direct consequence of the invalid parallel momentum equation.

2. **For vector basis functions, the weights should generally be arrays not scalars.** Eqs.

12 and 13 are vector equations of the form:

$$\mathbf{E} = \sum_j \eta_j \mathbf{\Phi}_j$$

$$E_x = \sum_j \eta_j \Phi_{xj}$$

$$E_y = \sum_j \eta_j \Phi_{yj}$$

$$E_z = \sum_j \eta_j \Phi_{zj}$$

This implies that all three components of the basis function get the same weight, which is an unusual restriction. To allow the three components to vary independently, the coefficients should be allowed to be different for the different vector components, i.e.

$$\mathbf{E} = \sum_{i=1}^{3} \sum_j \eta_{ij} \mathbf{\Phi}_j$$

$$E_x = \sum_j \eta_{1j} \Phi_{xj}$$

$$E_y = \sum_j \eta_{2j} \Phi_{yj}$$

$$E_z = \sum_j \eta_{3j} \Phi_{zj}$$

To be general, the three different components of η should be treated as three separate unknowns.

3. **The manuscript does not assume equipotential field lines and does not explain the rationale for allowing large variations in electric fields along a field line.** Past E-region neutral wind estimation techniques such as *Thayer* [1998] and *Heinselman and Nicolls* [2008] have always asserted that electric fields are invariant along field lines such that F-region measurements of the electric fields can be mapped into the E-region. The mapping of F-region electric fields into the E-region is crucial for all of these past studies of E-region neutral winds using ISR; without that assumption the ion momentum equation is unsolveable in the E-region. Past sounding rocket studies have demonstrated the reality of field line mapping using payloads that can measure electric fields independently of ion velocity [*Sangalli et al.*, 2009].

   In this manuscript the a priori standard deviation of the electric field gradient is allowed to be 20 mV/m per 2.5 km in all three directions, including along the field lines. This is equivalent to asserting that field-aligned mapping of the electric fields does not function between the F- and E-regions; fields of 50 mV/m in the F-region at 300 km can change by more that 100% over the distance to the E-region at 100 km.

Ignoring field-aligned mapping of electric fields between the E- and F-region makes the problem of estimating E-region neutral winds substantially more difficult, and it is clearly leading to unreasonable results in the examples presented. Figure 10 shows the algorithm estimates non-zero neutral winds at 125-135 km altitude in a truth model simulation where the true neutral wind is zero. This behavior is pathological and unreasonable. The results assert that the variance of the estimated electric fields at low altitudes is nearly infinitely large, when in reality electric field mapping should guarantee that electric fields at low altitudes should nearly match the fields at high altitudes.

4. **The justifications for the allowed magnitudes of $\nabla \cdot \mathbf{E}$ are inadequately justified.** Line 220 assumes a deviation from charge neutrality that is 10^{-6} with no justification. This leads to an assumed variance of $\nabla \cdot \mathbf{E}$ of 10^{-3} V/m^2. This is actually a very large value.

   An alternative way to estimate a typical value for $\nabla \cdot \mathbf{E}$ would be to start from the height-integrated current continuity equation [*Clayton et al.*, 2021].

   $$J_{\parallel} = \Sigma_P \nabla_{\perp} \cdot \mathbf{E} + \nabla_{\perp}\Sigma_P \cdot \mathbf{E} - \nabla_{\perp}\Sigma_H \cdot \left(\mathbf{E} \times \hat{b}\right)$$

   Ignoring the conductance gradient terms, this is approximately

   $$\nabla_{\perp} \cdot \mathbf{E} \approx \frac{J_{\parallel}}{\Sigma_P}$$

   Using typical values of $J_{\parallel} = 5 \times 10^{-5}$ A/m^2 and $\Sigma_P = 5$ S gives $\nabla_{\perp} \cdot \mathbf{E} = 10^{-5}$ V/m^2, which is two orders of magnitude smaller than what this manuscript assumes. Note that using the height-integrated current continuity equation provides and estimate of $\nabla_{\perp} \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}$, but if the field-aligned variation of $\mathbf{E}$ is small (i.e. $\frac{\partial E_z}{\partial z} \approx 0$), then $\nabla \cdot \mathbf{E} \approx \nabla_{\perp} \cdot \mathbf{E}$.

5. **The use of ground-based magnetometer data for constraining $\frac{\partial \mathbf{B}}{\partial t}$ is unjustified.** Ground-based magnetometers located at least 100 km below the current sources in the E-region are not necessarily going to capture realistic estimates of $\frac{\partial \mathbf{B}}{\partial t}$ in the F-region, particularly in cases where trapped Alfvén waves are bouncing around in the ionospheric Alfvén resonator. Observations from the rocket literature actually can justify large values of $\frac{\partial \mathbf{B}}{\partial t}$. For example, the observations from *Akbari et al.* [2022] describe standing Alfven waves with amplitudes of $\Delta E = \pm 40$ mV/m, $\Delta B = \pm 100$ nT, and frequencies of 0.25-0.5 Hz. In this case $\frac{\partial B}{\partial t} \approx 2\pi f \left|B\right| = 2\pi \times 0.5 \text{ Hz} \times 100 \text{ nT} = 314 \text{ nT/s}$.

   A caveat with this analysis is the 70 second integration time is going to average over fluctations associated with 0.5 Hz Alfven waves. Nonetheless, the integration is still not going to remove Ultra Low Frequency Pc5 waves (2-7 mHz), which can also have significant amplitudes (100s of nT on the ground, meaning they are even larger in the ionosphere).

6. **The assumptions about the relationship between electron density fluctuations and neutral density fluctuations is unjustified.** A more direct way to estimate neutral density variations is to look directly at lidar measurements of gravity waves. For example, *Vargas et al.* [2019] cite wave amplitudes ranging from 0.77 to 8.4% of the ambient sodium density, with an average of 2.7%. The assumed neutral density fluctuation of 50% at 100 km in the manuscript is unreasonably large.

7. **The use of a zeroth-order Tikhonov regularization is going to bias the neutral wind estimates low.** The assumed a priori variance is 200 m/s, but auroral neutral wind jets over 300 m/s have been observed, for example in the JETS rocket mission.

Minor Comments

1. The figure quality is generally low, with the text in the axis labels being highly pixelated.
2. An azimuth-elevation plot of the beam geometry would substantially clarify the beam geometry. Figures 3 and 4 have so many lines on them that the 3D geometry is hard to see.
3. Lines 209 and 210 should specify the interpulse period assumed for this experiment and specify how many independent estimates of the ACF/Spectra are obtained in 2 s of integration.

References

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