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Wavevector spectral signature of decay instability in space plasmas

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Abstract. Identification of a large-amplitude Alfvén wave decaying into a pair of ion-acoustic and daughter Alfvén waves is one of the major goals in the observational studies of space plasma nonlinearity. In this study, the decay instability is analytically evaluated in the 2-D wavenumber domain spanning the parallel and perpendicular directions to the mean magnetic field. The growth-rate determination of the density perturbations is based on the Hall-MHD wave-wave coupling theory for circularly-polarized Alfvén waves. The diagrams of the growth rates versus the wavenumber and propagation angle derived in analytical studies are replaced by 2-D wavenumber distributions and compared with the corresponding wavevector spectrum of density and magnetic field fluctuations. The actual study reveals a perpendicular-shape spectral pattern consistent with the result of a previous study based on 3-D hybrid numerical simulations. The wavevector signature of the decay instability observed in the two-dimensional wavenumber domain ceases at values of plasma beta larger than β=0.1. Growth-rate maps serve as a useful tool for predictions of the wavevector spectrum of density or magnetic field fluctuations in various scenarios for the wave-wave coupling processes developing at different stages in space plasma turbulence.

1 Introduction

Parametric instabilities driven by large-amplitude Alfvén waves have extensively been investigated by analytical studies or numerical simulations in one- or multi-dimensional approaches. A systematic analytical analysis of the multidimensional features of the parametric instabilities have been initiated by Viñas and Goldstein (1991a, b) by applying the Hall-magnetohydrodynamic (hereafter, Hall-MHD) theory to a large-amplitude field-aligned Alfvén wave with left-hand and right-hand circular polarization. Results of the two-dimensional predictions of Viñas and Goldstein (1991a, b) have successfully been confirmed by later numerical simulations. Obliquely propagating daughter waves excited by the decay of a field-aligned Alfvén wave have been observed in 2-D MHD numerical simulations by Ghosh et al. (1993) for a low-beta regime. Other studies on the nonlinear interaction of obliquely-propagating Alfvén waves confirm that the growth rate of the decay instability in direction oblique to the mean magnetic field is typically smaller than the field-aligned decay, see e.g., Mjølhus and Hada (1990), Laveder et al. (2002), Nariyuki et al. (2008). By using two-dimensional hybrid simulations, Matteini et al. (2010) discovered that a broad spectrum of Alfvén and density fluctuations is developing perpendicular to the direction of the mean magnetic field at the decay of a linear

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polarized Alfvén pump wave with oblique direction of propagation in low beta plasmas. Gao et al. (2013) reported by means of 2-D hybrid simulations that a linear polarized Alfvén pump wave with parallel propagation can also generate a perpendicular spectrum of daughter waves. Comişel et al. (2019) observed recently a perpendicular spectrum of daughter waves by using field-aligned Alfvén pump waves with circular left-hand polarization and 3-D hybrid simulations. This result was not predicted by previous 2-D numerical simulations. The three-dimensional setup has been also used by Comişel et al. (2020) for analyzing the evolution of large-amplitude Alfvén waves into the azimuthal (or transversal) plane with respect to the mean magnetic field in low-beta plasmas.

The purpose of this study is to recall the former Hall-MHD analytic approach developed by Viñas and Goldstein (1991a). The analytical predictions show that at very low beta values, the oblique decay of a circularly-polarized Alfvén wave becomes competitive with the field-aligned decay. We are looking whether the solutions of the dispersion equation provided by Viñas and Goldstein (1991a) model can or cannot drive a perpendicular spectrum of daughter waves in accordance with the prediction of the 3-D hybrid simulation. In the two-dimensional analytical analysis, the dispersion equation is typically solved by setting a priori the propagation direction of the daughter wave and the complex solution for the frequency is investigated in the wavenumber domain. Here we solve and display the imaginary part of frequencies (namely, growth rates) of the dispersion equation into a wavevector-spectrum like diagram along the parallel and perpendicular directions to the mean magnetic field. On basis on this study, the developed perpendicular spectrum of daughter waves can be considered as a signature for the decay of a left-handed circularly-polarized Alfvén pump wave in low beta plasmas. This pattern describing the oblique-decay process is vanishing for larger values of plasma beta parameter.

2 Method and results

We use the analytical analysis developed by Viñas and Goldstein (1991a, b) based on the two-fluid plasma model together with the generalized Ohm's law. The dispersive effects are driven by the ion inertia and the Hall term. Monochromatic parallel-propagating Alfvén waves are exact solutions of the nonlinear MHD equations describing a plasma system. Starting from this property, the set of equations for the evolution of density, flow velocity, and magnetic field is linearized by using a perturbation expansion in order to define a linear-mode wave (or eigenmode of the system) around the equilibrium of each of the above mentioned quantities. Each linear mode is specified by frequency, wavevector, and a state vector. The wave-wave coupling of the large-amplitude Alfvén pump wave with a density perturbation of wavevector $\boldsymbol{k}$ and frequency ω is conducting to side-band daughter waves expressed by the relations, $\boldsymbol{k}^{\pm} = \boldsymbol{k} \pm \boldsymbol{k_0}$, $\omega^{\pm} = \omega \pm \omega_0$, where $\boldsymbol{k}_0$ and ω_0 are the field-aligned wavevector and the frequency of the Alfvén pump wave, respectively. The daughter waves are allowed to propagate parallel and obliquely to the magnetic field. The linear dispersion analysis is restricted to excitation of the fundamental side-band daughter waves. The electron-inertia effect is neglected with limitation for the frequency domain. Thus the validity of the theory is limited at higher frequencies close or above the ion gyro-frequency. The Landau and cyclotron damping effects have been neglected. The general dispersion equation is derived in terms of 6x6 matrices and depends on six independent parameters: frequency, wavenumber and angle of propagation of the linear mode, amplitude and wavenumber of the pump wave, and plasma β parameter. The

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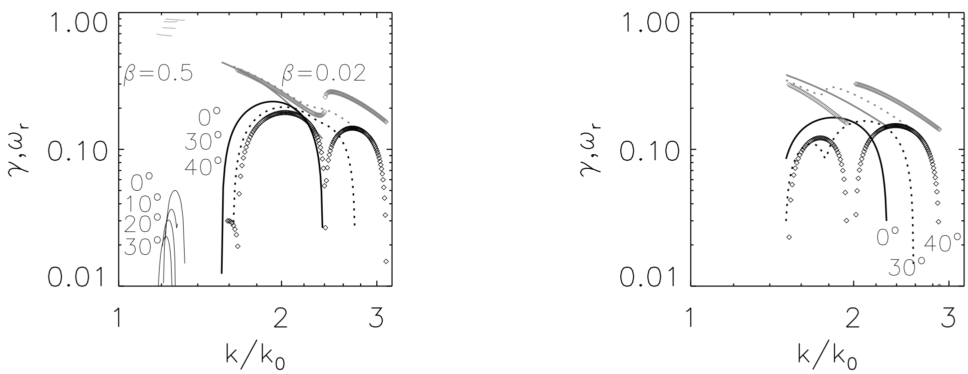

Figure 1. Solutions of the dispersion equation (growth rates in black and frequencies in gray) versus normalized wavenumber for right-handed (panel left) and left-handed (panel right) polarized Alfvén pump waves at propagation angles 0 deg, 30 deg, and 40 deg and plasma β=0.02. Left panel also shows the result obtained for right-hand polarization and plasma β=0.5.

frequency and growth rate are normalized according to Viñas and Goldstein (1991a) as $\omega_r = Re(\omega)/k_0 V_{\rm A}$, $\gamma = Im(\omega)/k_0 V_{\rm A}$, where $V_{\rm A}$ is Alfvén speed. The plasma beta is defined as $\beta = V_{\rm s}^2/V_{\rm A}^2$ with $V_{\rm s}$ - the sound speed.

The dispersion equation is implemented and solved by using the Mathematica software. We first investigate the solutions (ω_r, γ) for the decay of a right-handed polarized Alfvén wave with same parameters used by Viñas and Goldstein (1991b). Figure 1 (left panel) reports the growth rates (solid line) and frequencies (gray solid line) obtained for plasma β=0.5. The amplitude of the Alfvén pump wave is 0.2 (normalized to the background magnetic field) and its wavenumber (normalized in terms of ion inertial length) $k_0 V_{\rm A}/\Omega_{\rm p}$ is 0.3, where $\Omega_{\rm p}$ is the ion-gyrofrequency (for protons). The maximum growth rate for parallel propagation is obtained at $k/k_0 \approx 1.25$. The peaks of the growth rates determined at oblique-propagation angles of 10 deg, 20 deg, and 30 deg are decreasing and slightly moving to lower wavenumbers. Their corresponding frequencies are also reducing at larger propagation angles consistent with Viñas and Goldstein (1991b) result. In the same panel we show the solutions of the dispersion equation obtained at a lower plasma β value of 0.02. For a better visualization of both frequencies and growth-rate profiles, the values of the wavenumbers along horizontal axis are represented in a logarithmic scale. At small plasma-beta values, the maximum growth rate determined for parallel propagation is significantly larger and is located at $k/k_0 \approx 1.9$ (see black solid line). At an angle of 30 deg, the growth rate is slightly smaller (dotted line) while at a larger value (40 deg), the solutions are splitted into two peaks (thick diamond symbol). The dominant peak has a maximum close to that one derived at parallel propagation. The oblique decays have growth rates similar with that of the field-aligned decay (see also Fig. 3 in Viñas and Goldstein (1991b) at low β values). The result for a left-handed polarized Alfvén pump wave is given in the right panel of Fig. 1 calculated at the same low plasma beta value (β=0.02). The growth rates are slightly smaller for both parallel- and obliquely- propagating daughter waves than those analyzed by considering a right-handed polarized pump wave. At propagation angle of 30 deg, the splitting of the growth rate into two peaks is more pronounced and its maximum value is

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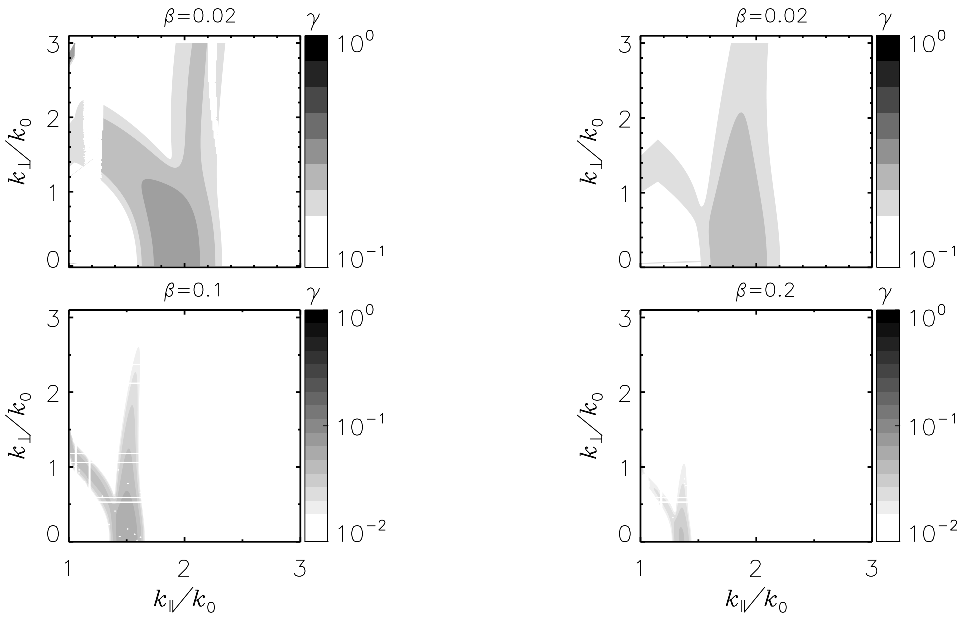

Figure 2. Top: Map of the growth rates in the $k_\parallel - k_\perp$ wavenumber domain corresponding to the analysis shown in Fig. 1 at plasma β=0.02. Bottom: Diagrams for left-handed polarized Alfvén pump wave at higher beta values.

shifted to larger wavenumbers. In contrast with the left panel of Fig 1, at larger propagation angles (30, 40 deg), the prominent peak of the growth rate (thick diamond) is located in the right-hand side of the plot at wavenumbers $k/k_0 > 2$.

The growth rates derived from the dispersion equation shown in Fig. 1 can be visualized in a different way by constructing a wavevector diagram or growth-rate map analogue to the representation of the 2-D wavenumber spectrum of density or magnetic field fluctuations. The mapping of the growth rates in such coordinates is helpful in our study for a two-folded purpose discussed bellow for: (i) establishing or finding of a specific pattern for the parametric decay in the wavevector domain and (ii) direct comparison of the analytical predictions with spectra of fluctuations obtained from numerical simulations (or presumably in-situ measurements).

The top panels of Fig. 2 are the arrangement of the plots shown in Fig. 1 into the parallel and perpendicular wavenumber domain for the right-handed and left-handed Alfvén pump waves. The growth rates in the new coordinates are obtained by solving the dispersion relation for k/k_0 wavenumbers spanning the domain (1,3) and (0,3) along the parallel and perpendicular direction with respect to the main magnetic field. The solution is determined for a given pair of values $(k_\parallel, k_\perp)$ which is then advanced by a discrete Δk step for each parallel or perpendicular direction. The resulted solutions are smoothed and represented in the wavevector domain. One may observe an "arc"-shape branch of solutions and a perpendicular one for the right-handed polarized Alfvén pump wave. For left-handed polarized waves, the perpendicular branch clearly dominates the "arc"-shape branch in accordance with the profiles drawn in Fig. 1. The bottom panels of Fig. 2 present the map of growth rates determined at larger beta values for left-handed polarized Alfvén pump waves. While the parallel wavenumber of the

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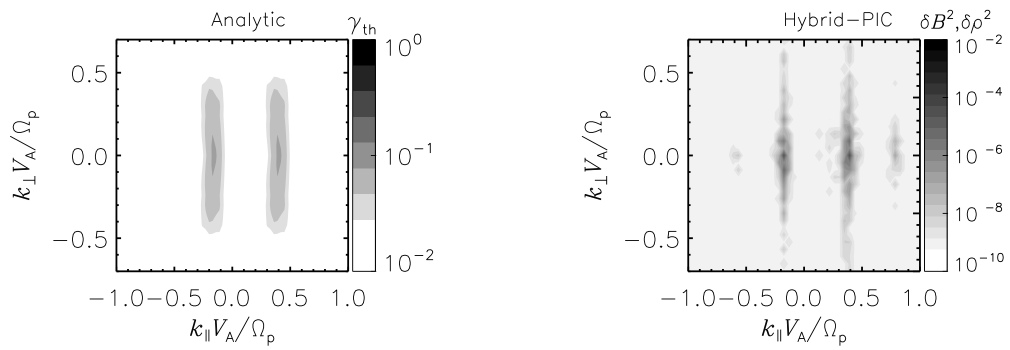

Figure 3. Comparison between the extended map of the growth rates in the $k_\parallel - k_\perp$ wavenumber domain (left panel) and 2-D wavenumber spectrum of density and magnetic field field fluctuations (right panel) from Comişel et al. (2019).

maximum growth rate is shifting to lower values, the perpendicular branch becomes weaker and at larger beta values (β >0.1) the oblique-decay becomes insignificant with respect to the lowest-beta analyzed case. From this analysis we conclude that a perpendicular spectral pattern of the decay products can be associated with the decay process of a circularly polarized Alfvén wave in low-beta plasma.

In Fig. 3 we compare the result of the actual study for the left-handed polarization and beta value of 0.02 with the 2-D wavenumber spectrum of density and magnetic field fluctuations from a former 3-D hybrid simulation (Comişel et al., 2019). First, the growth-rate map given in Fig. 2 is extended towards both positive and negative perpendicular wavenumbers. Second, the growth rates for the lower side-band daughter waves ($\omega_- = \omega - \omega_0$) describing Alfvén waves with backward propagation are added at (negative) wavenumbers $k_\parallel^- = k_\parallel - k_0$ on the basis that $Im(\omega_-) \equiv Im(\omega)$ according to the wave-wave coupling scheme. The magnetic field and density fluctuations given in right panel of Fig. 3 are obtained from a former 3-D hybrid simulation based on AIKEF code (Müller et al., 2011) and a similar scenario with the current study. The spectral analysis shows the Alfvén daughter modes at $k_\parallel V_\mathrm{A}/\Omega_\mathrm{p} \sim -0.2$ and the sound daughter waves at $k_\parallel V_\mathrm{A}/\Omega_\mathrm{p} \sim 0.4$. There is a good qualitative match between the two panels. As we already mentioned, the analytical model does not consider the wave damping or the harmonics of the excited daughter modes. The actual study and former hybrid simulation suggest that the field-aligned decay is accompanied by an oblique-decay process developing a perpendicular spectrum of density and magnetic field fluctuations.

3 Outlook

In conclusion, the analytic method developed by Viñas and Goldstein (1991a) prescribes that in low beta plasmas, circularly polarized Alfvén waves decay into parallel and obliquely-propagating daughter waves. The growth-rate values of the decay process plotted in the two-dimensional domain of the wavevector parallel and perpendicular to the mean magnetic field evince a displacement of the solutions into two branches: a perpendicular one predominant for both left- and right- hand polarization and an "arc"-shape one which is stronger for the right-hand polarization. The oblique decay significantly decreases at beta values larger than β >0.1. The theoretical prediction for the left-handed polarized Alfvén pump wave derived in the 2-D

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wavenumber domain is consistent with the 2-D spectrum of density and magnetic field fluctuations resulted from 3-D hybrid simulations.

Growth-rate maps as those discussed above can be conveniently obtained for various values of the input parameters describing the dispersion equation. A catalogue of maps realized by a systematic analyzing for plasma beta, amplitude of pump wave, polarization or amount of dispersion can provide valuable information for further investigations of parametric instabilities by using hybrid or full-particle numerical simulations in two- or three- dimensional approaches. The growth-rate maps derived by analytical models and subsequently confirmed by numerical simulations can be helpful in future studies as predictions for the spectrum of density or magnetic field fluctuations expected from in-situ measurements. Furthermore, the particular signature of the oblique decay can serve as an evidence of wave-wave coupling processes acting at different evolution stages in space plasma turbulence.

Code and data availability. Data sets and software code can be obtained by writing to the following email addresses: h.comisel@tu-braunschweig.de or comisel@spacescience.ro.

Author contributions. HC worked on theory implementation and manuscript writing. YN worked on representation of wave nonlinearities and application to space plasmas. UM worked on discussion and supervised of the study. All authors read and approved the final manuscript.

Competing interests. The authors declare that they have no conflict of interest.

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