SCHUMANN RESONANCE IN THE MODEL OF THUNDERSTORM ACTIVITY UNIFORMLY DISTRIBUTED OVER THE GLOBE*

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Interest to the Schumann resonance phenomenon is explained by its availability to investigating the lower ionosphere characteristics, while the measurements of these by the direct methods is extremely difficult: the satellites fly at higher heights and the balloons drift at lower altitude. The phenomenon itself arises due to electromagnetic waves of the extremely low frequencies (ELF) that travel around the globe, and the global thunderstorm activity is the source of these waves. When solving the inverse problems, one must have the most adequate mathematical model of the phenomenon combined with the possibility of matching the experimental data by varying the model parameters in the theoretical description. The paper presents the rigorous methods of the ELF fields computations in the regular isotropic Earth–ionosphere waveguide with an emphasis on the power spectra of the vertical electric and horizontal magnetic field components.

KEY WORDS: *extremely low frequencies, Schumann resonance, global thunderstorm activity, ionosphere conductivity profiles, Legendre functions, Monte Carlo method*

1. INTRODUCTION

Many publications compare the experimental data of the spectral parameters of the extremely low frequency (ELF) noise driven by the global thunderstorm activity with the theoretical model computations (e.g. [1–3] and reference therein). Such a problem was not solved yet in the general formulation that takes into account the anisotropy of ionospheric plasma, the difference of ambient day and night propagation conditions, a real spatial distribution of the global thunderstorms, etc. As a rule, the approach applies an extremely simplified physical model of the Earth–ionosphere cavity that allows obtaining an analytical formal solution. The objective of the present paper is an

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estimate for the error in such a solution that applies the approximate method for computing propagation parameters in comparison with the rigorous results.

2. PHYSICAL MODEL

We assume that natural ELF noise is caused by electromagnetic radiation from independent vertical lightning strokes uniformly distributed over the globe. Important for the ELF radio propagation ionosphere characteristics depend on the vertical conductivity profile $\sigma_i(r)$, which does not vary with the position of observatory. Such a model has the following shortcomings: the real lightning activity depends on the coordinates, it is concentrated in the vicinity of equator; dependence of the conductivity profile on the solar zenith angle is not taken into account, e.g., the "day–night" non-uniformity; the anisotropy is ignored of the ionosphere plasma caused by the influence of geomagnetic field.

3. MATHEMATICAL MODEL

We use in what follows the spherical polar coordinate system $\{r, \theta, \varphi\}$ with the origin at the center of the Earth and the $\theta = 0$ axis directed to the observer.

Formulas for computing vertical electric E_r and horizontal magnetic H_φ field components are the generalized form of equations (4.19) and (4.20) from paper [2]. These were derived for the ionosphere whose properties are described by the surface impedance referred to a certain "effective" waveguide height h:

$$E_r(\omega) = \frac{M_c(\omega)}{4ha^2\varepsilon_0} \frac{i\nu(\nu+1)}{\omega} \frac{P_\nu(\cos[\pi-\theta])}{\sin(\pi\nu)},$$

$$H_\varphi(\omega) = -\frac{M_c(\omega)}{4ha} \frac{P_\nu^1(\cos[\pi-\theta])}{\sin(\pi\nu)}. \tag{1}$$

Here, a is the Earth's radius; ε_0 is the permittivity of free space; $\omega = 2\pi f$ is the angular frequency; M_c (ω) is the current moment of the source (lightning stroke), it is a δ-function in the time domain, so that its spectral density is frequency-independent. Parameters h (the effective height of the ionosphere above the ground) and $\nu(\omega)$ (the dimensionless complex propagation constant) depend on the ionosphere model applied. Quite satisfactory approximate analytic solutions exist for these parameters in some models of ionosphere profiles, such as flat multi-layered impedance model or the exponential ionosphere [1,3].

Dependence of the field on the angular distance θ between the source and the observer is described by the Legendre function $P_\nu(\cos[\pi-\theta])$ of complex index ν.

Formal generalization of the equations (1) to the case of isotropic ionosphere with the conductivity $\sigma_i(r)$ depending only on the height r, is constructed by replacing the effective height of the ionosphere h by the so-called "normalizing integral" N^0 (in general complex):

$$E_r(\omega) = \frac{M_c(\omega)}{4a^2 N^0 \varepsilon_0} \frac{i\nu(\nu+1)}{\omega} \frac{P_\nu(\cos[\pi-\theta])}{\sin(\pi\nu)},$$

$$H_\varphi(\omega) = -\frac{M_c(\omega)}{4aN^0} \frac{P_\nu^1(\cos[\pi-\theta])}{\sin(\pi\nu)}. \tag{2}$$

By using procedure described in [4], we find the propagation parameters such as the eigen-value ν and the normalizing integral N^0 that appears in (2).

4. COMPUTING PROPAGATION PARAMETERS

Parameters ν and N^0 are found from the solution of the problem for the eigen-values ν of the differential operator:

$$\frac{d}{dr}\delta(r) - i\kappa\varepsilon(r)\delta^2(r) + i\kappa + \frac{\nu(\nu+1)}{i\kappa r^2 \varepsilon(r)} = 0, \tag{3}$$

where $\varepsilon(r) = 1 + \frac{\sigma_i(r)}{i\omega\varepsilon_0}$ the boundary conditions $\delta(a) = 0$ and $\delta(\infty) = 0$.

In this model, the ionosphere conductivity $\sigma_i(r)$ depending on the height r only, is a part of the expression for the relative permittivity $\varepsilon(r)$. The eigen-value ν is derived by using the iteration procedure. A proper initial approach is $\nu \approx ka$ for the zero-order propagation mode (normal wave).

5. COMPUTING THE LEGENDRE FUNCTIONS

The problem is not elementary of computing the Legendre functions $P_\nu^m(z)$ having the complex argument z, and complex indices ν and m that are used in (1) and (2). It is impossible to propose an algorithm being equally efficient for all possible combinations of parameters θ and ν. This difficulty might be recognized in particular from the fundamental work [5] where more than 100 formulas are suggested for expanding this function. A variant is given in [1,2] of the algorithm for computing the ELF fields in the spherical coordinate system, which is based on the acceleration of the zonal harmonic series representation. Another algorithm is used in what follows, which has a broader area of possible applications, in a wider frequency band in particular. The argument z is real, and it is denoted as $z = \cos\theta$ in the case, while the domain of its definition is the interval [–1; 1]; the index ν is a complex number.

- For large $|\nu\theta|$ and $|\nu(\pi-\theta)|$, the asymptotic expansion is used 3.5 (5) from [5]:

$$\Gamma\left(\nu+\frac{3}{2}\right)P_\nu^\mu(\cos\theta)=\sqrt{\frac{2}{\pi\sin\theta}}\Gamma(\nu+\mu+1)\sum_{l=0}^{\infty}(-1)^l\frac{\left(\frac{1}{2}+\mu\right)_l\left(\frac{1}{2}-\mu\right)_l}{l!(2\sin\theta)^l\left(\nu+\frac{3}{2}\right)_l}\times$$

$$\times\sin\left[\left(\nu+l+\frac{1}{2}\right)\theta+\left(\frac{\mu}{2}+\frac{1}{4}\right)\pi+\frac{1}{2}l\pi\right].$$

One can use the expansion 6.1.47 [6] when computing the ratio of two Γ - functions used in the above equation, which have the arguments deviating by the predetermined constant (–1/2 in our case):

$$z^{b-a}\frac{\Gamma(z+a)}{\Gamma(z+b)}\sim 1+\frac{(a-b)(a+b-1)}{2z}+\frac{1}{12}\left(\frac{a-b}{2}\right)\left(3(a+b-1)^2-a+b-1\right)\frac{1}{z^2}+\ldots$$

- The hypergeometric series 3.5 (9) [5] is used in the source antipode neighborhood:

$$\Gamma(1-\mu)P_\nu^\mu(\cos\theta)=\left(\operatorname{cotan}\frac{\theta}{2}\right)^\mu F\left[-\nu,\nu+1;1-\mu;\left(\sin\frac{\theta}{2}\right)^2\right],\ 0<\theta<\pi.$$

- One applies formulas 3.4 (14) [5] in the source vicinity:

$$P_\nu^\mu(-x)=P_\nu^\mu(x)\cos\left[\pi(\nu+\mu)\right]-\frac{\pi}{2}Q_\nu^\mu(x)\sin\left[\pi(\nu+\mu)\right],\quad 0<x<1,$$

and the problem is reduced to calculation of spherical functions for the angles, symmetrically arranged in respect to the equator, i.e., the angle θ is replaced by (π–θ) and the convergence is improved as a result. The increasing intricacy of computations is explained by a necessity to calculate the Q_ν functions in addition to functions P_ν. These functions are calculated by using formula 3.6.1 (11) [5]:

$$\frac{2\pi}{\sin(\nu\pi)}Q_\nu^m(z)=\frac{\pi}{\sin(\nu\pi)}P_\nu^m(z)\left[\ln\left(\frac{z+1}{z-1}\right)-2\gamma-\psi\left(\nu+m+1\right)-\psi\left(\nu-m+1\right)\right]-$$

$$-e^{im\pi}\left(\frac{z+1}{z-1}\right)^{\frac{m}{2}}\sum_{r=0}^{m-1}\Gamma(r-\nu)\Gamma\left(r+\nu+1\right)\Gamma(m-r)\frac{\cos(r\pi)}{r!}\left(\frac{1}{2}-\frac{z}{2}\right)^r-$$

$$-\left(\frac{z+1}{z-1}\right)^{\frac{m}{2}}\sum_{l=1}^{\infty}\frac{\Gamma\left(m+l-\nu\right)\Gamma\left(m+l+\nu+1\right)}{l!(m+l)!}\sigma(l)\left(\frac{1}{2}-\frac{z}{2}\right)^{m+l}-$$

$$-\frac{\Gamma(\nu+m+1)}{\Gamma(\nu-m+1)}\left(\frac{z-1}{z+1}\right)^{\frac{m}{2}}\sum_{r=0}^{\infty}\frac{\Gamma(r-\nu)\Gamma(r+\nu+1)}{r!(r+m)!}\sigma(m+r)\left(\frac{1}{2}-\frac{z}{2}\right)^r,$$

where $\sigma(l)=1+\frac{1}{2}+\cdots+\frac{1}{l},\quad \sigma(0)=0.$

This formula acquires the following form when $m = 0$:

$$Q_\nu(z)=\frac{1}{2}P_\nu(z)\left[\ln\left(\frac{z+1}{z-1}\right)-2\gamma-2\psi\left(\nu+1\right)\right]-$$

$$-\frac{1}{\pi}\sin(\nu\pi)\sum_{l=1}^{\infty}\Gamma\left(-\nu+l\right)\Gamma\left(\nu+l+1\right)\frac{\sigma(l)}{(l!)^2}\left(\frac{1}{2}-\frac{z}{2}\right)^l,$$

Simultaneously, the logarithmic singularity at the source point $\ln[(z+1)/(z-1)]$ is written explicitly in this representation. One can calculate the function $\psi(\nu+1)$ from 3.6.1(11) in [5] by using equation 6.3.18 [6]:

$$\psi(z)\sim\ln z-\frac{1}{2z}-\sum_{n=1}^{\infty}\frac{B_{2n}}{2nz^{2n}}=\ln z-\frac{1}{2z}-\frac{1}{12z^2}+\frac{1}{120z^4}-\frac{1}{252z^6}+\dots$$
$$\left(z\to\infty\ in\ \left|\arg z\right|<\pi\right).$$

6. COMPUTATION OF RESULTANT NOISE

The concept of energy spectral density is widely used in radio, which is introduced as a portion of the signal energy brought in the 1 Hz bandwidth per 1 second. In actual practice to ensure the stability of the results, the much longer accumulation times are used than 1 second, while the result obtained is divided by the accumulation period. Integration in time might be replaced by computing the modulus quadrate of the corresponding Fourier component of the signal when the length of the signal time realization is 1 second.

To compute the energy spectrum of natural noise produced by the global thunderstorm activity, one has to specify in addition to the $M_c(\omega)$ parameter (being the average the current moment of the lightning strokes) the average number of lightning strikes L occurring in a second on the whole Earth. Estimates of these values might be found in [2]: $M_c(\omega) \approx 10^5$ A m s and $L \approx 100\ \mathrm{s}^{-1}$.

Since the lightning strokes occur at uncorrelated moments of time, one has to sum the power spectra of individual strokes instead of direct summing of fields arriving from the globally distributed lightning discharges. Thus, the problem of computing the noise power is reduced to the calculation of the integrals of the functions $|E_r|^2$ and $|H_\tau|^2$ given by formula (2) over the surface of the Earth.

One can implement computing of these integrals in three different ways: to sum the zonal harmonic series; to use the quadrature formulas for numerical integration; or the Monte Carlo method. Each method has its advantages.

6.1 Monte Carlo Method of Statistical Tests

This method seems to be the most natural, because it actually emulates a realistic thunderstorm activity. It might be readily generalized to the case of the real distribution of lightning strokes in their coordinates, waveform, and amplitude.

The following sums are computed in the Monte Carlo method:

$$< E_r^2(f) > = \frac{L|M_c(\omega)|^2}{M\, 4^2 a^4 \varepsilon_0^2} \frac{|\nu(\nu+1)|^2}{\omega^2 |N^0|^2} \sum_M \left| \frac{P_\nu\left(\cos[\pi-\theta]\right)}{\sin(\pi\nu)} \right|^2, \quad (4a)$$

$$< H_\tau^2(f) > = \frac{L|M_c(\omega)|^2}{M\, 4^2 a^2 |N^0|^2} \sum_M \cos^2(\varphi) \left| \frac{P_\nu^1\left(\cos[\pi-\theta]\right)}{\sin(\pi\nu)} \right|^2, \quad (4b)$$

where $< E_r^2 >$ and $< H_\tau^2 >$ are the sought average power spectra of vertical electric and horizontal magnetic components of EM noise; θ and φ are the angular coordinates of lightning strokes relative the observation point (the drawing random variables); the $\cos\varphi$ factor accounts for the azimuth between the propagation path and orientation of the horizontal magnetic antenna core (it is used only in the magnetic field computations); M is the number of statistical tests.

The random numbers with a given distribution function are obtained in the Monte Carlo method from the pseudo-random numbers, uniformly distributed in the interval [0; 1]. These are regarded in what follows *Random*. Coordinates of the points uniformly distributed over the surface of a sphere are given by formulas: $\varphi = Random * 2\pi$ and $\theta = \arccos(2 * Random - 1)$.

One can also use a slightly modified formula when drawing the lightning strokes:

$$< E_r^2(f) >= \frac{2}{\pi}\frac{L|M_c(\omega)|^2}{M 4^2 a^4 \varepsilon_0^2}\frac{|\nu(\nu+1)|^2}{\omega^2 |N^0|^2}\sum_M \sin(\theta)\left|\frac{P_\nu\left(\cos[\pi-\theta]\right)}{\sin(\nu\pi)}\right|^2, \quad (5a)$$

$$< H_\tau^2(f) >= \frac{2}{2\pi}\frac{L|M_c(\omega)|^2}{M 4^2 a^2 |N^0|^2}\sum_M \sin(\theta)\left|\frac{P_\nu^1\left(\cos[\pi-\theta]\right)}{\sin(\nu\pi)}\right|^2, \quad (5b)$$

$\varphi = Random*2\pi$, $\theta = 2\pi*Random$. Formulas (4) and (5) deviate by the way of the coordinate θ drawing, by presence of the $\sin\theta$ factor, which accounts for varying length of parallels at different latitudes, and by the coefficient $2/\pi$ compensating the difference of the surface area on the sphere of the unit radius (4π) and a rectangle having the random θ, φ coordinates ($\pi*2\pi=2\pi^2$). The $\cos^2\varphi$ term in (5b) is replaced by 1/2 since the relationship $\sum\cos^2(2\pi*Random) = M/2$ is valid.

6.2 Direct Numerical Integration

The formulas for the numerical integration of the noise power are similar to expressions used in the Monte Carlo method, only, the summation over the random events must be replaced by integration over the surface of the sphere

When deriving the appropriate formulas, one should take into account the following: the rate of the global lightning strokes in 1 s is equal to the parameter L, therefore, the median current moment density per 1 km^2, is equal to $LM_c(\omega)/S$ where S is the total surface of the Earth $4\pi a^2$.

$$< E_r^2(f) >= \frac{L}{S}\frac{|M_c(\omega)|^2 |\nu(\nu+1)|^2}{4^2 a^4 \varepsilon_0^2 \omega^2 |N^0|^2}\oint_S \left|\frac{P_\nu\left(\cos[\pi-\theta]\right)}{\sin(\pi\nu)}\right|^2 dS,$$

$$< H_\tau^2(f) >= \frac{L}{S}\frac{|M_c(\omega)|^2}{4^2 a^2 |N^0|^2}\oint_S \cos^2(\varphi)\left|\frac{P_\nu^1\left(\cos[\pi-\theta]\right)}{\sin(\pi\nu)}\right|^2 dS. \quad (6)$$

Element of the surface is equal to $dS = a^2\sin\theta d\theta d\varphi$, and the surface integration is split into integration over the φ and the θ variables. The integral over φ is calculated using formula $\int d\varphi = 2\pi$, $\int\cos^2\varphi d\varphi = \pi$, so that expressions (6) take the form:

$$< E_r^2(f) >= \frac{L|M_c(\omega)|^2}{2a^4\varepsilon_0^2}\frac{|\nu(\nu+1)|^2}{4^2\omega^2|N^0|^2}\int_0^\pi \sin(\theta)\left|\frac{P_\nu\left(\cos[\pi-\theta]\right)}{\sin(\nu\pi)}\right|^2 d\theta, \quad (7a)$$

$$< H_{\tau}^{2}(f) >= \frac{L\left|M_{c}(\omega)\right|^{2}}{4a^{2}4^{2}\left|N^{0}\right|^{2}}\int_{0}^{\pi}\sin(\theta)\left|\frac{P_{\nu}^{1}\left(\cos[\pi-\theta]\right)}{\sin(\nu\pi)}\right|^{2}d\theta. \tag{7b}$$

The additional factor of 1/2 in the magnetic field component takes into account the integral of $\cos^2(\varphi)$).

Integrals might be computed using any quadrature formula, in particular, by the Simpson (Cotes) method. Difficulty arises when calculating the power of the magnetic noise component $<H_\varphi^2>$. It is caused by a logarithmic singularity of the Legendre function in the vicinity of the field source while its derivative with respect to the θ is included into expression for the magnetic field being equal to $1/\theta$. Since we integrate the square modulus of the field component, the magnetic integral diverges in spite of presence of the factor $\sin\theta$ in the integrand. It is clear that no infinity is met in the real records of magnetic field. The possible explanation for this fact is the shortage of the model of uniform distribution of lightning strokes over the surface of the globe. Global thunderstorms are centered mainly around the equator [1,2], and the observatories are usually placed at the middle latitudes where thunderstorms occur less frequently. Besides, fields from the local thunderstorms (closer than 100 km) are almost independent of the ionosphere properties, simultaneously they can overload the input circuits of receivers. This is why signals from the nearby lightning are excluded from the processing in real experiments.

On the other hand, equations (1) and (2) based on the zero-mode approximation become invalid when the source–observer distance is as small as a few ionospheric heights (being N^0 in our notation), i.e., closer than 100–200 km. The easiest way of avoiding this problem is eliminating the source vicinity from the integration. The same consideration is valid when using the Monte Carlo method. An expulsion from the integration procedure of the area having the 200 km radius does not reduce substantially the reliability of obtained results, as the excluded area occupies less than 0.03% of the complete Earth's surface.

6.3 Summation of Zonal Harmonic Series

The method is based on the Dougall decomposition (see 3.10 (6) in [2]) of the Legendre function into a series of the Legendre polynomials:

$$P_{\nu}^{-\mu}(\cos\theta) = \frac{\sin(\nu\pi)}{\pi}\times\sum_{n=0}^{\infty}(-1)^{n}\left(\frac{1}{\nu-n}-\frac{1}{\nu+n+1}\right)P_{n}^{-\mu}(\cos\theta),$$

$$-\pi < \theta < \pi,\ \mu \ge 0.$$

Since we are interested only in the Legendre function $P_\nu(\cos\theta)$ involved in equation (7a), we can put the upper index $\mu = 0$ in the above formula and obtain the presentation sought:

$$\frac{P_\nu(\cos\theta)}{\sin(\nu\pi)} = \frac{1}{\pi}\sum_{n=0}^{\infty}(-1)^n \frac{2n+1}{\nu(\nu+1)-n(n+1)} P_n\left(\cos\theta\right). \quad (8)$$

After substituting equation (8) into (7a) and raising the expression for $P_\nu(\cos\theta)$ to quadrate, one obtains the dual infinite series over the indices of Legendre polynomials. If we recall the orthogonality of the Legendre polynomials

$$\int_0^\pi \sin(\theta) P_n\left(\cos\theta\right) P_m\left(\cos\theta\right) d\theta = \frac{2}{2n+1}\delta_{nm}$$

we obtain after integration with respect to θ an infinite series over the matching indices n only:

$$< E_r^2(f) > = \frac{L\left|M_c(\omega)\right|^2}{2a^4\varepsilon_0^2}\frac{1}{2\pi^2}\frac{|\nu(\nu+1)|^2}{4^2\omega^2\left|N^0\right|^2}\sum_{n=0}^{\infty}\frac{2(2n+1)}{\left|n(n+1)-\nu(\nu+1)\right|^2}. \quad (9)$$

As it was already mentioned, the eigen-value of the zero-mode ν in the Schumann resonance frequency band (below 50 Hz) is approximately equal to $\approx ka$ that does not exceed 7. Therefore, the series (9) converges rather fast in the entire frequency range.

Formula (9) might be also obtained by substituting into (7a) the representation for the product of two Legendre functions 3.10 (8) [2]:

$$P_\nu^{-\mu}\left(\cos\theta\right)P_\nu^{-\lambda}\left(\cos\theta'\right) = \frac{\sin(\nu\pi)}{\pi}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{\nu-n}-\frac{1}{\nu+n+1}\right)P_n^{-\mu}\left(\cos\theta\right)P_n^{-\lambda}\left(\cos\theta'\right),$$

$$-\pi < \theta+\theta' < \pi, -\pi < \theta-\theta' < \pi,\ \mu \ge 0, \lambda \ge 0,$$

and setting $\mu = 0$, $\lambda = 0$, and $\theta' = \theta$.

Summation of the zonal harmonics representation (9) is possible only for the power of vertical electric field component of natural noise. Similar series is divergent for the energy spectrum of the horizontal magnetic field component.

7. COMPARISON OF DIFFERENT METHODS

We must acknowledge that results obtained in the three different ways may slightly deviate when compared with each other. One must also specify the desired accuracy of computations in every algorithm, and the time of computations will be inversely proportional to the given precision:

• The accuracy of computations by the Monte Carlo method is inversely proportional to the number of statistical tests M; this is a rather weak convergence: usually, it takes a few tens or hundreds of thousands of drawings.

• When implementing formulas (7), the accuracy of computations depends on the particular method of numerical integration: for the most commonly used Simpson method, it is proportional to the fourth power of the integration step s^4.

• General term of zonal harmonics series (9) that determines accuracy of computations is inversely proportional to the third power of numbers n^3, which guaranties the rapid convergence.

The computational results obtained by the Monte Carlo method (4), (5) and of the numerical integration (7) depend both on the accuracy of the Legendre functions computations and on the size of the source area, which is excluded from the integration.

The results obtained by summing the zonal harmonic series should be considered as the most reliable from the most general considerations. However, these imply computations of the noise power in the vertical electric field component and only in the model of thunderstorms uniformly distributed over the globe. The rest of methods, in spite of their greater complexity, are deprived of this limitations.

When comparing spectral data, we used the ionosphere conductivity profile $\sigma_i(h)$ shown in Fig. 1.

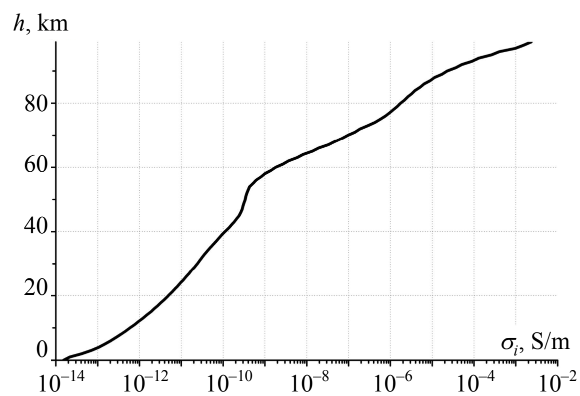

FIG. 1: Profile of the ionosphere conductivity

Figures 2 and 3 present energy spectra (in arbitrary units) of the vertical electric and horizontal magnetic field components.

The plots do not depart qualitatively when computed by using different methods, therefore we show only single line that corresponds to one of the methods. In implementation of these methods, we specified the following parameters:

– The relative accuracy of computations of Legendre functions is 10^{-5}.

– The accuracy of the summation of zonal harmonics series is 10^{-5}.

– The number of integration steps in the method of Simpson is 400.

– The radius of the area excluded from the integration in the vicinity of the source is 200 km.

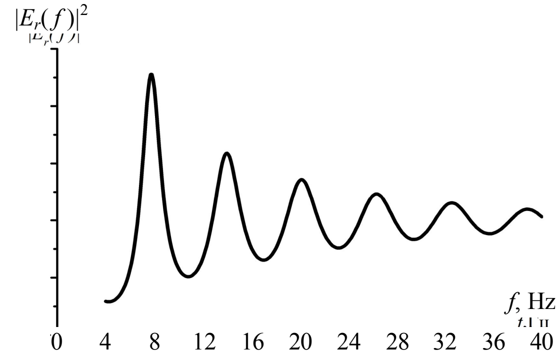

FIG. 2: Power spectrum of vertical electric field components

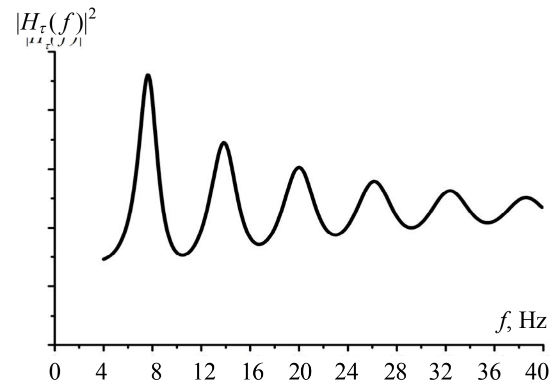

FIG. 3: Power spectrum of horizontal magnetic field component

Impact of the drawing number in the Monte Carlo method on the computation accuracy is demonstrated in Figs. 4 and 5. The bold line here presents the result of direct integration; dashed lines show three variants of Monte Carlo computations for the drawing number of 10.000; solid line is the Monte Carlo method with 100.000 events.

It is obvious that in spite of the relatively casual appearance of the results obtained by the Monte Carlo method, the output accuracy is increasing with the number of tests, although the relevant growth is slow, but sufficient. The computation accuracy in the magnetic component of ELF noise power is lower than in the electric component. This apparently might be explained by the fact that this field is expressed via derivative of the Legendre function having a stronger singularity at the source.

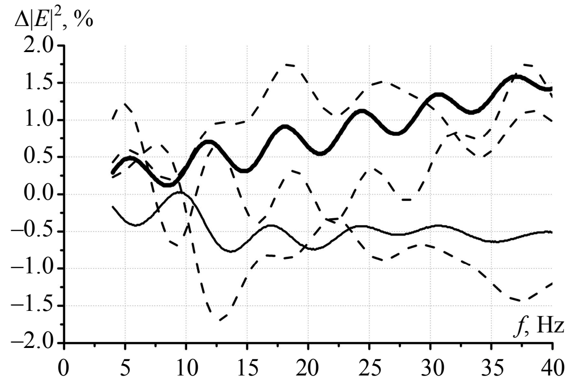

FIG. 4: Accuracy of the power spectrum computations of the vertical electric field components $|E_r(f)|^2$ (relative the summation of zonal harmonic series)

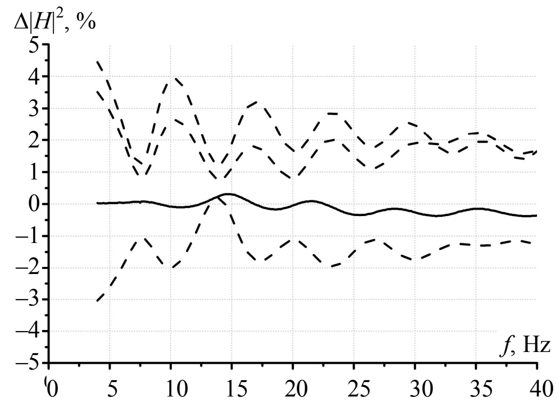

FIG. 5: Accuracy of the power spectrum computations of the horizontal magnetic field component $|H_\tau(f)|^2$ (relative the direct integration)

8. CONCLUSIONS

We have demonstrated that application of modern computer technology and rigorous numerical methods allows calculating the propagation characteristics and the fields, which are no more intricate in realization than the approximate analytical methods. A further improvement in correspondence between the theory and experiment might be expected in the framework of more complicated propagation models accounting for the ionosphere irregularity “day-night” and the anisotropy. The method of two-dimensional telegraph equation [7] is among the most promising methods for solving such complicated problems.

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