Response to reviewers

Reviewer #1 Evaluations:

Summary: The work deals with improving the quantification of one of the main processes acting in the radiation belts: radial diffusion. It provides a variety of content of potential importance: It briefly describes how electric and magnetic field measurements from the three THEMIS inner probes (A, D, E) were processed to compute products equated with radial diffusion coefficients, DLLs. It discusses several dependencies of the database related to spatial location and magnetic activity. It compares and contrasts the database outputs with various published models. It also shows two numerical simulations of outer radiation belt dynamics: one where radial diffusion is parameterized by the data products introduced in this manuscript, and the other where radial diffusion is parameterized by the published model that best compares with the database (l.261-262). One of the main findings is that "all models underestimate the DLL during quiet times and at low L* values, while they overestimate the DLL during high levels of geomagnetic activity and at high L* values" (l.279-281).

General Comments: The work claims to provide a database of "accurately calculated" radial diffusion coefficients (l.4, l.12, l.71, l.216, l.246, l.265). Yet, it fails to be convincing. A much more rigorous treatment of both data processing and scientific presentation is required to demonstrate the validity and significance of the work.
Response: We agree with the reviewer that the expression "accurately calculated" may lead to misinterpretations. The term "accurate" intended to describe the detailed process we followed for the calculation of the DLL database, from the pre-processing of the data to final scientific product. Nevertheless, we acknowledge that any calculation of the DLL is an estimation based on several assumptions and, thus we have removed this term from the manuscript. Furthermore, we have included in the manuscript a detailed description concerning the entire data processing chain. To that end we have also discussed several assumptions DLL database which arise from the theoretical approach used in this study and the inherent limitations of the in-situ data.

Specific Comments:
Major comments:
1. The database does not provide radial diffusion coefficients:
A radial diffusion coefficient quantifies the long-term phase-averaged effect of small electromagnetic fluctuations on trapped particles' third adiabatic invariant (e.g., Schulz and Lanzerotti, 1974). Thus, a radial diffusion coefficient is independent of magnetic local time by definition. In this work, the products resulting from THEMIS data processing present significant variations with magnetic local time (section 3.2, Figure 4). This feature is enough to demonstrate that the database does not provide a time series of radial diffusion coefficients.
Response: We would like to thank the Reviewer for noting these points in the calculation of the radial diffusion coefficients. Indeed, the radial diffusion coefficient, DLL, quantifies the mean square displacement of radiation belt electrons across Roederer's L* as a result of fluctuations in the magnetic and electric fields. In the classic electromagnetic diffusion formulas proposed by Falthammar (1965), particle perturbations leading to diffusion result from variations in the magnetic field along the drift orbit and the electric fields induced by these magnetic field fluctuations as well as electric potential fluctuations, DLL,m and DLL,e.

In this manuscript, for the calculation of DLL, we have adopted the newer formulas for radial diffusion coefficients proposed by Elkington et al. (2003) and further developed by Fei et al. (2006) that consist of a component that quantifies radial diffusion driven by magnetic field disturbances in the direction of the background magnetic field, DLLB and a second component that quantifies radial diffusion driven by azimuthal electric field disturbances, DLLE. Since no coupling between wave magnetic and electric fields through Faraday's law is assumed, there are uncertainties introduced in the derivation of radial diffusion coefficients by Fei et al.(2006). We have noted that Lejosne (2019) has estimated that, in the presence of magnetic field disturbances, adopting the approach of Fei et al. (2006) leads to underestimation of the total radial diffusion coefficients by a factor of 2. However, as Sandhu et al. (2020) have suggested and as we demonstrate in section 4.1, this discrepancy is comparatively minor relative to the large variability of the calculated values which span orders of magnitude especially during magnetic storms.

Furthermore, spatial variations in the power of magnetic and electric field perturbations have been found to impart local time dependencies to calculated diffusion coefficients. In the following figure we demonstrate that wave power calculated based on measurements from three spacecraft of the THEMIS constellation is highly dependend on the limited MLT sector sampled.

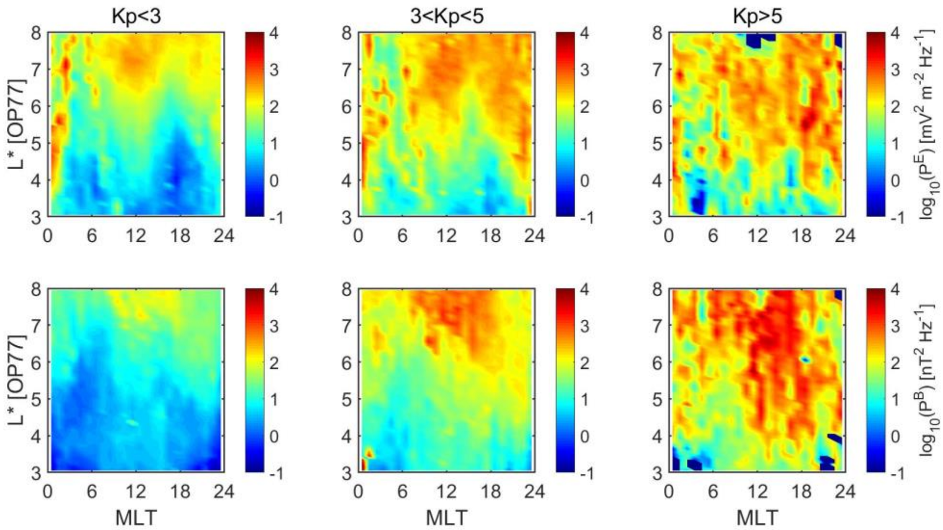

Figure 1: Logarithms of the mean ULF power with 1-min as a function of MLT (dMLT=1 hour) and L* (dL*=0.1) for three levels of geomagnetic activity: (left column panels) Kp<3, (middle column panels) 3<Kp<5 and (right column panels) Kp>5. Top and bottom row panels correspond to the power in the azimuthal electric field component and in the compressional magnetic field component, respectively.

Local time variations in wave power indicate sources of wave activity both internal (coupling with ring current ions and substorm particle injections) as well as external (solar wind driving). However, using measurements from a single spacecraft or from a single mission that sample a specific MLT sector can result to under- or over-estimates of radial diffusion coefficients, since spatial variations are neglected. In our case, the maximum MLT coverage from all three spacecraft does not exceed 6 hours per hour and per L*. This means that our DLL (and of course any other estimated by in-situ measurements) employs a small fraction of the full MLT coverage which would be required. Therefore, figure 4 in the manuscript reflects exactly the features presented in the above mentioned figure 1.

We emphasize that radial diffusion is a drift-averaged process and radial diffusion coefficients should describe an average over all local times and the possibility of combining measurements from missions and spacecraft sampling different parts of the magnetosphere needs to be explored. Our efforts have been currently focused on quantifying the magnitude of radial diffusion due to ULF waves observed solely by the THEMIS spacecraft since combining measurements from different missions will need intercalibration of measurements which is beyond the scope of this study.

A brief description has been added in the revised section 2 of the manuscript as follows: "Equations 2 and 3 also implicitly assume a uniform distribution of wave power in azimuth. In reality, the azimuthal distribution of the wave power in the Pc4-5 range depends on their generation mechanism, e.g. the wave power due to the Kelvin-Helmholtz instability is expected to be greater near dawn and dusk sectors, while due to the pressure pulses from the solar wind is expected to be greater near noon. Furthermore, the maximum MLT coverage from all three spacecraft does not exceed 6 hours per hour and per L*. This means that our DLL--and of course any other estimated by in-situ measurements [Jaynes et al. 2018, Olifer et al. 2019, Sandhu et al. 2021]--employs a small fraction of the full MLT coverage which would be required. We note that radial diffusion is a drift-averaged process and radial diffusion coefficients should describe an average over all local times. Nevertheless, in order to achieve a full MLT coverage, one would need a large multi-satellite dataset which would span several years. Our efforts have been currently focused on quantifying the magnitude of radial diffusion due to ULF waves observed solely by the THEMIS spacecraft since combining measurements from different missions will need inter-calibration which is beyond the scope of this study."

Another paragraph has been added in section 3.2 describing the physical meaning of figure 4 as follows: "We must note that the aforementioned MLT dependence reflects directly the azimuthal distribution of power for both the magnetic and the electric component of the DLL. This means that even though the radial diffusion coefficient is calculated with the drift-averaging assumption, in practice, the limited MLT coverage from single mission in-situ data introduce an azimuthal structure, which accounts for the coupling of external and internal ULF generation mechanisms and may be quite important for future modelling efforts."

Moreover, we have added a paragraph in the end of section 4.1 discussing how the aforementioned assumptions could affect the comparison of our database with the results of the semi-empirical models.

Finally, we have clarified in section 2 that our database consists of two parts: a) the ULF wave PSD, which is stored in daily CDF files with 1-min resolution for each spacecraft separately, and b) the drift-averaged DLL (grouped in bins with dt=1 hour and dL*=0.1).

2. THEMIS data processing, and its presentation, need improvement:
- Quantifying radial diffusion using satellite measurements is a challenging task. For instance, it requires differentiating spatial and temporal variations from a time series of field measurements sampled along spacecraft trajectory, often in the presence of strong spatial gradients. How this is achieved remains unclear.
Response: As mentioned in the previous comment response, we have included in the manuscript a detailed description concerning the entire THEMIS data processing chain.

Concerning the spatial gradients, we are not entirely sure what exactly the reviewer is referring to. We have included a paragraph in section 2.1 which refers to the magnetic field gradients as the spacecraft moves close to the Earth. This paragraph is as follows: "Note that as the satellites move inbound and outbound with high velocities at low L-shells, the magnetic field measurements exhibit, not only orders of magnitude increase, but very large gradients as well. These large gradients make it quite difficult to estimate the background trend, which has to be removed. Even if we filter the magnetic field time-series, the filtered signal's amplitude still grows significantly near perigee, which renders any PSD calculations erroneous. Therefore, we manually remove the corresponding part of the spectrum."

Furthermore, Fei's expressions are based on the assumption that the asymmetric background magnetic field leads to enhanced radial diffusion in the presence of broadband ULF waves. Following previous studies (Ozeke et al., 2012, Ali et al., 2015, Liu et al., 2016, Jaynes et al.,2018 and others), we have determined the power spectrum of ULF waves along each spacecraft orbit but, due to the limited coverage of THEMIS spacecraft measurements, it is not possible to determine the waves mode structure. In this light, we opted to use the weighted averaged power over the whole frequency range under study (i.e. Pc4-5 frequency range) in the place of waver power at a specific frequency. This procedure and its benefits are discussed in detail in section 2.2.

Finally, in order to extract field perturbations in the Pc4-5 frequency range, the background electric and magnetic field was identified by taking a running average over a 30 min sliding window (see also section 2.1). The power of magnetic field perturbations was calculated in the direction of the background magnetic field (compressional perturbations) and power in local electric field perturbations in the azimuthal direction. However, due to spacecraft motion, separation between spatial and temporal variations is not possible. Temporal variations may be introduced by the dynamics of the different wave sources and these have not been separated from spatial variations of ULF wave components due to weakening of the local magnetic field strength associated with an enhanced ring current population or an increase in the local plasma mass density.

- Fei et al. formulas apply at the magnetic equator only. Yet, THEMIS probes do not necessarily sample the magnetic equator. The manuscript does not explain how this feature is taken into account in the data processing.

Response: As mentioned in the revised section 2, Fei's approach considers equatorially mirroring particles only, while THEMIS satellites do not necessarily sample the magnetic equator. Nevertheless, they remain very close to the magnetic equator throughout their trajectories in the outer belt [Angelopoulos 2008, Turner et al. 2012] something that allows us to assume that the uncertainty in the DLL calculation will be rather small. This is also supported by the results shown in the following figure where we have plotted the mean power versus the Beq/Blocal ratio for different values of L*. As shown, both in the electric and magnetic field power, the variation of power versus Bratio is up to a factor of 2, at least for Bratio values larger than 0.8 (note that this threshold in Bratio is used in our study so all results correspond to DLL values at points with Bratio>0.8). Nevertheless, we should note that there is no straightforward comparison with the dataset used by Sandhu et al. 2020, since we have no information about whether they have sorted their dataset based on magnetic latitude or about the model used for the calculation of the magnetic ephemeris data.

Note that we have revised the corresponding paragraph of the manuscript as follows: "Finally, we emphasize the fact that our results on the MLT asymmetry are in good agreement with Sandhu et al. 2020 who used Van Allen probes data (different magnetic latitude) to infer the radial diffusion coefficients. This agreement also indicates that the uncertainty introduced by the magnetic latitude (and already discussed in section 2) is insignificant, even though there is no straightforward comparison with the dataset used by latter authors, since we have no information about whether they have sorted their dataset based on magnetic latitude or about the model used for the calculation of the magnetic ephemeris data."

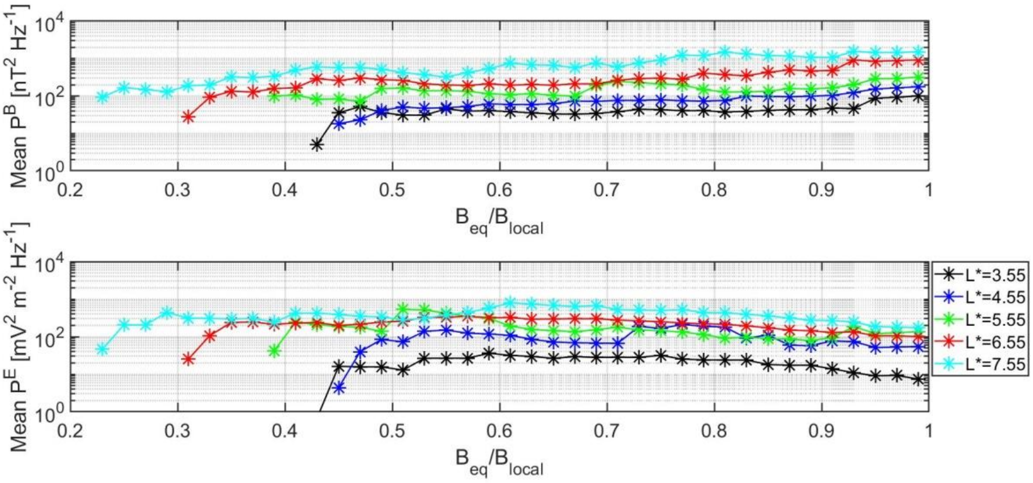

Figure 2: Logarithms of the mean ULF power with 1-min as a function of Beq/Blocal (dB=0.05) for 5 L* bins with dL*=0.1.

- Not all choices made during data processing are well explained or well justified. For instance, why the equation (4)? Why "cdelta" =0.76? What is the definition of "dj"?
Response: Indeed the data processing section was insufficient. Please note that we have revised section 2 in order to provide many more details to the reader concerning the entire data processing chain. Especially for the calculation of the ULF wave PSD, we have used the wavelet analysis as described in Torrence and Compo [1998]. The latter authors provide an in-depth analysis of the use of wavelet functions. This analysis is quite long and we believe that including it in the manuscript would be out of the scope of our study and would render the reading of the manuscript unnecessary difficult. Nevertheless, we have included the definitions of specific parameters.

For example, dj is the sampling scale of the wavelet analysis, which of course depends on the frequency range under study. Cdelta is a smoothing factor which depends on the non-dimensional frequency ω_0 of the Morlet wavelet:

$$\psi_0(\eta) = \pi^{-\frac{1}{4}} \cdot e^{i\omega_0\eta} \cdot e^{-\frac{\eta^2}{2}}$$

where η is a non-dimensional time parameter. For the Morlet wavelet ω_0 is taken as 6 to satisfy the admissibility condition (Farge 1992) and then Cdelta is empirically derived as 0.776.

- Farge, M., 1992: Wavelet transforms and their applications to turbulence. Annu. Rev. Fluid Mech., 24, 395–457.
- Torrence, C., and Compo, G. P.: A practical guide to wavelet analysis. Bulletin of the American Meteorological Society, 79(1), 61–78, 1998.

3. The claim that the data products are accurate is not justified:
Before being able to make any claim regarding the accuracy of the approach, it seems necessary to discuss the extent to which the outputs depend on the variety of choices made during data processing. Yet, this has not been done.
Response: As stated before the term "accurate" intended to describe the detailed process we followed for the calculation of the DLL database, from the pre-processing of the data to final scientific product, and in order to avoid any misinterpretations we have replaced it in the manuscript. Also we have included the subsection 2.3 (Assumptions) where we discuss

all the assumptions used for the calculation of the PSD and DLL including the theoretical approach (Fei et al 2006) and the inherent limitations of our dataset. Furthermore, all the results presented in this study have been further discussed in the basis of these assumptions.

Minor comments:

* Table 1: Units are missing. The list of limitations provided is incomplete.

Response: Duly amended. We have now revised the table in order to included L* and Kp limitations for each model.

* While Falthammar's (1965) framework was developed in the non-relativistic case, the extension to relativistic particles is straightforward (e.g. Schulz and Eviatar, 1969). Thus, the claim that Falthammar's formulation is "valid for sub-relativistic particles, only" (l.32) is misleading.

Response: We thank the reviewer for the valuable information. The sentence has been removed from the manuscript.

References:

Schulz and Eviatar (1969), Diffusion of equatorial particles in the outer radiation zone, https://doi.org/10.1029/JA074i009p02182

Schulz and Lanzerotti, 1974, Particle Diffusion in the Radiation Belts, https://doi.org/10.1007/978-3-642-65675-0