

Reviewer 1

The manuscript “Reconstruction of Mercury’s internal magnetic field beyond the octupole” by Toepfer et al. presents and compares different existing inversion techniques to reconstruct the internal magnetic field data of Mercury. The authors use a simulated magnetospheric model to get the synthetic magnetic field data for the inversions, to mimic the BepiColombo (MPO) mission data acquisition in a non current free environment. In this way, the authors are capable of evaluating the different methods in retrieving the known coefficients given a priori to the simulations. This study is of great importance specially that BepiColombo is on its way to Mercury. Besides some concerns detailed below, the manuscript is well organized and well written.

Reply: Thank you very much for reviewing the paper.

Main comments:

My main comment concerns the application to simulated Mercury magnetic field data. The authors make use of known Gauss coefficients from literature (Anderson et al. 2012, Thébault et al. 2018 and Wardinski et al. 2019), where the different models represent the Hermean internal magnetic field with Gauss coefficients of $m=0$. Those models are evidently constrained by the available data over a single hemisphere.

As the authors are doing a theoretical exercise using a synthetic model, I wonder if the authors shouldn’t test the different inversion techniques for a more complex internal magnetic field, where $m \neq 0$ coefficients are also considered. For example, I have in mind the Jupiter’s internal magnetic field case where the magnetic field from a hemisphere is axisymmetric but not in the other. In the case Mother Nature surprises us with a more exotic Hermean internal magnetic field, are the different inversion methods described in this manuscript capable of representing the non-axial Gauss coefficients? I suggest to run a couple of simulations using non-axisymmetric coefficients to test the limits of the different inversion techniques combined with the Gauss-Mie parametrization.

Reply: Agreed. We will perform additional simulation where the reconstruction of $m \neq 0$ coefficients is considered. The simulations are currently running on the high performance computer.

My second main comment is related to the effectiveness of the different techniques to retrieve the Gauss Coefficients. This issue is actually highlighted when the authors used two different values for coefficient g^0_5 . Depending

on the strength of this coefficient the authors show that the inversion method perform differently. It should be mentioned that the used models (Anderson et al. 2012, Thébault et al. 2018 and Wardinski et al. 2019) give Gauss coefficients with a strong covariance, because there are no data available in the Southern Hemisphere. This means that several sets of coefficients are still valid to represent the internal field. Given that, I wonder what is the impact in changing slightly the many coefficient values (as done for g^{0_5}). How well are the different inversion techniques retrieving the given coefficient, and how much that value should vary to notice that change? I suggest the authors to try other g_1^0 , g_2^0 , g_3^0 , ... values not too different from those already used, and that can potentially be also a solution for Mercury's magnetic field, to check if there are limitations in discerning one from the other.

Reply: Agreed. We will perform an additional simulation with slightly different axisymmetric internal Gauss coefficients. The simulation is currently running on the high performance computer.

Moderate comments:

Introduction: I find that a description of the state of the art of the existing internal magnetic field models using MESSENGER data is missing. The lack of an introduction on this topic is enhanced later in section 4.1. I would also specify the limitations of those models, for example, the coefficients that are covarying.

Reply: Agreed. We will add a description about the current knowledge of Mercury's internal magnetic field resulting from the MESSENGER data analysis.

Lines 96 - 100 Please, add a sentence to describe how the thin shell approximation is affecting the results.

Reply: Agreed.

Section 4.2 After running the simulations but before selecting the data for inversions, is there a noise added to each data point? What is the error level? Is there more than a value considered? How it affects the inversions? There is a brief mention on this topic at lines 368-371 but it is not satisfying.

Reply: Agreed. We will disturb the data synthetically and add the resulting errors of the coefficients.

Lines 309 - 312 It might help display a figure with the spherical grid points used for the inversions. Please, indicate the grid resolution used. Also, how

much time shall we wait for BepiColombo to acquire enough data for your inversions?

Reply: Agreed. We will add a figure where the synthetically generated data points are illustrated.

Conclusion: An important aspect on internal field modeling is the time variation (or secular variation). This should also be mentioned here, even if this is not the scope of this manuscript.

Reply: Agreed. We will add this aspect as an outlook.

Minor comments:

Lines 14-17 there is a lack of citations.

Reply: Agreed. We will add a citation.

Line 26 Wardinski et al 2019 also estimate the size of the core.

Reply: Agreed. We will add this aspect in the introduction section.

Lines 34-36 There is also a disadvantage, the authors are modeling the external sources that are given by the simulations only, which could not be a full representation of the real magnetosphere currents.

Reply: Agreed. We will clearly emphasize that the simulated data are a proxy for the not yet available MPO data.

Lines 87 - 89 Please, define O in the text from equations 8 and 9.

Reply: Agreed.

Table 1 and 2: You have described in section 3.1 the Least Square Fit method, I would also add a column for that method in the given tables for comparison purposes.

Reply: Making use of the truncated singular value decomposition, the Least Square Fit estimator transits into the TSVD estimator. Thus, comparing the LSF estimator with Capon's method (with truncated singular values) and the Tikhonov regularization may be declared as „unfair“.

Typos:

line 176 Imagenary -> Imaginary

Reply: Agreed.