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Knee model: Comparison between heuristic and rigorous solutions for the Schumann resonance problem

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ABSTRACT

Rapid development of computers allows for application of the direct numerical solution of the global electromagnetic resonance problem in the Earth-ionosphere cavity. Direct numerical solutions exploit the cavity models with the given conductivity profile of atmosphere such as exponential or the knee profiles. These profiles are usually derived from the knee model by Mushtak and Williams (2002) developed for obtaining the realistic ELF propagation constant. It is usually forgotten that profiles of the knee model are only a convenient approximate interpretation for the heuristic relations used in computations. We demonstrate that the rigorous full wave solution of the electromagnetic problem for such profiles deviates from that obtained in the knee model. Therefore the direct numerical solutions must also depart from the heuristic one. We evaluate deviations of the heuristic knee model data from those pertinent to equivalent profile of atmospheric conductivity.

1. Introduction

Owing to rapid development of computing resources, the direct modeling of radio propagation in the Earth–ionosphere cavity is widely used in the studies of global electromagnetic resonance (e.g. Kirillov, 1993, 1996, 1998; Kirillov et al., 1997; Kirillov and Kopeykin, 2002; Hayakawa and Otsuyama, 2002; Otsuyama et al., 2003; Pechony and Price, 2004; Pechony, 2007; Ando et al., 2005; Morente et al., 2003; Yang and Pasko, 2005, 2007; Yang et al., 2006; Molina-Cuberos et al., 2006; Toledo-Redondo et al., 2010, 2013). The Maxwell's equations are solved numerically in this case. Typically, modern versions are used of the two-dimensional transmission line and relevant telegraph equations (regarded as 2DTU) or the grid methods. The most popular among the latter is the Finite Difference in Time Domain (FDTD) technique. A direct numerical solution demands enormous amount of operations, but its crucial advantage is in the flexibility. It allows for numerical simulation of the real Earth–ionosphere cavity with all its features, including the global ionospheric irregularities such as polar non-uniformity or the day–night asymmetry.

The FDTD technique became especially popular, so much that it is included into the MATLAB software. Typically, the electrical properties of ionospheric plasma are described by either an exponential vertical conductivity profile or a bended-knee profile (bended-knee model). These models were developed to obtain a realistic frequency dependence of the ELF propagation constant $\nu(f)$. The propagation constant is used in the classical solution of the Schumann resonance problem in the form of zonal harmonics series representation (ZHSR) (e.g. Nickolaenko and Hayakawa, 2002, 2014). It turns out that the FDTD solution deviates from that obtained by ZHSR in the knee model in spite of the application of the same field source and the "same" vertical conductivity profile $\sigma(h)$.

The ELF radio propagation in a uniform Earth–ionosphere cavity has been investigated for a long time. Field representations in the form of zonal harmonic series are given in the literature for both the frequency and the time domains (see e.g. Nickolaenko and Hayakawa, 2002, 2014). Knowledge of the complex radio propagation constant $\nu(f)$ is sufficient for computing the Schumann resonance fields. The $\nu(f)$ function is determined by the global properties of the lower ionosphere. However, owing to poor knowledge of the plasma parameters at altitudes from 40 to 100 km, the propagation constant cannot be derived from the ionospheric data, and the heuristic $\nu(f)$ models are used instead. The majority of these is based on the experimental observations. Ishaq and Jones (1977) suggested the most reliable model, which is a nonlinear function of frequency. There are also other models

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(Nickolaenko and Hayakawa, 2002, 2014). The formal solution of the inverse electromagnetic problem is still due: finding the profile of atmospheric conductivity $\sigma(h)$ from the $\nu(f)$ dependence based on the Schumann resonance records.

Knowledge of the vertical profile of atmosphere conductivity is necessary for investigating the possible electrical activity on other planets. Electric activity might be a source for the global electromagnetic resonance, and therefore an estimate is desirable of the expected resonant frequencies for the known parameters of planetary atmosphere (see Nickolaenko and Rabinowicz, 1982, 1987; Sentman, 1990a; Pechony and Price, 2004; Yang et al., 2006; Molina-Cuberos et al., 2006 and references therein). The height profile also becomes necessary when one wants to evaluate the impact of various disturbances in the lower ionosphere on the $\nu(f)$ dependence and, consequently, on the resonance pattern. These problems are related to the seismic activity and the space weather (Nickolaenko and Hayakawa, 2002, 2014).

Determining the $\nu(f)$ function is difficult, which is relevant to a particular profile $\sigma(h)$. We disregard the cases when unrealistic conductivity profiles allow for formal solving the electromagnetic problem in certain special functions: these cases were listed in Bliokh et al. (1980). The horizontally uniform layered atmosphere is assumed in the general formulation of the problem to be an isotropic medium, so that the $\sigma(h)$ profile is introduced as a set of thin vertically uniform layers. The rigorous solution of such a problem is multi-parametric. It is necessary to write the solutions of the wave equation for each of N layers and satisfy the $2N$ boundary conditions. Thus, a system of $2N$ linear equations appears for the wave transition and reflection coefficients at each layer (Wait, 1970). This problem can be reformulated to the first order differential equation for the surface impedance of the field. The equation itself becomes nonlinear, and the solution may be built only numerically. The approach is well known, which is regarded as the Full Wave Solution (FWS) (see Wait, 1970; Hynninen and Galuk, 1972; Bliokh et al., 1977). Application of FWS is associated with performing time-consuming calculations, although less massive as in the FDTD technique.

Greifinger and Greifinger (1978) suggested the approximate expressions for the propagation constant ν at a fixed ELF frequency in terms of the exponential conductivity profile $\sigma(h)$. The exponential conductivity profiles were used in the very low frequency band (VLF, 3–30 kHz) long before the Greifinger's publication (see e.g. Wait and Spies, 1964). However, as Greifinger and Greifinger (1978) have demonstrated, application of an exponential profile is different when we turn to the extremely low frequencies (ELF, 3–3000 Hz). They proposed approximate relations for computing the complex propagation constant ν involving the characteristic "electric" and "magnetic" heights together with the relevant height scales. The lower characteristic (electric) height is derived at the given frequency from the equality of the conductivity and the displacement currents. At the fixed frequency f the height h_E corresponds to the condition:

$$\sigma(h_E) = \sigma_E = 2\pi \cdot f \cdot \varepsilon_0, \tag{1}$$

where $\omega = 2\pi \cdot f$ is the circular frequency and $\varepsilon_0 = 8.859 \cdot 10^{-12}$ F/m is the permittivity of vacuum.

After finding the electric height, one turns to the upper, magnetic height h_M, where the wavelength in the plasma at the given frequency is equal to the local scale height ζ_M:

$$\sigma(h_M) = \sigma_M = [4\mu_0 \cdot \omega \cdot {\zeta_M}^2]^{-1}, \tag{2}$$

here μ_0 is the permeability of vacuum and ζ_M is the scale height of profile in the vicinity of upper characteristic height.

With the help of two characteristic heights h_E and h_M and two height scales ζ_E and ζ_M of the classical profile by Cole and Pierce (1965) the values of the propagation constant were obtained by the measurements of ELF radio signals transmitted by the Wisconsin Test Facility. Thus the Greifinger and Greifinger (1978) model proved to be convenient and rather efficient. This is why a desire emerged to adapt it to the calculation of Schumann resonance parameters. However, there were two obstacles to overcome. The first was the fact that formulas by Greifinger and Greifinger (1978) were obtained for the flat Earth–ionosphere duct. The global resonance is possible only in a spherical cavity and at the particular frequencies when the radio waves have traveled around the planet meet in phase. Thus, spherical geometry is a requisite feature. This first obstacle was overcome by demonstrating that the formulas derived in the flat cavity are also held in the spherical geometry, provided that the signal frequency exceeds a few hertz.

The second problem is that unlike the ELF radio transmissions, the natural signals of global electromagnetic resonance cover a broad band approximately a decade. So, the second obstacle was a frustrating prospect of multiple $\sigma(h)$ plots for different values of signal frequency for finding graphically the new characteristic heights and height scales. This difficulty was overcome by deriving formulas for the electric and magnetic height as functions of frequency. For this purpose the reference height and reference frequency were introduced. All this has been done in the works by Nickolaenko and Rabinowicz (1982, 1987) devoted to estimates of feasible global resonance on other planets of Solar system. The Earth-ionosphere cavity acted in these papers as a test for assessing the accuracy of the approach. Later, similar formulas were published by Sentman (1990a, b) and Fullekrug (2000).

Advantage of approximate solutions for the propagation constant relevant to the exponential conductivity profile does not lie only in its simplicity. In addition, the approach allows for reasonable interpretation of observations in terms of rather realistic parameters of the lower ionosphere.

Further development of the approach was associated with elaboration of more sophisticated model profiles. In particular, the knee $\sigma(h)$ profiles were suggested with a bend (or kink) at an altitude between 50 and 60 km. This is the region where the conductivity switches from ionic conductivity dominating below ~50 km to a more rapidly varying electronic conductivity dominating above ~60 km (see Kirillov, 1993, 1996, 1998; Kirillov et al., 1997; Kirillov and Kopeykin, 2002; Mushtak and Williams, 2002; Pechony and Price 2004; Pechony 2007; Greifinger et al., 2007). In these works a method was suggested for determining the $\nu(f)$ propagation constant. An alternative is obtaining the effective R, L, C parameters of the cells in artificial transmission line used in the two-dimensional telegraph equations (2DTU).

A set of heuristic knee models was suggested by Pechony and Price (2004) and Pechony (2007). However, it was not emphasized that such an efficient and rather convenient approach only approximately matches the results obtained in the rigorous solution for the actual conductivity profile $\sigma(h)$. In other words, if we use the real parts of complex characteristic heights of the knee model together with the relevant scale heights for constructing the real function $\sigma(h)$ and find the complex propagation constant $\nu(f)$ from the rigorous full wave solution, the result will deviate from that based on the knee model equations. These deviations were demonstrated for the exponential profile by Jones and Knott (1999, 2003). In order to do this, the expected resonance frequencies and the Q-factors were estimated. It has been shown that the results of the exponential model deviate from the FWS for the Schumann resonance. That is, the resonant frequencies remained almost unchanged (deviations ranged between 0.15% and 1.2%), while the Q-factors or the wave attenuation departed by more than 10%.

Similarly to the exponential profile, the popular knee models remain a convenient procedure for obtaining the heuristic $\nu(f)$

dependence. They also must lead to departures in the wave attenuation rate from the rigorous values based on the FWS. In this paper, we use the knee model by Mushtak and Williams (2002) and check its correspondence to the rigorous FWS. This discussion is opportune, since exponential and knee profiles are usually applied in the FDTD technique.

2. Knee profiles

We consider three model profiles and two heuristic models of propagation constant. Fig. 1 shows these conductivity profiles. Traditionally, the height h being the argument of $\sigma(h)$ function is plotted on the ordinate in kilometers and the abscissa depicts the logarithm of the air conductivity (S/m).

Profile 1 (the solid thick line) in Fig. 1 presents the altitude profile introduced by Cole and Pierce (1965); it is often used when demonstrating the conductivity of regular ionosphere below 100 km. Profile 2 (the line with asterisks)is the knee profile by Mushtak and Williams (2002). The profile combines two exponential height factions $\sigma(h)=\exp(-h/\zeta)$ in its lower part. These lines intersect at the knee height $h_{KNEE}=55$ km. Since the knee reference frequency is $f_{KNEE}=10$ Hz (see Table 1), the air conductivity at the knee height is readily found from Eq. (1) being equal to $\sigma_E=5.5663\cdot10^{-10}$ S/m. The scale height is equal to $\zeta_b=8.3$ km below the knee, and it is $\zeta_a=2.9$ km above the knee altitude. The characteristic electric height h_E is found in the knee area, and it is used in the heuristic formulas for deriving the ELF propagation constant $\nu(f)$. The upper, or the magnetic height h_M was postulated to be 96.5 km (Mushtak and Williams, 2002). Here, the air conductivity is $\sigma_M=0.9895$ S/m is found from Eq. (2) for the magnetic reference frequency $f_m^*=8$ Hz, and the height scale is $\zeta_M=4$ km. Parameters of the knee profile are listed in Table 1. We call the part above the kink the magnetic, and below, the electric part of the profile.

When the logarithmic scale is used along the ordinate, the profile 2 is a piecewise-linear line with two kinks. Its lower section of profile 2 forms the knee, while its upper part must pass through the magnetic conductivity σ_M at the 96.5 km height and have the tilt corresponding to "magnetic" scale height of 4 km. As Fig. 2 shows, the straight line drawn from this "magnetic" point intersects with the upward going "electric" part of profile at around 83 km. We use the profile with 'magnetic' parameters above the 83 km altitude, and the 'electric' parameters below it.

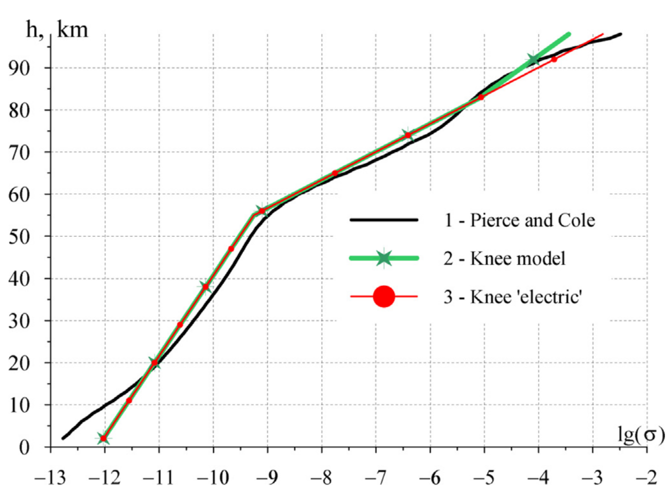

Fig. 1. Conductivity profiles of atmosphere used in calculations of propagation constant. The smooth curve 1 shows the profile by Cole and Pierce (1965) being the regular altitude variations of the air conductivity, the broken line 2 is the knee profile by Mushtak and Williams (2002), and line 3 is the "electric" part of the knee profile, which is often used in the FDTD simulations.

Table 1
Parameters of the knee profile.

Knee frequency f_{KNEE} Hz	10
Knee height h_{KNEE} km	55
Scale height above the knee ζ_a km	2.9
Scale height below knee ζ_b km	8.3
Magnetic reference height h_m^* km	96.5
Magnetic reference frequency f_m^* Hz	8
Magnetic scale height at reference frequency ζ_m^* km	4
Reference frequency of magnetic scale height f_m^* Hz	8
Parameter of frequency dependence of magnetic scale height b_m km	20

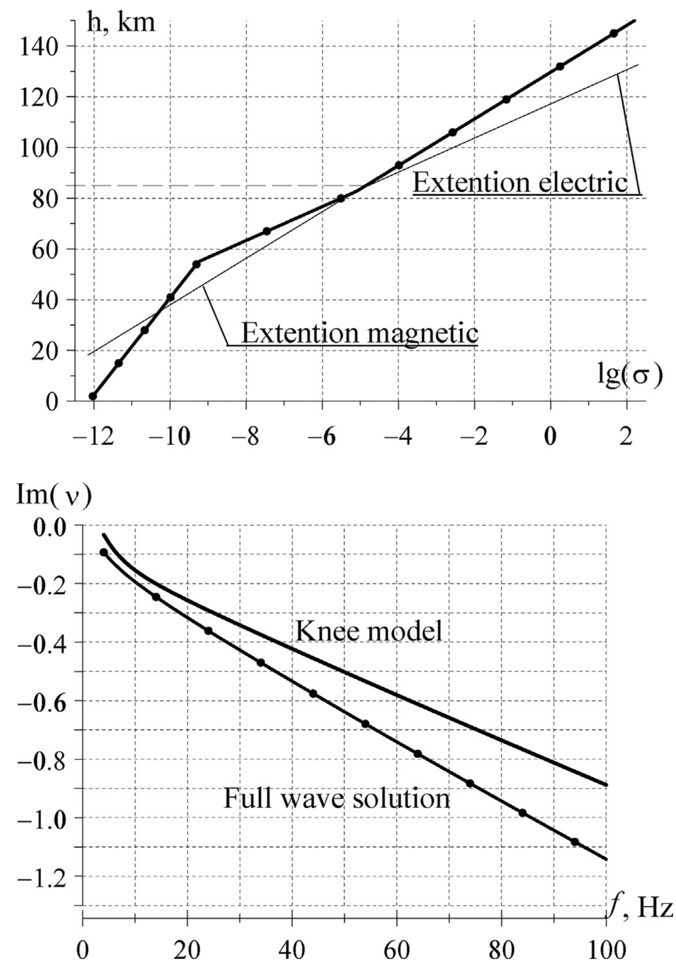

Fig. 2. Model profile and the attenuation rate. Upper frame shows the conductivity profile relevant to the knee model by Mushtak and Williams (2002). The lower frame shows the attenuation rate: the smooth black line depicts the heuristic knee model data, and line with dots is the full wave solution for the knee conductivity profile.

The piecewise-linear line with circles 3 in Fig. 1 is a simplified profile that was built exclusively on the knee parameters. It is formed by two straight lines of different slope, which intersect at the knee altitude. Profiles 2 and 3 of Fig. 1 coincide below the 83 km, while Profile 3 continues with the electric scale height of 2.9 km above 83 km. Therefore, it is found somewhat below the Profile 2 linked to the magnetic height and having the magnetic scale height of 4 km.

It is important to note that in contrast to the smooth classical plot 1 reflecting the aeronomy data, the profiles 2 and 3 correspond to a convenient interpretation of the heuristic formulas for the ELF propagation constant. This becomes especially clear if we recall that all parameters of the knee model depend on the frequency. Moreover, the electric parameters are based on the reference frequency of 10 Hz, while the magnetic characteristics refer to the reference frequency of 8 Hz.

3. Rigorous solution of the problem

In a rigorous treatment of ELF radio propagation in a horizontally stratified medium, one obtains a system of linear algebraic equations for the wave transition and reflection coefficients in the adjacent layers. The problem formulation in terms of the surface impedance is more convenient than a direct application of boundary conditions for the horizontal electric and magnetic field components at each boundary. Thus, the problem is reduced to a first-order differential equation (Wait, 1970), the solution of which is constructed numerically. Description of particular procedures and the results obtained for a realistic conductivity profile can be found in the papers by Hynninen and Galuk (1972), Bliokh et al. (1977), and Galuk and Ivanov (1978). We only outline the relevant process below.

One can obtain the following first order nonlinear equation for the spherical surface impedance $\delta(r)$ when treating the eigen-value problem in the Earth–ionosphere cavity formed by the stratified plasma of the a complex dielectric constant $\varepsilon(r) = 1 + i\frac{\sigma(r)}{\omega\varepsilon_0}$:

$$\frac{d}{dr}\delta(r) - i\kappa\varepsilon(r)\delta^2(r) + i\kappa + \frac{\nu(\nu+1)}{i\kappa r^2\varepsilon(r)} = 0. \tag{3}$$

Here the exp $(+i\omega t)$ time dependence is assumed, $\delta(r)$ is the spherical surface impedance at the interfaces of the adjacent layers of ionospheric plasma; ν is the complex eigen-value (the propagation constant) sought; κ is the wave number $\kappa=\omega/c$; c is the light velocity in vacuum; r is the radius of the spherical polar coordinate system (r, θ, φ); $\varepsilon(r)$ is the dielectric constant of the plasma, a function of the height.

The radial operator is defined on the semi-infinite interval $[a; \infty)$, it has the sought eigen-values $\lambda=\nu(\nu+1)$. The boundary conditions for the surface impedance must be formulated in addition to Eq. (3).

The surface impedance (3) depends on the radius r (altitude above the ground), and it is determined by the conductivity of the soil at the air-land interface. The Earth might be considered as perfectly conducting at ELF, so that the first boundary condition takes the form $\delta(a)=0$, where a is the Earth's radius. Since radio waves of a given frequency ω strongly attenuate with the altitude inside the ionospheric, the plasma properties above a few skin-depths provide no impact on the result obtained.

The penetration depth of ELF radio wave into plasma depends on frequency ω, but it never exceeds the 100 km altitude. Therefore, the plasma was regarded as uniform along the radius from the 100 km height: i.e., we used $\varepsilon(r)=const$ when $r \geq r_1=100$ km. At the 100 km boundary we use the surface impedance of the uniform highly conducting plasma, so that the second boundary condition takes the form $\delta(r_1)=[\varepsilon(r_1)]^{-1/2}$, where $|\varepsilon(r_1)| \gg 1$.

In fact, the complex eigen-value problem is reduced to finding the parameter λ from the nonlinear equation $\delta(a, \lambda)=0$. The function $\delta(a, \lambda)$ is obtained by numerical integration of Eq. (3) from the height r_1 to $r=a$. The function $\delta(a, \lambda)$ is analytic with respect to the parameter λ, so that its roots can be found by iterations or by the Newton's method. Let λ^l be the l order iteration to the sought eigen-value λ, then the $(l+1)$-th iteration in accordance with the Newton's method is:

$$\lambda^{l+1} = \lambda^l - \frac{\delta(a;\lambda^l)}{\frac{\partial}{\partial\lambda}\delta(a;\lambda^l)} \tag{4}$$

After obtaining the λ^l iteration, the integration of Eq. (3) is repeated with the new eigen-value thus providing the next, λ^{l+1} iteration. The process is repeated until the old and new eigen-values deviate by less than 10^{-7}. The derivative $\frac{\partial}{\partial\lambda}\delta(a;\lambda)$ is obtained by integrating the differential Eq. (3) for $\delta_1(r) \equiv \frac{\partial}{\partial\lambda}\delta(r,\lambda)$ together with the $\delta(r)$ function. The equation for this surface impedance function and the boundary value at $r=r_1$ is obtained by differentiating Eq. (3) in respect to parameter λ:

$$\frac{d}{dr}\delta_1(r) - 2i\kappa\varepsilon(r)\delta(r)\delta_1(r) + \frac{1}{i\kappa r^2\varepsilon(r)} = 0 \tag{5}$$

and

$$\delta_1(r_1) = 0 \tag{6}$$

The height r_1 was chosen by exhaustive search: solutions were construed for several r_1 values, and the value of 100 km was chosen since the variation of the eigen-value did not exceed the 10^{-7} level when r_1 was set above this height. This method allows finding both the zero mode (the propagating mode) eigen-value and also solutions for the localized waves trapped in the Earth–ionosphere duct or the transverse resonance. The Newton's procedure converges to the root that is closest to the initial approximation. Therefore, we eliminated the already found root with the help of Bézout's theorem: instead of searching the roots of the initial function $\varphi(\lambda)=0$, we use the auxiliary function $\phi_n = \frac{\phi}{\Pi_{i=1}^{n}(\lambda-\lambda_i)}$, where λ_i denote the already found roots of the function φ, and n is the number of these roots. It is easy to see that the φ_n function has the same roots as the original function φ, except for the already known first n roots.

When calculating the field components, one has to derive both the eigen-values and the so-called integral norm N^0. Physical meaning of this quantity is clear from the relation $N^0 = \int_a^\infty E_r(r)dr$ where a is the Earth's radius (Kirillov, 1993, 1996, 1998, Kirillov et al., 1997, Kirillov and Kopeykin, 2002, Galuk and Ivanov, 1978). This norm does not require additional computations in our scheme, since

$$N^0 = ika^2\frac{\partial}{\partial\lambda}\delta(a;\lambda) = ika^2\delta_1(a). \tag{7}$$

4. The ELF propagation constant

The altitude profile of atmospheric conductivity $\sigma(h)$ does not depend on the radio wave frequency. The knee profile, as well as the exponential models that are used in computations of ELF propagation constant are nothing else, but a convenient interpretation for the heuristic dependence $\nu(f)$. It would be naive to expect that the approximate equations of Mushtak and Williams (2002) provide the same complex ELF propagation constant as the rigorous FWS of the electrodynamics problem, even when one applies the same profile, say, the profile 2 of Fig. 1.

Fig. 2 shows the elements of a solution to the problem we treat. The upper frame shows the vertical changes of atmosphere conductivity from 0 km to 150 km, this is the knee profile relevant to paper by Mushtak and Williams (2002). The kink is clearly visible in the altitude dependence around the knee height of 55 km. The real part of the complex electric characteristic height introduced by Mushtak and Williams (2002) is located in its vicinity for the Schumann resonance frequencies. The upper part of the profile is associated with the magnetic characteristic height of 96.5 km. The scale height around magnetic altitude is different from that at the electric, therefore we observe an opposite kink around 83 km. To make it more vivid, we added the thin lines extending the 'electric' and 'magnetic' dependence.

The lower panel in Fig. 2 shows the frequency dependence of the wave attenuation rate Im[$\nu(f)$]. The abscissa shows the frequency in Hz. Curve 2 was calculated by using formulas from Mushtak and Williams (2002). The knee model (see Table 1) gives the following complex electric and magnetic heights h_E and h_M:

$$h_E(f) = h_{KNEE} + \zeta_a ln(f/f_{KNEE}) + ln[1 + (f_{KNEE}/f)^2]$$
$$(\zeta_a - \zeta_b)/2 + i[\zeta_a \pi/2 - (\zeta_a - \zeta_b)\, tan^{-1}(f_{KNEE}/f)], \quad (8)$$

and

$$h_M(f) = h_m^* - \zeta_m ln(f/f_m^*) - i\zeta_m(f)\pi/2 \quad (9)$$

Here h_{KNEE} and f_{KNEE} denote the knee height and knee frequency correspondingly. The coordinates of the lower kink of line 1 in Fig. 2 satisfy Eq. (1). We remind that the conductivity profile has a kink at 55 km owing to different scale heights ζ_a and ζ_b above and below the knee altitude.

In the vicinity of the magnetic height of 96.5 km, the profile depends on the parameters h_m^*, f_m^*, ζ_m^*, and b_m. The scale height ζ_m is the function of frequency:

$$\zeta_M = \zeta_m^* + b_m(1/f - 1/f_m^*), \quad (10)$$

The profile is described by a single exponential function around the magnetic characteristic height, which is shown in the upper panel of Fig. 2 by the thin line running down from the point h_M=96.5 km; σ_M=0.9895 S/m with the ζ_M=4 km tilt. The upward going thin line is the continuation of the electric part of the profile above the knee. The propagation constant $\nu(f)$ is obtained from the regular expression in the heuristic knee model:

$$\nu(\nu + 1) = (ka)^2 h_M/h_E, \quad (11)$$

or from this equation

$$\nu(f) = [1/4 + (ka)^2 h_M/h_E]^{1/2} - 1/2 \quad (12)$$

When the time dependence $\sim\exp(+i\omega t)$ is used, the positive sign of the root is chosen, which guaranties the attenuation of propagating radio waves. Concluding, we should remind that the parameters involved in formulas (8)–(12) were selected by Mushtak and Williams (2002) to match the Schumann resonance observations.

We use the knee profile that was introduced by Mushtak and Williams (2002) model. Therefore, one may hope in the idealistic case that computations by Eq. (8)–(12) give the data coincident with the rigorous FWS of the electrodynamics problem (1)–(3). Unfortunately, the lower panel in Fig. 2 indicates that this is not so. The real parts of propagation constant are almost coincident when found by the heuristic formulas and from the rigorous solution. However, the imaginary parts (wave attenuation) are different. Attenuation rate of the exact solution is higher than that obtained from the formulas (8)–(12). Jones and Knott (1999, 2003) reported a similar result, but it was relevant to the exponential profile of Greifinger and Greifinger (1978).

5. Model Schumann resonance spectra

The propagation constants $\nu(f)$ and the ZHSR allow calculating the resonance fields for an arbitrary position of the observer and the field source. To eliminate influence of the source–observer distance on the resonance pattern, we apply the globally uniform spatial distribution of lightning strokes (Bliokh et al., 1980, Nickolaenko and Hayakawa, 2002, 2014, Williams et al., 2006). This means that independent random vertical lightning strokes occur with the same probability at any point of the globe: $w(\theta,\varphi)=1/4\pi$.

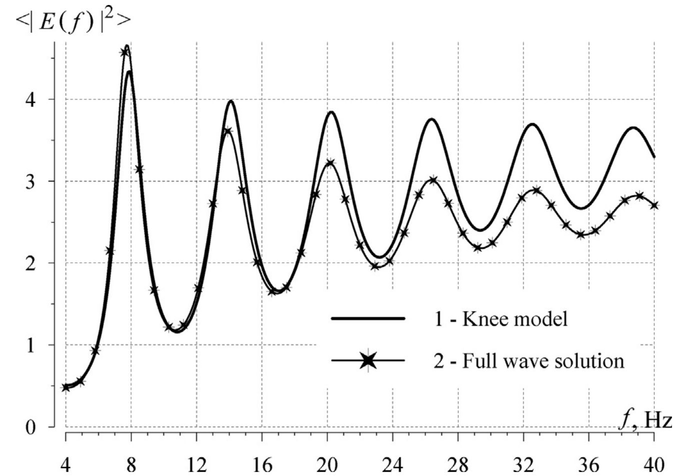

Fig. 3. Power spectra of the vertical electric field for the uniform global distribution of thunderstorms over the Earth. Curve 1 is the solution obtained from the heuristic formulas by Mushtak and Williams (2002) and curve 2 corresponds to the full wave solution.

The thunderstorms form a random succession of pulses at an observatory. The delay of the arrival times is supposed to have an exponential distribution. In this case we have a Poisson random process, and the mutual interference of pulses disappears in the averaged power spectrum: the individual power spectra of pulses are summed (Bliokh et al., 1980, Nickolaenko and Hayakawa, 2002, 2014, Williams et al., 2006). The resulting power spectrum of the vertical electric field is:

$$<|E(f)|^2> \propto <|M_C|^2> \left|\frac{\nu(\nu+1)}{2\pi f}\right|^2 \sum_{n=0}^{\infty} \frac{2n+1}{|n(n+1) - \nu(\nu+1)|^2}. \quad (13)$$

Here $<|M_C|^2>$ is the average current moment of vertical lightning strokes supposed to be independent of frequency (white source), n=0, 1, 2, 3… is the Schumann resonance mode number. The series in Eq. (13) was obtained by integrating the electric field power with the uniform probability density of the source coordinates $w(\theta,\varphi)=1/4\pi$ (see Bliokh et al., 1980, Nickolaenko and Hayakawa, 2002, 2014 for details). It converges rather fast, and the computations are simple.

We show in Fig. 3 the power spectra in relative units of the vertical electric field calculated for the uniform distribution of thunderstorms and the ionospheric conductivity profiles described by the knee model. We used in computations the current moment of the vertical dipole source $<|M_C|^2>$=const=1. Spectrum 1 in Fig. 3 was obtained for the rigorous propagation constant, while spectrum 2 was computed by using heuristic Eqs. (8)–(12).

Obviously, both spectra are close to each other, but there are clear distinctions. The height and the width of spectral maxima are different, especially of the higher resonance modes. The difference of resonance power at frequencies around 40 Hz reach 20–30%, and these arise from deviations in the wave attenuation (curves 2 and 3 in Fig. 2). Departure is small, but it can play a significant role when the spectra of the heuristic model are applied in the interpretations of experimental records. For example, if we try to evaluate the spectrum of the source current moment via dividing the observed amplitude spectrum by the heuristic model spectra of Schumann resonance, the estimated amplitude of the source current moments $M_C(f)$ decreases with the frequency: the source spectrum turns from the "white" into a "red" one. Concurrently, the magnitude of the source current moment is underestimated. The spectral departures arise due to deviations in the propagation constant, specifically, in the wave attenuation. This effect will become more pronounced in the spectra of ELF pulses arriving from the distant strokes. Therefore, application of spectra based on the heuristic model in the solution of the inverse problem (Shvets et al., 2010, Shvets and Hayakawa, 2011) will reduce the source distances in comparison with the actual values. The inaccuracy will deform the global distribution of thunderstorms when it is obtained by the ELF radio tomography based on the Schumann resonance data.

6. Discussion

The heuristic knee model was elaborated for an adequate description of Schumann resonance observations. However, the rigorous solution based on the equivalent vertical profile of atmospheric conductivity might feebly correspond to experimental data. We showed that deviations arise from the approximated character of the heuristic model.

As can be seen from Fig. 2, rigorous and the heuristic attenuation factors noticeably depart in the Schumann resonance band. This result remains valid for any knee model and the relevant profile. Deviations are small in the real part of the propagation constant, or in the phase velocity of radio waves. Therefore, all the models more or less appropriately describe the observed peak frequencies of the Schumann resonance spectra, as it was noted by Jones and Knott (1999, 2003). Deviations are more distinct in the imaginary part of the propagation constant, or in the attenuation factor.

We must remind that the original knee model by Mushtak and Williams (2002) has never operated with the actual conductivity profile of the air. Instead, they introduced the optimal parameters (8)–(10) being the functions of frequency: the complex electric $h_E(f)$ and magnetic $h_M(f)$ heights together with the electric and magnetic scale heights $\zeta_E(f)$ and $\zeta_M(f)$. These four parameters were tuned to match the Schumann resonance data in the best possible way (Williams et al., 2006). The choice is a delicate task, since the original profile $\sigma(h)$ is independent of frequency. However, the success in finding the optimal h_E, h_M, ζ_E, and ζ_M variables does not mean that the results will be coincident both in the rigorous and the heuristic solutions for the $\nu(f)$ dependence.

The noted deviations in the power spectra might result in uncertainty when performing computations of Schumann resonance by direct methods. Let us imagine that someone has constructed the conductivity profile based on the knee model and applied it in the direct computations. Of course, the results obtained should be compared with the original knee model data, and the comparison will show small, but distinct deviations. It is clear now that it is useless to look for a bug in the direct computation algorithm: departures arise from the approximate character of the heuristic knee model.

From the other hand, errors in the wave attenuation reduce accuracy of the inverse problem solution (Shvets et al., 2010, Shvets and Hayakawa, 2011). Obviously, underestimated wave attenuation will result in the overrated level of global thunderstorm activity when derived from the experimental records. In case such a model solution is used in the tomographic reconstruction of global lightning activity, the amount of distant strokes will be overestimated, etc.

7. Conclusion

The knee model by Mushtak and Williams (2002), similarly to the exponential model by Greifinger and Greifinger (1978) does not provide the actual height dependence of the air conductivity: it is just a convenient instrument for calculating the appropriate frequency dependence $\nu(f)$. This is why the relevant profiles should be applied with caution when including them into direct numerical algorithms for computing Schumann resonance.

The spectral data computed in the knee model plausibly agree with the Schumann resonance data. The reciprocity is reduced when computations imply the rigorous full wave solution based on the atmosphere conductivity relevant to the knee model. Deviations in the real part of propagation constant (the phase velocity) do not exceed 1%. However, application of the profile provides overestimated wave attenuation by $\sim$10%.

The modest deviations of propagation constant result in visible departures of the Schumann resonance spectra: the heuristic knee model provides the higher peak frequencies and the higher Q-factors of resonance modes in comparison with the rigorous full wave solution. This result might explain departures of the update direct numerical solutions from the knee model.

Parameters of the heuristic knee model were developed for fitting observational Schumann resonance data, however, these do not grant a realistic conductivity profile of atmosphere. Additional efforts are necessary for obtaining equally efficient $\sigma(h)$ dependence admissible for direct computational algorithms.

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