

Response to reviewers

Response to Reviewer #1

We thank the reviewer for their examination of our manuscript and their comments. Please find below, in bold, our response to your concerns, and the changes added to the revised manuscript. The lines mentioned are pointing to the manuscript with tracked changes, that the reviewer can find at the end of the responses.

”The authors have written an article about a long-standing problem of whether mirror modes or ion cyclotron waves should dominate in the Earth’s magnetosheath. This topic has a long history and there have been many papers(which were not referenced) on this topic. However more importantly, there have been some recent works that appear to have solved the problem. These papers unfortunately supercede the work that you have done. I therefore cannot accept your work to AG in anything resembling the present version. [...] The authors should also cite some of the earlier papers quoted in the 2009 and 2013 papers that address the issue of MM/EMIC growth rates. This will give more context to your work.”

We thank the reviewer for pointing out the lacking connections to previous work about the question of mirror mode versus ion cyclotron waves in the magnetosheath and for providing valuable references about this topic. We would like to emphasise that the intention of this paper is primarily to help future global hybrid-Vlasov simulations to save resources when simulating and investigating magnetosheath waves, and secondarily to investigate the mirror mode waves and the ion cyclotron waves in term of those resolutions, not the competition between these modes. We have added the suggested references in the revised manuscript and revised the manuscript in terms of the physics the reviewer suggested (Lines 50 to 54). We have added more clarity and emphasis to the primary focus of the paper (Abstract).

”The second sentence in the abstract may be incorrect. Please note that there are electron cyclotron waves in the magnetosheath called lion roars.”

We have emphasised in the revised manuscript that we focus on

ion-scale waves (Abstract and Lines 50 to 54).

”These earlier papers state that the growth rate of the ion cyclotron wave is higher than the MM waves, contrary to your results. You will need to reconcile this point as well.”

Fig. 6 of our manuscript shows the growth rate of the proton cyclotron (Panel a) and mirror (Panel b) instabilities at the three different resolutions: 300 km (blue), 600 km (red), 900 km (orange). As can be seen, the proton cyclotron growth rate is higher than the mirror growth rate in all resolutions, consistent with the previous works mentioned by the reviewer. We have added references the reviewer provided to compare our results to previous work (Lines 249 to 258).

”MM waves have also been detected at Jupiter’s and Saturn’s magnetosheaths as well. There are many references to this, including the original 1982 paper. Line 30: See introduction of 2011 JGR paper for more references to MM detection in interplanetary space (solar wind), in the Earth’s geotail, and at comets. MMs have also been detected at heliopause. These references should be included. Line 255. I suggest that you add more references to some of the earlier work that focused on the issue of the growth of the two competing modes. This was mentioned earlier.”

Thank you very much for pointing this out. We have provided more references about the detection of these waves in these other setups and to emphasise previous work on the topic (Lines 39 to 52 and 57 to 67).

Response to Reviewer #2

We thank the reviewer for their careful examination of our manuscript and their constructive comments. Please find below, in bold, our response to your concerns, and the changes added to the revised manuscript. The lines mentioned are pointing to the manuscript with tracked changes, that the reviewer can find at the end of the responses.

”Major comments: Plasma β is one very important parameter to two instabilities. If different solar wind speed, solar wind temperature, and IMF field strength are used in the simulation, the plasma β in the magnetosheath will be very different. So my concern is that the concluded spatial resolution very likely depends on the magnetosheath plasma β or solar wind parameters. Therefore, it is necessary to justify that the conclusion is true for all the possible solar wind

parameters or how the conclusion depends on the solar wind parameters. Otherwise, the importance of this work to future simulations will be very limited.”

This is an excellent point. While running an entire new set of simulations with different solar wind parameters would be too expensive, we plotted the growth rate of the proton cyclotron instability with three different β (the reviewer can find the figure attached to this document). An increased plasma β gives a higher value of the maximum growth rate γ_{\max} , but does not change significantly the value of the corresponding wave vector k . Our conclusions remain that the low resolution run does not resolve a large enough spectrum of wave vectors to allow the proton cyclotron instability to develop. A higher beta value would slightly improved runs with resolution between 300 and 600 km. We have added this to the discussion in the revised manuscript (Lines 276 to 283).

”Under this certain solar wind condition, the authors conclude that $\Delta r \approx 440 \text{ km} = 0.6 \text{ di}$ would be adequate. However, I am not convinced by this number which is based on the growth rate profile. For example, where the growth rate is calculated may not be the source region. So I wonder whether the authors can run several more cases, e.g., with Δr around 400 km, to show that the results are indeed close enough to the case with $\Delta r = 300 \text{ km}$ ”

We thank the reviewer for pointing this out. We agree that this conclusion is purely made based on the growth rate profile is probably too specific, since many other parameters, such as the source region of the wave in the global simulation, may affect it. As mentioned earlier, running additional Vlasator simulations is difficult because of their large computational cost (a few million CPU hours for a 400 km run similar as those presented in this study). In the revised manuscript, we have reformulated our conclusions as follows (Abstract and Lines 354 to 358):

The currently available runs allow us to conclude that the wave modes of interest here are properly resolved at a resolution of 300 km = 1.32 di. The growth rate profiles suggest that larger cell sizes, between 300 and 600 km, may still be sufficient to resolve those wave modes in the simulations. This hypothesis could be tested in running additional global simulations with a range of spatial resolutions. Such a parametric study is however not currently achievable because of the large computational costs of global Vlasator runs at these relatively high resolutions.

Incidentally, we have realised that there was an error with the value of the resolution in term of ion skin depth in the manuscript. The values should be $\Delta r = 300 \text{ km} = 1.32 d_i$, $\Delta r = 600 \text{ km} = 2.64 d_i$, $\Delta r = 900 \text{ km} = 3.96 d_i$ and $\Delta r = 400 \text{ km} = 1.76 d_i$, as the ion skin

depth in the solar wind is 228 km. This does not however affect our conclusions. This has been corrected throughout the manuscript.

”Minor comments: Please rephrase “magnetosheath waves” in the abstract and conclusion as there are not just mirror mode and AIC waves in the magnetosheath.”

We have rephrased to focus on ion-scale instabilities in the abstract (Line 3) and the in conclusions (Lines 360 to 362). We have also clarified our conclusions regarding the availability of other runs (Lines 354 to 358).

”Why the position of quasi-perpendicular bow shock in Figure 1c is more outward than Figures 1a and 1b?”

We thank the reviewer for raising this discussion. As can be seen on Fig. 7, due to the AIC waves not being resolved, the temperature anisotropy is greatly increased in the 900 km run. The lack of mechanism to reduce the perpendicular temperature leads to an increased pressure behind the shock, “pushing” it further away. We have added this to the discussion (Lines 298 to 303) in the revised manuscript as an additional argument toward the importance of setting the resolution to 300 km.

”In Figure 2f, there are signals along two blue lines. What are they?”

We believe it to be due to some numerical artefacts. The low resolution and the fact that these features appear in the perpendicular direction make us doubt that this would be related to Alfvén waves. We currently apply a Hanning window filtering in both time and spatial dimension. We have tested other filters and the features are not disappearing.

”In Figure 5, it would be better if there are similar panels of other two runs for comparison.”

We have updated Fig. 5 as requested, and added the related discussion (Lines 182 to 184).

”There are some typos such as line 221, ‘were the data were taken’ → ‘where the data were taken’ and line 225, ‘more efficient tp’ → ‘more efficient to’”.

We thank the reviewer for picking up these typos, we have corrected them.

Relevant changes made to manuscript

Abstract

- Added and modified a few sentences to clarify the goal of the paper and to highlight the focus on ion-scale waves.

Introduction

- Added references provided by the reviewers.
- Added a paragraph citing additional previous work on the topic of the paper.
- Added a few sentences to clarify the goal of the paper.

Results

- Added two panels to Figure 5. Added the related description in section 3.1.

Discussion

- Added the discussion related to Figure 5.
- Added a paragraph comparing the HYDROS results to previous studies and their agreement with our work.
- Added a paragraph discussing the relevance of the β parameter in the study.
- Added a paragraph discussing the influence of the temperature anisotropy on the position of the bowshock.

Conclusions

- Modified the conclusion regarding the acceptable limits of the resolution.

Other relevant changes

- Corrected the values of the resolution in term of ion inertial length throughout the manuscript.
- Added two notations to clarify the difference between the ion inertial length in the solar wind and in the magnetosheath. Added a column in Table 1 for the additional notation.

Tracked changes Manuscript

Following is the revised manuscript. The discussions and minor revisions added for clarity can be found in bold font. The sentences removed from the manuscript can be found crossed.

Resolution dependence of magnetosheath waves in global hybrid-Vlasov simulations

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Abstract. Plasma waves are ubiquitous in the Earth's magnetosheath. **Kinetically driven plasma waves are fundamental for a description of the thermodynamical properties of the Earth's magnetosheath.** The most commonly observed waves arise from **ion-scale instabilities** are generated by temperature anisotropy of the ions, such as the mirror and proton cyclotron instabilities. We investigate here the spatial resolution dependence of the mirror and proton cyclotron instabilities in a global hybrid-Vlasov simulation using the Vlasiator model, **in order to find optimal resolutions and help future global hybrid-Vlasov simulations to save resources when investigating those instabilities in the magnetosheath.** We compare the proton velocity distribution functions, power spectra and growth rates of the instabilities in a set of simulations with three different spatial resolutions but otherwise identical set-up. We find that the proton cyclotron instability is absent at the lowest resolution and that only the mirror instability remains, which leads to an increased temperature anisotropy in the simulation. We conclude that the proton cyclotron instability, **its saturation and the reduction of the anisotropy to marginal levels** are resolved well enough at the highest spatial resolution. **An increase of resolution does not improve the resolution of the instability enough to A further increase in resolution does not lead to a better description of the instability to an extent that would justify this increase at the cost of numerical resources in future simulations.** We also find that a **spatial resolutions around 0.6 between 1.32 and 2.64 times** the inertial length in the solar wind present acceptable **minimum spatial limits for the** resolution within which the proton cyclotron is still correctly resolved **velocity distribution functions resulting from the proton cyclotron instability are still bi-maxwellian and reach marginal stability levels. Our results allow us to determine a range of spatial resolution suitable for the modelling of the proton cyclotron and mirror instabilities and** These results should be taken into consideration regarding the optimal grid spacing for the modelling of magnetosheath waves **these two instabilities,** within available computational resources.

20 *Copyright statement.* TEXT

1 Introduction

The Earth's magnetosheath is permeated with several kinds of ion-kinetic waves, which are an important source of energy transfer and dissipation within the magnetosheath plasma (Schwartz et al., 1997). The most commonly observed waves arise from instabilities generated by temperature anisotropy of the ions. The mirror instability (Chandrasekhar et al., 1958; Hasegawa, 1969; Southwood and Kivelson, 1993; Kivelson and Southwood, 1996) and the proton cyclotron instability (Davidson and Ogden, 1975; Gary et al., 1993) are excited by a temperature anisotropy where the ions' perpendicular temperature T_{\perp} is larger than the parallel temperature T_{\parallel} .

The mirror instability gives rise to compressional, linearly polarised waves characterised by zero frequency in the plasma frame, anti-correlation between the plasma density and the magnetic field, and magnetic perturbations which are mostly parallel to the background magnetic field. They create magnetic mirror-like structures trapping particles (Soucek et al., 2008). ~~Mirror modes are dominant in high plasma beta (Tsurutani et al., 1982).~~ The proton cyclotron instability has maximum growth rate around the ion cyclotron frequency, and produces waves propagating in the direction parallel to the background magnetic field. The magnetic perturbations of the waves are perpendicular to the background magnetic field and produce left-handed circularly polarised waves in the plasma frame (Davidson and Ogden, 1975; Lacombe et al., 1994). ~~The proton cyclotron instability is dominant in lower beta plasma (Gary, 1992).~~ **Both instabilities isotropize the ion populations of the magnetosheath by pitch angle scattering the protons (Hasegawa, 1969; Tanaka, 1985), thus reducing the temperature anisotropy of the population. The proton cyclotron instability isotropizes ions faster than the mirror instability (McKean et al., 1992).**

The mirror and proton cyclotron instabilities have been observed in the Earth's magnetosheath (Tsurutani et al., 1982; Anderson et al., 1996; Gary et al., 1993; Soucek et al., 2008) as well as in the solar wind (Hellinger et al., 2006), **heliosheath (Liu et al., 2007; Tsurutani et al., 2011)**, and sheath regions driven by coronal mass ejections (Ala-Lahti et al., 2019). **The mirror instability has also been observed in the magnetosheaths of Jupiter and Saturn (Tsurutani et al., 1982; Erdős and Balogh, 1996), distant magnetotails (Tsurutani et al., 1984), cometary sheaths (Russell et al., 1987) and interplanetary space (Tsurutani et al., 1992).**

The competition between the mirror and proton cyclotron instabilities has been a topic of many studies for the past 45 years (Price et al., 1986; Winske and Quest, 1988; Brinca and Tsurutani, 1989; Gary, 1992; Gary et al., 1993; Gary and Winske, 1993; Anderson and Fuselier, 1993; Lacombe and Belmont, 1995; Schwartz et al., 1997; Shoji et al., 2009, 2012; Remya et al., 2013). The proton cyclotron instability is dominant in lower beta plasma (Gary, 1992), while mirror modes are dominant in high plasma beta (Tsurutani et al., 1982). Price et al. (1986) showed that the presence of He^{++} tends to lower the growth rate of the proton cyclotron instability, while Brinca and Tsurutani (1989) showed that such ions would have less effect on the growth rate of the mirror instability. **The mechanisms of the competition between the two instabilities have also been extensively studied using numerical simulations of the magnetosheath plasmas (Shoji et al., 2009, 2012; Remya et al., 2013).** However, in this study, instead of investigating which mode is dominant, we rather evaluate the influence of the spatial resolution of a simulation in the development of those instabilities. The goal of this study is to determine an acceptable resolution where both instabilities are resolved correctly.

55 Both instabilities isotropize the ion populations of the magnetosheath by pitch angle scattering the protons (Hasegawa, 1969; Hasegawa, 1975) thus reducing the temperature anisotropy of the population. The proton cyclotron instability isotropizes ions faster than the mirror instability (McKean et al., 1992). The properties of **both instabilities** have been studied through simulations by, e.g. McKean et al. (1994); Gary and Winske (1993); Seough et al. (2014); Hoilijoki et al. (2016). However, these simulations were either one-dimensional (Gary and Winske, 1993) or used a particle-in-cell approach (Seough et al., 2014), and apart from Hoilijoki et al. (2016), none of them studied the instabilities in a global simulation of Earth's magnetosheath. **Gary and Winske (1993) showed that the introduction of Helium ions reduces the growth rate of the proton cyclotron instability. Seough et al. (2014) found good agreement between quasi-linear theory and the simulation. Hoilijoki et al. (2016) focused only on the mirror instability presented the first study of mirror modes in a global hybrid-Vlasov simulation and found that their properties were consistent with that obtained from previous studies. Kunz et al. (2014) also showed in a hybrid-kinetic simulation that trapped particles govern the nonlinear evolution of the mirror instability to maintain the pressure anisotropy to a marginal level. They also found that energy is removed below the Larmor scales by what appears to be Kinetic Alfvén Waves turbulence.** These simulations did not study the impact of the spatial resolution on the description of the instabilities.

Modern plasma physics is increasingly relying on the support of numerical simulations in understanding waves and instabilities. Whether it is used for the study of laboratory plasmas (Revel et al., 2018), nuclear fusion (Görler et al., 2011) or space plasmas (McKean et al., 1994), numerical modelling of instabilities is crucial for the understanding of the physics of the system. However, no matter what kind of numerical model is chosen, it is difficult to model the entire system at a numerical resolution capturing both large-scale and small-scale physical processes involved without incurring a very high computational cost. The issue is even more relevant when global simulations of large systems are carried out, for physical understanding or for space weather forecasting (Palmroth et al., 2018; Pomoell and Poedts, 2018). The choice of resolution is a central parameter in numerical models, and often presents a tradeoff between accuracy and computational cost. To be able to make an informed choice about this tradeoff, a firm understanding of the impact of the models' resolution on the physical processes at play in the system is required.

In order to model the processes involved in energy transfer and dissipation, the understanding of the instabilities generating them is essential. In this study, we investigate the impact of the spatial resolution on the ion-scale waves produced by the mirror and proton cyclotron instabilities in a 2D global hybrid-Vlasov simulation of the Earth's magnetosphere using the Vlasiator model (Palmroth et al., 2013; von Althaus et al., 2014; Palmroth et al., 2018). A study by Pfau-Kempf et al. (2018) showed that, for 1D simulations of oblique shocks, a coarse resolution in Vlasiator such as cells of size $\Delta r = 1000$ km was still sufficient to describe correctly most of the kinetic effects related to shocks. Despite not resolving the ion skin-depth **inertial length in the solar wind** of 228 km in this simulation, the results were similar to a simulation with a spatial resolution of $\Delta r = 200$ km, where the ion skin-depth **inertial length** was resolved. Ion velocity distribution functions obtained by the Vlasiator model at coarse resolution are also consistent with observations (Kempf et al., 2015). However, the effect of spatial resolution on the description of plasma instabilities in 2D global kinetic simulations is still an open question. The spatial resolution of a simulation impacts the evolution of the fields during the simulation. Therefore, it will modify the development of the different

90 instabilities present in the magnetosheath and their effect on the velocity distribution functions. In this paper, we determine the lowest possible spatial resolution which can still be used to model the mirror and proton cyclotron instabilities in a 2D global simulation. This allows computational resources to be used more efficiently when global hybrid-Vlasov simulations of near-Earth space are expanded to the third dimension. We focus this investigation on the magnetosheath waves downstream of the quasi-perpendicular shock, as they are well defined and less perturbed by the shock processes than downstream of the quasi-parallel shock. We decided to focus on the proton cyclotron and mirror instabilities as their properties are well documented and are a good proxy for their dependence on the resolution.

2 Global hybrid-Vlasov model

We performed this study using the Vlasiator model. Vlasiator is a global hybrid-Vlasov model (Palmroth et al., 2013; von Alfthan et al., 2014; Palmroth et al., 2018). Currently, it consists of a cartesian 2D spatial grid containing the nightside and dayside of the Earth’s magnetosphere, magnetosheath, bow shock and foreshock. A cartesian 3D velocity space grid is coupled with each of the ordinary space cells. The model solves the time evolution of the protons in phase space by solving the Vlasov equation, coupled with the electric and magnetic fields. The fields are propagated using Maxwell’s equations. Closure of the system is performed with the generalised Ohm’s law including the Hall term. In each grid cell, the protons are discretised as velocity distribution functions (VDFs). Electrons are considered a cold, massless, charge-neutralising fluid. The Vlasiator model, and global hybrid-Vlasov simulations in general, have the advantage to be noise-free (Palmroth et al., 2018).

The Vlasiator model can be run in 1D, or 2D in ordinary space. In this study we investigate the ion-scale waves produced by the proton cyclotron and mirror instabilities in three 2D simulations with different spatial resolutions but otherwise identical set-up. Typically, in 2D Vlasiator simulations, the spatial resolution of the grid in ordinary space is set to $\Delta r = 300$ km (e.g. Blanco-Cano et al., 2018; Grandin et al., 2019; Hoilijoki et al., 2019), which corresponds to $\Delta r = 1.32 \mathbf{d}_{i,SW}$, ~~proton-skin~~ depth **where $\mathbf{d}_{i,SW}$ is the ion inertial length in the solar wind**. Simulations with resolution of $\Delta r = 228$ km = $1 \mathbf{d}_{i,SW}$ have also been used (e.g. Palmroth et al., 2018; Turc et al., 2018). When extending simulations to 3D, such a high resolution may become unfeasible, even when using adaptive mesh refinement for regions of interest.

In order to study the effect of spatial resolution on magnetosheath waves, we conducted three simulations using the same set-up, with different spatial resolution: $\Delta r = 300, 600, \text{ and } 900$ km, which corresponds to $\Delta r = \mathbf{1.32, 2.64}$ and $\mathbf{3.96} \mathbf{d}_{i,SW}$ in the solar wind, respectively. The system is in the geocentric solar ecliptic (GSE) coordinate system, assuming a zero magnetic dipole tilt. All runs are 2D, describing the noon-midnight meridional plane (X-Z) of near-Earth space. The real-space boundaries of the simulations extend from $X = -48 R_E$ in the nightside to $X = 64 R_E$ in the dayside, and from $Z = -60 R_E$ to $Z = 40 R_E$ in the north-south direction, asymmetrical to accommodate the foreshock in the negative Z-direction, with $R_E = 6371$ km the Earth radius. The north, south and nightside boundaries all apply Neumann boundary conditions. The inner boundary is located at $4.7 R_E$ from the centre of the Earth and consists of a perfectly conducting sphere. The homogeneous and constant solar wind is flowing from the dayside boundary in the $-X$ direction with a velocity of 750 km/s, interplanetary magnetic field (IMF) strength of 5 nT, and temperature of 0.5 MK. The IMF makes an angle of 45° with respect to the X

direction, southward. The solar wind protons are represented by a Maxwellian distribution function, with density 1 cm^{-3} , and a velocity space resolution of 30 km s^{-1} . This setup is identical to the one used in Blanco-Cano et al. (2018).

125 Figure 1 displays a global overview of the magnetic field magnitude in the dayside of near-Earth space in the three different runs. One can identify the upstream solar wind (in dark blue), the bow shock, the magnetosheath and the magnetosphere (mostly yellow). The white circle of radius $4.7 R_E$ represents the inner boundary of the simulation. The white square indicates the portion of the simulation we will focus on in this study. One can already notice differences in the magnetosheath wave properties as a function of the resolution of the three different setups. For example, we can observe stripes of roughly constant
 130 magnetic field strength in the 300 km run. These structures are larger in the 600 km run, and have almost disappeared in the 900 km run.

3 Results: Ion-scale waves

3.1 Alfvén waves and mirror modes

In order to identify the resolved wave modes in the different runs, we use the 2D Fast Fourier Transform (FFT) analysis.
 135 Figure 2 displays the wave power of the electric field component in the GSE y direction (i.e. the out-of-plane direction), in the simulation frame, as a function of the frequency ω normalised to the **local** ion cyclotron frequency **in the magnetosheath** $\Omega_c = qB/m_p$, where m_p is the proton mass and q the proton charge, the wave vector \mathbf{k} parallel to the average magnetic field over the time and space intervals in panels (a), (c) and (e), and perpendicular in panels (b), (d) and (f). This analysis is performed in a square extending from $X = 3 R_E$ to $X = 6 R_E$ and from $Z = 15 R_E$ to $Z = 18 R_E$, depicted in red in
 140 Fig. 1, during a time interval from 800 s to 1200 s of the simulation. The maximum possible \mathbf{k} , the Nyquist wave number, depends on the spatial resolution Δr of the simulation as $k_{\text{max}} = \pi/\Delta r$, hence a smaller k_{max} at lower resolution. The x-axis is normalised to the **local** ion inertial length **in the magnetosheath** given by $\mathbf{d}_{i,M} = \sqrt{m_p \epsilon_0 c^2 / (n q^2)} \approx 135 \text{ km}$, where ϵ_0 is the vacuum permittivity, c the speed of light, and n the local proton number density in the magnetosheath. The solid black lines indicate the Courant-Friedrichs-Lewy (CFL) condition (Courant et al., 1967). The CFL condition is a necessary condition for
 145 the convergence of the solution in a model and depends on its spatial and temporal resolutions, implying that no signal can propagate more than one spatial cell within one time interval of the simulation. This means that all features found in between the two black lines are beyond the resolution of the simulation and probably result of numerical features.

At the highest resolution, the dominant wave mode observed in Fig. 2a (i.e. $k_{\parallel} \mathbf{d}_{i,M} \approx -0.4$, $\omega/\Omega_{ci} \approx 1.0$) matches the Alfvén velocity $v_A = B/\sqrt{\mu_0 \rho_m}$ (Alfvén, 1942), indicated by solid blue lines, where B is the magnetic field, μ_0 the vacuum
 150 permeability and ρ_m the mass density of protons. This wave mode is propagating almost entirely in the anti-parallel direction, as evidenced by the much smaller wave power along the Alfvén velocity in the perpendicular direction (Fig. 2b). Since the plasma flow in the magnetosheath is super-Alfvénic, two curves describing the Alfvén velocity appear on the left side of Fig. 2a: the upper solid blue curve describes the waves propagating in the direction anti-parallel to the magnetic field in the plasma frame, while the lower solid blue curve describes the waves propagating in the direction parallel to the magnetic field
 155 in the plasma frame. These ones appear on the $k_{\parallel} < 0$ side of the plot, because of the Doppler shift $\omega' = \omega - \mathbf{k} \cdot \mathbf{V}$, where \mathbf{V} is

the plasma bulk velocity. As in observations from Zhao et al. (2020), we see both parallel and anti-parallel propagating waves at the same time. ~~The anti-parallel propagating waves are dominant, as an instability will generate waves propagating in both directions.~~ The wave mode in Fig. 2a extends up to the proton cyclotron frequency. Figures 2c and 2d show similar features: the observed waves are the same as in Fig. 2a, matching the Alfvén velocity, except that their excitation seems to be constrained to frequencies below $\approx 0.7 \omega_{ci}$. Now for the lowest resolution case, displayed in Fig. 2e and 2f, it appears that the features present in the two higher-resolution simulations are completely absent. The waves around the cyclotron frequency are ~~most likely~~ not resolved at this resolution.

Figure 3 displays the wavelet power spectra obtained from wavelet analysis (Torrence and Compo, 1998) of the out-of-plane component of the magnetic field during the interval of time when the FFTs were performed, taken at the virtual spacecraft locations indicated by a black and white circle in Fig. 1. The frequency of the main wave mode at the highest resolution in panel (a) fluctuates around the proton cyclotron frequency $f_{ci} = \Omega_c/2\pi$ shown with a black line, as observed in Fig. 2a. In panel (b), the waves are still present around the cyclotron frequency but the wave power is lower, as observed in Fig 2c, most likely due to the lower resolution. They are completely absent at the lowest resolution in panel (c), as only very low frequency waves below the cyclotron frequency can be observed.

The polarisation of the magnetic field taken at the virtual spacecraft locations indicated in Fig. 1 is also analysed using the minimum variance analysis (Sonnerup and Scheible, 1998), and the results are displayed in Fig. 4a-d. These hodograms display the magnetic field fluctuations during 18 s. The wave vector is along the minimum variance direction δB_N . Figures 4a and 4b highlight that, at $\Delta r = 300$ km, the wave displays few to no perturbations in the parallel direction to the magnetic field (panel (a)), and is left-handedly polarised (panel (b)) in the simulation frame. The angle between the wave vector \mathbf{k} , obtained from minimum variance analysis, and the ambient magnetic field is $\theta_{kB} = 15^\circ$. This can be assumed to be a nearly parallel propagation. Based on Fig. 2, we find that these waves move along the plasma flow. The frequency of the waves in the plasma frame is given by $\omega' = \omega - \mathbf{k} \cdot \mathbf{V}$. Therefore, a Doppler shift will not change the sign of ω and the polarisation is the same in the plasma frame. Figures 4c and 4d highlight an identical behaviour of the wave at lower resolution $\Delta r = 600$ km. We don't perform a minimum variance analysis on the third run at $\Delta r = 900$ km because there is no significant wave activity around the ion cyclotron frequency. However, we display the fluctuations of the magnetic field and the proton density at the position indicated in Fig. 1c for the time interval considered in the study in Fig. 5c. This indicates that they are anti-correlated and would suggest the presence of mirror waves (Hoilijoki et al., 2016). **Fig. 5a and Fig. 5b show the fluctuations at the other two resolutions. Fig. 5a highlights the presence of higher frequency waves on top of the lower fluctuations. These higher-frequency waves are greatly attenuated in Fig. 5b.**

To further analyse how the spatial resolution impacts the different wave modes, we investigate the growth rates of the waves. We use the numerical dispersion solver HYDROS (HYbrid Dispersion RelatiOn Solver) (Told et al., 2016), designed for hybrid kinetic plasmas. The solver assumes a bi-Maxwellian proton distribution function, and we input the ion parallel temperature, the ion temperature anisotropy, and the ion parallel beta in the magnetosheath of the different simulations, taken at the same locations as the data for the wavelet analysis, indicated in Fig. 1, averaged over the time range of the study. The electrons are modelled as a fluid. The propagation angle between the wave vector and the magnetic field vector is set to zero for parallel

propagation. Figure 6a displays the growth rate γ of the proton cyclotron instability for the three different resolutions. The theoretical maximum wave vector is $k_{\max} = \pi/\Delta r$. However, a signal modelled by such k_{\max} would be described with only two points per wavelength. Slightly more realistic is the assumption that a wave needs to be modelled with at least four cells per wavelength. Hence, we consider that the minimum wavelength the model can resolve at each resolution is $\lambda_{\min} = 4\Delta r$, which
195 corresponds to a maximum wave vector $k_{\max} = \pi/2\Delta r$. This wave length is displayed by vertical dashed lines. The growth rates are consistent with what is observed in Fig. 2: at $\Delta r = 300$ km, the growth rate for the proton cyclotron instability is almost fully within the resolved wave length domain. At $\Delta r = 600$ km, only the low-wavenumber edge of the growth rate curve is resolved, below $k_{\parallel} \mathbf{d}_{i,M} \approx 0.35$, which is consistent with where the wave power vanishes in Fig. 2c. At $\Delta r = 900$ km, the growth rate curve is completely outside the resolved domain. This would explain why the waves around the proton cyclotron
200 frequency are not observed in this run. We also notice that the maximum growth rate is dependent on the resolution of the simulation, the maximum being higher at lowest resolution, because of higher temperature anisotropy. Figure 6b displays the growth rate of the mirror instability for the three different resolutions. We used the same input parameters as for the proton cyclotron instability, except that we set the propagation angle to 45° , and frequency to zero. At $\Delta r = 300$ km, the growth rate is fully within the resolved domain, but at the very low rate **very low for these conditions**. It is higher for the two other
205 resolutions, even though only partially resolved.

Figure 7 displays the temperature anisotropy in the three different runs considered in the study. The anisotropy grows downstream of the quasi-perpendicular bow shock. For $\Delta r = 300$ km in panel (a), waves reduce the anisotropy quickly in the middle of the magnetosheath. We have found that waves with a frequency around the proton cyclotron frequency are present at this resolution. For $\Delta r = 600$ km in panel (b), the anisotropy is reduced but at a much slower rate, **resulting in larger**
210 **anisotropy levels in the magnetosheath**. At this resolution, the waves at the proton cyclotron frequency have a lower wave power. In panel (c), at $\Delta r = 900$ km, a strong anisotropy persists for more than $5 R_E$, and even after some isotropization has taken place **due to the mirror instability**, a high anisotropy, with $T_{\perp}/T_{\parallel} \sim 3$, remains.

3.2 Velocity distribution functions

Figure 8 displays the magnitude of the magnetic field in a zoomed portion of the simulation in the magnetosheath, downstream
215 of the quasi-perpendicular shock, indicated by the white square in Fig. 1, with velocity distribution functions (VDF) taken from the point marked by the black and white circle (see Fig. 1), for the three different resolutions. In Fig. 8a, small wavelength waves (of the order of $0.2 R_E$) are distinguishable. These are the waves with frequency around the proton cyclotron frequency identified in the previous section. In addition, larger wavelength structures (of the order of $1 R_E$) are observed, becoming larger in Fig. 8e, and becoming distinct magnetic field enhancements in Fig. 8i. These structures appear to be convected with
220 the plasma flow, as shown by the animated version of Figure 1 (see Supplementary Video), and are consistent with the mirror modes identified in the previous section. On the right, three slices of the VDFs through the velocity space in different planes are presented. All velocities are transformed to the local plasma frame. Panels (b), (f) and (j) display the slice in the $(\mathbf{v}_B, \mathbf{v}_{B \times V})$ plane. Panels (c), (g) and (k) display the $(\mathbf{v}_{B \times V}, \mathbf{v}_{B \times (B \times V)})$ plane. Panels (d), (h) and (l) display the $(\mathbf{v}_B, \mathbf{v}_{B \times (B \times V)})$ plane. On panels (b), (c) and (d), corresponding to the $\Delta r = 300$ km resolution, one can identify nearly Maxwellian VDFs in all three

225 directions, which is consistent with observations (Williams et al., 1988). In panels (f), (g) and (h), at $\Delta r = 600$ km, the VDFs
have a nearly Maxwellian shape in all three directions, with the beginning of the development of a small loss cone in the parallel
direction. At the lowest resolution $\Delta r = 900$ km, in panel (i), the smaller wavelength structures which can be observed in the
background of panel (a) have disappeared, with only large structures remaining. Moreover, the associated VDFs in panels (j)
and (l) have an "hourglass" shape, in contrast to the nearly Maxwellian shape in panels (b), (c) and (d). The 600 km case in
230 panels (e)-(h) can be considered an intermediate case.

4 Discussion

In this paper, we use Vlasiator simulations of near-Earth space with three different spatial resolutions to investigate the be-
haviour of the proton cyclotron and the mirror instabilities and their dependence on these resolutions. We used 2D-FFT and
wavelet analysis in order to identify the waves produced by these instabilities, their different properties at the different reso-
235 lutions, and the impact of these different properties on the velocity distribution functions of the protons. The growth rate of
the proton cyclotron instability at the different resolutions is calculated and compared with resolution-dependent minimum
wavelengths. The temperature anisotropy in the magnetosheath of the different run is analysed.

As Fig. 2, 3 and 4 illustrate, the higher frequency waves propagate with the Alfvén velocity, in the parallel direction, with
perpendicular perturbations which are left-hand polarised, with frequency around the ion cyclotron frequency. This suggests
240 that the wave mode present at the 300 km and 600 km resolutions is the Alfvén ion cyclotron wave mode (AIC waves)
(Anderson et al., 1996; Rakhmanova et al., 2017), or also known as electromagnetic ion cyclotron (EMIC) waves. In the
900 km case, the AIC waves are not present. In addition, the magnetic field and density perturbation analysis shown in Fig. 5c
suggest that mirror modes are present in the 900 km case, which appear to be the dominant wave mode at this spatial resolution.
The anti-correlation between density and magnetic field strength is less pronounced in panels (a) and (b) for the two
245 **higher resolutions, as the AIC waves are resolved and are dominant in the magnetosheath.** A more detailed study about
mirror modes in Vlasiator has been conducted by Hoilijoki et al. (2016).

The growth rate analysis shown in Fig. 6a suggests that the AIC waves are well resolved in the 300 km resolution run, at
the highest resolution used in this study. ~~Along with the proton cyclotron instability,~~ Hoilijoki et al. (2016) showed that the
mirror instability was also resolved in a different simulation. **The observations of mirror modes in this study are consistent**
250 **with the work of Hoilijoki et al. (2016). The results obtained with HYDROS show that the maximum growth rate of the**
proton cyclotron instability should be higher than that of the mirror instability in all three simulations, in agreement
with previous numerical and theoretical studies (Price et al., 1986; Gary, 1992; Gary et al., 1993; Gary and Winske,
1993; Shoji et al., 2009; Remya et al., 2013). The Vlasiator simulations show similar results, except for the lowest
resolution, where the proton cyclotron instability does not develop. Note that the inclusion of anisotropic electrons, not
255 **present here in Vlasiator, could slightly reduce the growth rate of the proton cyclotron instability (Remya et al., 2013).**
The work of Shoji et al. (2009) and Shoji et al. (2012) show that the mirror instability becomes dominant when adding
the third spatial dimension in hybrid-PiC simulations. Future 3D global hybrid-Vlasov simulations can be employed to

further probe this balance question. The 600 km resolution case seems to limit the frequency of the waves below the ion cyclotron frequency, while still partially resolving the AIC waves, whereas the 900 km run highlights that only mirror modes are present in the magnetosheath and shows no sign of the AIC waves, as evidenced by Fig. 2 and 3, since the resolution of the simulation does not allow the Alfvén mode to grow sufficiently (Fig. 6). Figures 8a and 8i can be compared with Fig. 3 and 6 of McKean et al. (1994), although the disappearance of the proton cyclotron instability is not due to the spatial resolution in their study. They included Helium ions in their simulation, which tend to suppress the proton cyclotron instability, with only the mirror instability remaining, resulting in similar magnetosheath wave properties as in our 900 km resolution run. The growth rate analysis of the mirror instability shown in Fig. 6b suggests that the mirror modes barely grow in the middle of the magnetosheath, where the data were taken, for $\Delta r = 300$ km. Since Hoilijoki et al. (2016) have shown **in a different simulation** that they are still present **in the middle of the magnetosheath**, ~~they probably~~ **we can expect that the modes** grow near the quasi-perpendicular bow shock, where the temperature anisotropy is higher, and then travel with the plasma flow. The proton cyclotron instability however grows much faster and isotropises the ions (Davidson and Ogden, 1975; McKean et al., 1992). At $\Delta r = 600$ km, the mirror instability has a larger maximum growth rate than at $\Delta r = 300$ km, but still lower than that of the proton cyclotron instability. Both instabilities grow, the proton cyclotron instability is more efficient to isotropise ions than the mirror instability (McKean et al., 1992), but cannot develop completely, hence the beginning of a loss-cone observed in Fig. 8f and Fig. 8h. At the lowest resolution, ~~both instabilities should grow. However,~~ the spectrum of wave vectors triggered by the instabilities is not broad enough to scatter particles and thermalise the plasma. Therefore, we believe no instability grows in the middle of the magnetosheath at this resolution.

The plasma β is an important parameter to the development of these instabilities (Tsurutani et al., 1982; Gary, 1992). With different initial parameters, such as solar wind speed or IMF strength, the plasma β in the magnetosheath would be different. One could then expect different results regarding the correlation between the spatial resolution and the instabilities. Using the plasma solver HYDROS, we investigated how the plasma β affects the growth rate of the two instabilities (not shown). We found that an increased value of β leads to a higher value of the maximum growth rate γ_{\max} , but does not change significantly the value of the corresponding wave vector k . A higher value of β could slightly improve runs with resolution between 300 and 600 km. The low resolution run would still not resolve a large enough spectrum of wave vectors to allow the proton cyclotron instability to develop.

The absence of the Alfvén mode at lower resolution leads to the discrepancies on the VDFs depending on the spatial resolution. Panels (b)-(d) of Fig. 8 show that, in the higher resolution case, the VDFs appear to have a nearly bi-Maxwellian shape, which is still partially present in panels (f)-(h) at $\Delta r = 600$ km, until this shape is deformed into an "hourglass" shape at the lowest resolution in panels (j)-(l). This shape suggests the presence of a loss-cone instability (Ichimaru, 1980), produced by mirror modes like structures. However, the growth rate of this instability is too slow to develop further. Particles cannot be scattered by the AIC waves, which are absent at low resolution, and hence the particles are trapped within the mirror modes. Therefore we observe a loss-cone in pitch-angle in the VDFs at this resolution. This loss-cone does not appear at the highest resolution, as the AIC waves dominate the wave-particle interaction when both instabilities are present (McKean et al.,

1994). The VDFs shown in Fig. 8 are representative of those observed throughout the studied time range, as can be seen in supplementary videos.

Figure 7 displays the temperature anisotropy of the global simulation at the three different resolutions. The consequence of the absence of the AIC waves can be observed as a higher temperature anisotropy of the magnetosheath at the lowest resolution. Fig. 7 indicates that the temperature anisotropy grows larger as the spatial resolution of the simulation decreases. The AIC waves isotropize VDFs faster than the mirror modes, reducing the temperature anisotropy (Davidson and Ogden, 1975; McKean et al., 1992). **Moreover, one can notice that the position of the quasi-perpendicular bow shock is more outward at the resolution $\Delta r = 900$ km than the others. Due to the AIC waves not being resolved, the temperature anisotropy is greatly increased behind the shock. The lack of mechanism to reduce the perpendicular pressure leads to an increase in the downstream perpendicular pressure, which further leads to a lower shock compression of the plasma. This in turn forces the shock to move outward faster. Therefore, a resolution higher than $\Delta r = 900$ km is necessary to model the quasi-perpendicular bow shock accurately.**

Unresolved waves lead to energy transfer processes which are not being properly simulated, and hence lead to larger temperature anisotropies. One could argue that an easy way to get rid of this issue and to resolve physics beyond the ion inertial length (e.g. kinetic Alfvén waves) would be to use a higher spatial resolution. We conducted a similar study for a spatial resolution of $\Delta r = 227$ km (this simulation was also used in Hoilijoki et al. (2016)), which resolves the ion inertial length, and shows no evidence of new phenomena or wave modes, nor a better modelling of the ones already present at $\Delta r = 300$ km. High-resolution simulations are numerically costly, and therefore are not feasible globally in this fine resolution for the entire volume. This is especially true in large simulations such as global 6D simulations (3D real-space grid and 3D velocity-space grid) which will require adaptive mesh refinement allowing to focus resolution on regions of interest and decrease the resolution and computational cost significantly elsewhere. For our solar wind driving parameters, the ion skin-depth inertial length is 227.7 km, **and around 135.0 km in the magnetosheath for all simulations.** We find that, despite not fully resolving the inertial length, the resolution $\Delta r = 300$ km leads to well resolved proton cyclotron and mirror instabilities. Since they are the two main competing instabilities in magnetosheath plasmas (Anderson and Fuselier, 1993; Gary, 1992; Soucek et al., 2015), we find that the resolution $\Delta r = 300$ km is sufficient to correctly resolve these waves in the magnetosheath. We also find that even at the intermediate resolution $\Delta r = 600$ km, the proton cyclotron instability still produces left-handed circularly polarised waves, and almost bi-Maxwellian VDFs. **While slightly anisotropic, the distributions reach marginal stability levels.** Therefore we believe that an acceptable minimum spatial resolution in a simulation to study magnetosheath waves would lie between $\Delta r = 300$ and 600 km. ~~Based on Fig. 6, we suggest that a spectrum of wave vectors $k \cdot \mathbf{d}_{i,M}$ up to 0.5 would be large enough to allow the proton cyclotron instability to grow sufficiently to produce well resolved AIC waves and correctly model the isotropization processes. This would correspond to a maximum resolution of $\Delta r \approx 440$ km, around $1.76 \mathbf{d}_{i,SW}$ in the solar wind.~~

It is evident that one can make a choice of spatial resolution depending on which waves are wanted in the simulation. However, in case of large-scale simulation volumes where the entire simulation box cannot be represented with a uniform grid resolution, it is interesting to contemplate whether one can use a sub-grid model to reproduce the most important wave

modes at the coarser grid volumes. A future topic of study would be to design an empirical model based on the results we have presented here in this article, in order to modify the VDFs to a more Maxwellian shape, or to solve the Vlasov equation with adding a diffusion term at lower resolution in order to mimic the energy dissipation mechanisms at work at smaller scales.

330 5 Conclusions

This paper presents an investigation into the spatial resolution dependence of two proton instabilities in a global hybrid-Vlasov model. Three 2D simulations of the near Earth-space at different spatial grid resolutions are carried out, and the effects on the produced magnetosheath waves and velocity distribution functions downstream of the quasi-perpendicular shock are investigated.

335 The first simulation uses a resolution of $\Delta r = 300 \text{ km} = 1.32 d_{i,SW} = 2.22 d_{i,M}$. The proton cyclotron instability is identified by the production of left-hand circularly polarised waves around the ion cyclotron frequency, with properties consistent with those of Alfvén Ion Cyclotron waves. The VDFs have a nearly bi-Maxwellian shape, indicating isotropization of the species. We also observe mirror modes in the middle of the magnetosheath, although with a lower growth rate, indicating that they grow further upstream. This resolution allows the proton cyclotron and mirror instabilities to grow adequately.

340 The second simulation uses a resolution of $\Delta r = 600 \text{ km} = 2.64 d_{i,SW} = 4.44 d_{i,M}$. The AIC waves are still present at this resolution, yet not completely resolved. The VDFs are still nearly bi-Maxwellian, with a small loss-cone starting to appear, due the fact that the AIC are only partially resolved. The temperature anisotropy is hence larger than at the previous resolution. Even though the growth rate is larger than at $\Delta r = 300 \text{ km}$, the resolution does not allow the maximum growth rate of the proton cyclotron instability to be reached.

345 The third simulation uses a resolution of $\Delta r = 900 \text{ km} = 3.96 d_{i,SW} = 6.67 d_{i,M}$. Large structures are observed, and the VDFs display a significant loss-cone in the parallel directions. The anti-correlation of the fluctuations in magnetic field and density highlights the presence of mirror modes. In this simulation the temperature anisotropy is much larger than at higher resolutions. This is because the AIC waves are not present anymore. At this resolution, the spectrum of wave vectors is not large enough to allow the instability to grow.

350 This work shows that the proton cyclotron instability does not develop at low spatial resolution. Energy dissipation processes are missing and thus the velocity distribution functions are not isotropised. Larger simulations with inhomogeneous spatial resolution scale should include a sub-grid model, like velocity space diffusion. This would account for the effects of the proton cyclotron instability without a significant increase of numerical resources.

355 **The currently available runs allow us to conclude that the wave modes of interest here are properly resolved at a resolution of $300 \text{ km} = 1.32 d_{i,SW} = 2.22 d_{i,M}$. The growth rate profiles suggest that larger cell sizes, between 300 and 600 km, may still be sufficient to resolve those wave modes in the simulations. This hypothesis could be tested by running additional global simulations with a range of spatial resolutions. Such parametric study is however not currently achievable because of the large computational costs of global Vlasov runs at these relatively high resolutions.** At the highest resolution, the proton cyclotron instability is well resolved. The proton cyclotron instability and the mirror instability

360 are the two competing instabilities in this simulation. **Based on our results, we conclude that, if the focus of the simulation is to evaluate the effects related to the proton cyclotron and mirror instabilities in the magnetosheath**, there is no need to increase the spatial resolution of a simulation beyond $\Delta r = 300 \text{ km} = 1.32 d_{i,\text{SW}} = 2.22 d_{i,\text{M}}$ at the cost of numerical resources. We also conclude that the resolution $\Delta r = 440 \text{ km} = 1.76 d_{i,\text{SW}} = 3.26 d_{i,\text{M}}$ would still give an adequate description of the proton cyclotron instability and therefore is an acceptable minimum spatial resolution for a simulation to
365 study magnetosheath waves ~~the proton cyclotron and mirror instabilities.~~

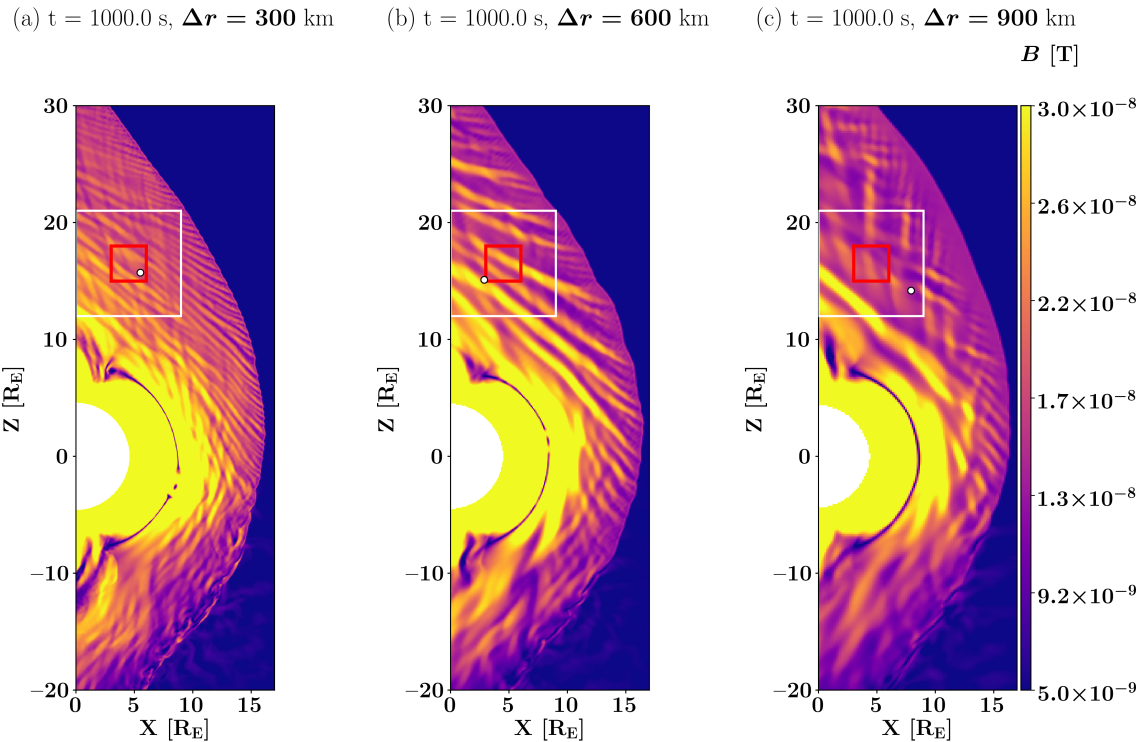


Figure 1. Global overview of the simulation setup with three different spatial resolutions: (a) $\Delta r = 300$ km, (b) $\Delta r = 600$ km and (c) $\Delta r = 900$ km. The colormap in each run is the magnitude of the magnetic field. The white square displays the area we focus on in the magnetosheath in the rest of the study. The red square displays the area where the FFT in Fig. 2 is performed. The black and white dot displays the location where data are taken for Fig. 3, 4, 5, 6, and the VDFs in Fig. 8.

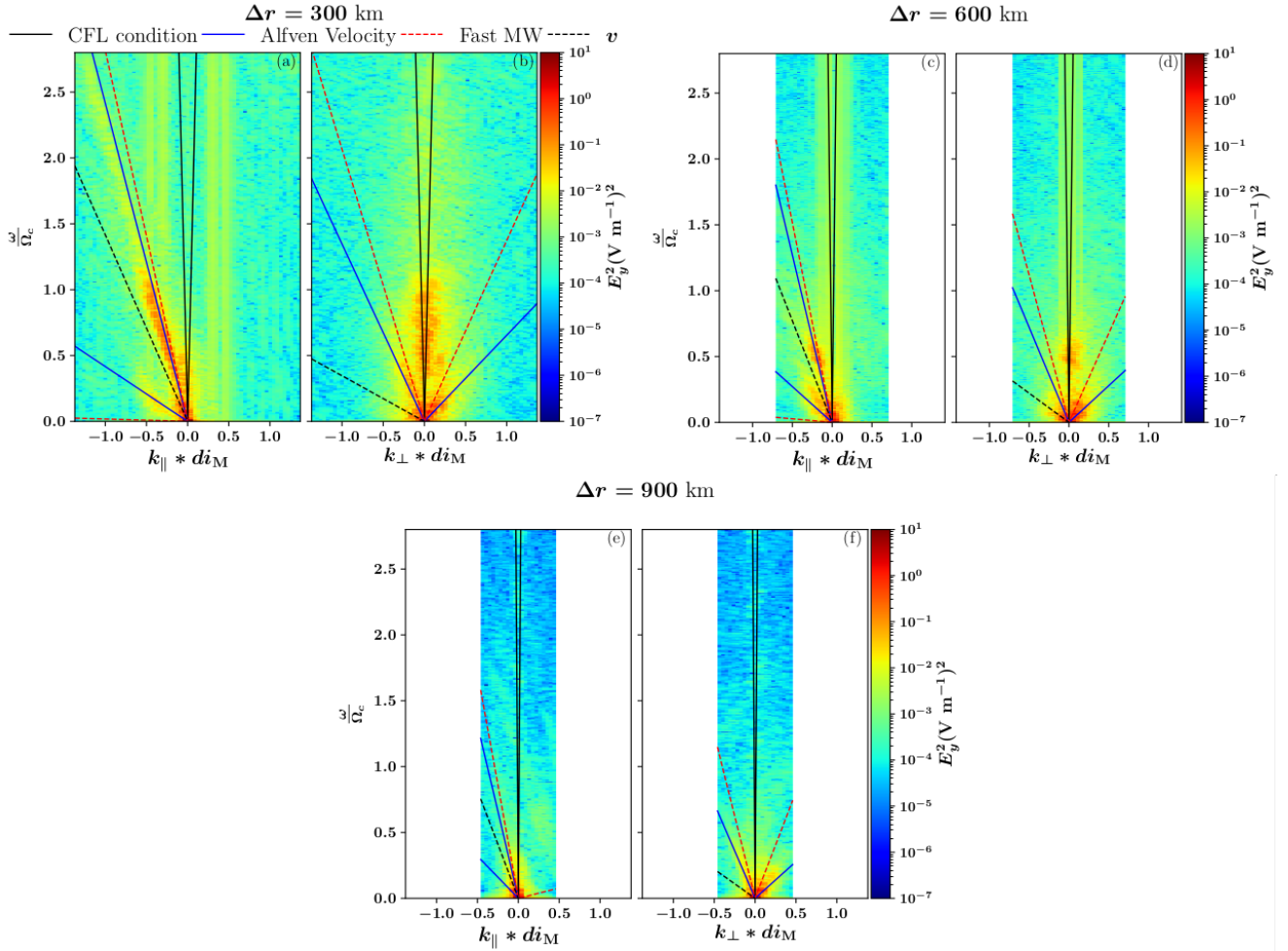


Figure 2. 2D Fast Fourier Transform of the y-component of the electric field in the direction parallel and perpendicular to the background magnetic field, at the location depicted by a red square in Fig. 1. Panels (a) and (b) display the results for the run at resolution $\Delta r = 300$ km, panels (c) and (d) the results for the run at resolution $\Delta r = 600$ km, and panels (e) and (f) the results for the run at resolution $\Delta r = 900$ km. The solid black lines represent the Courant-Friedrichs-Lewy condition, the solid blue lines the Alfvén speed, the dashed red lines the fast magnetosonic speed (labelled MW), with all wave frequencies shown in the simulation frame. The dashed black lines show the Doppler shift due to the plasma bulk flow.

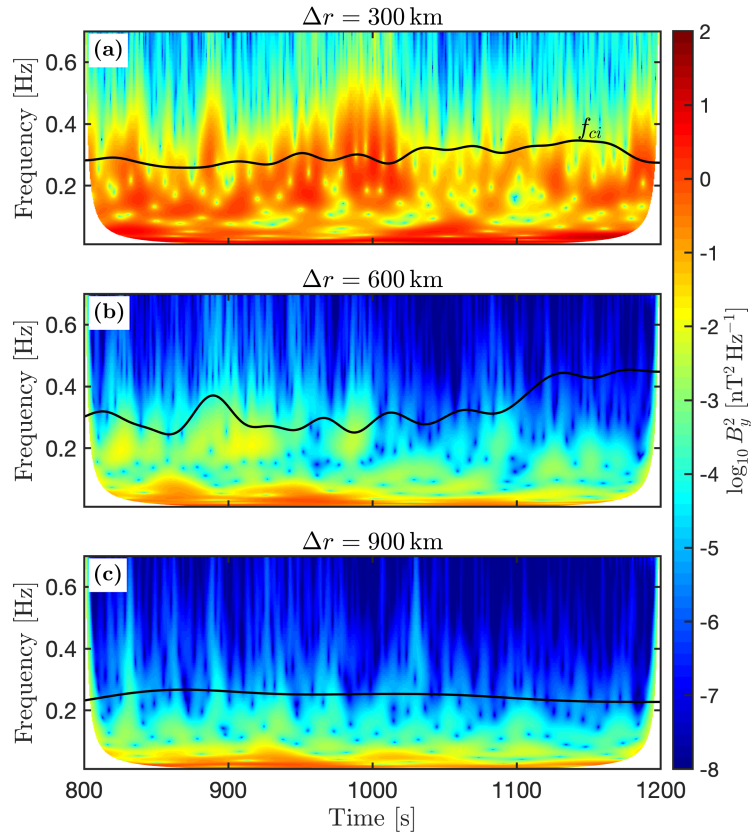


Figure 3. Wavelet analysis of the magnetic field for the virtual space craft locations given in Fig. 1. The colour background represents the power spectrum density of the y-component of the magnetic field. Panels (a) displays the results for the run at resolution $\Delta r = 300$ km, panel (b) the run at resolution $\Delta r = 600$ km, and panel (c) the run at resolution $\Delta r = 900$ km. The black curve on each plot indicates the proton cyclotron frequency.

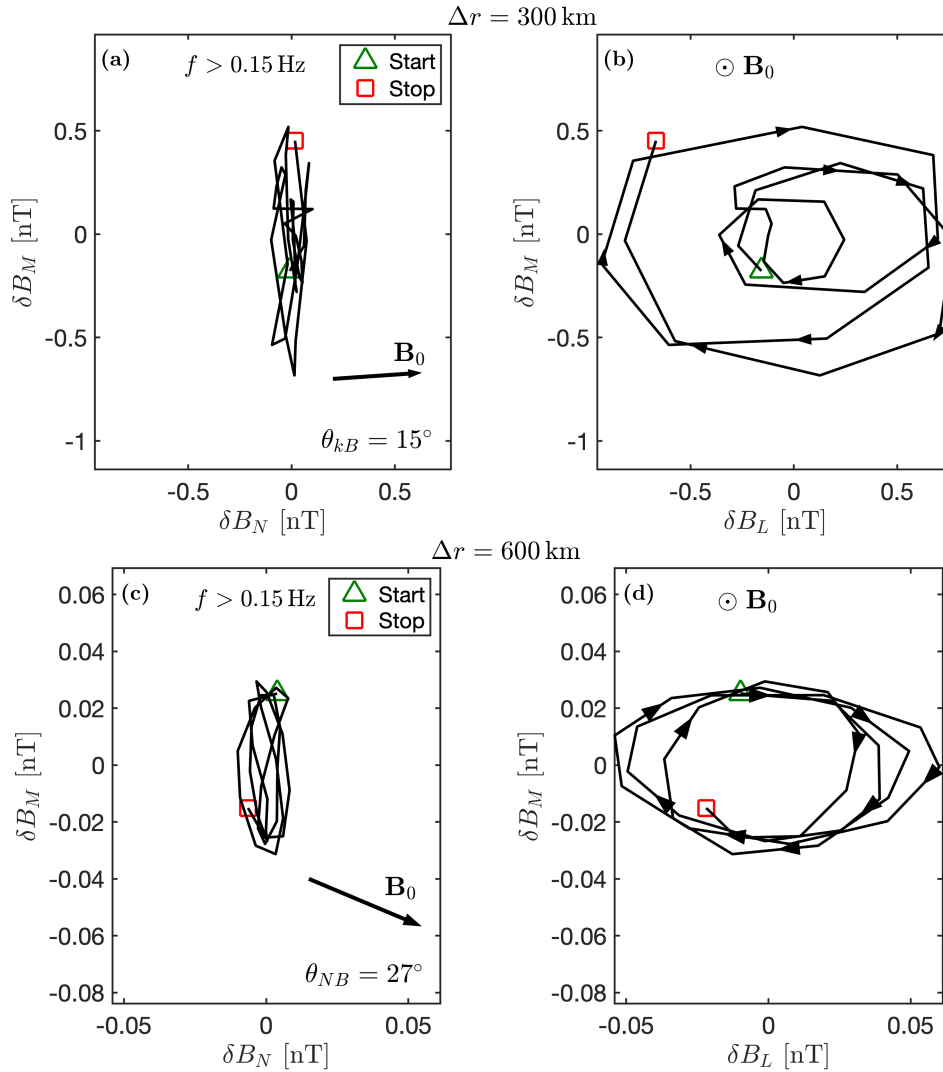


Figure 4. Hodogram of the magnetic field fluctuations. Panels (a) and (b) display the case $\Delta r = 300 \text{ km}$, taken between $t = 996.5 \text{ s}$ and $t = 1013.5 \text{ s}$. Panels (c) and (d) display the case $\Delta r = 600 \text{ km}$, taken between $t = 910.0 \text{ s}$ and $t = 928.0 \text{ s}$. Panels (a) and (c): intermediate (δB_M) and minimum (δB_N) variance directions. An arrow marks the average background field \mathbf{B}_0 . Panels (b) and (d): intermediate (δB_M) and maximum (δB_L) variance directions. Arrows show the time evolution of the fluctuations. A green triangle marks the start of the interval and a red square marks the end.

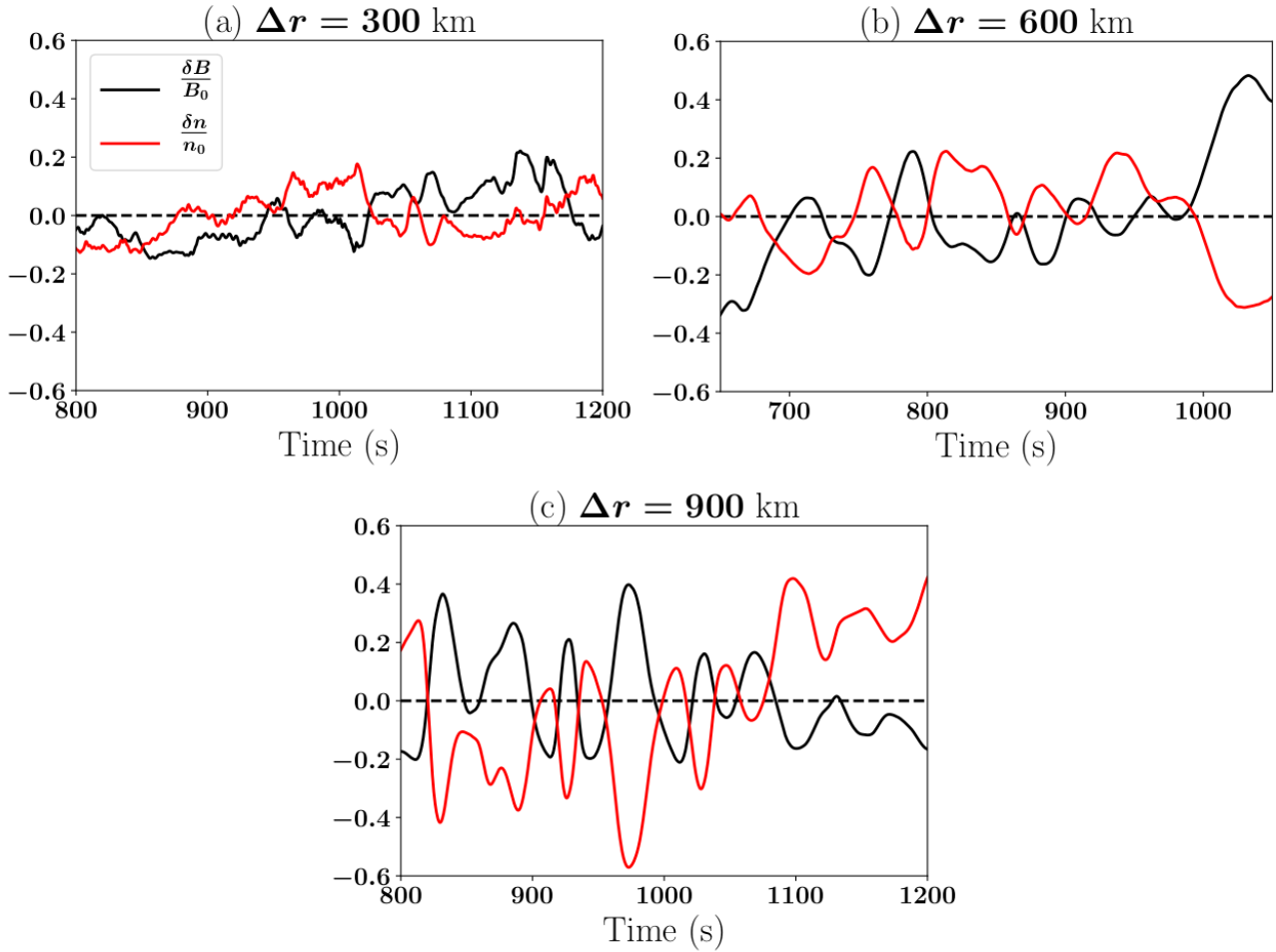


Figure 5. Magnetic field (black) and density (red) fluctuations for the **three simulations** measured at the virtual spacecraft locations indicated by a black and white dot in Fig. 1.

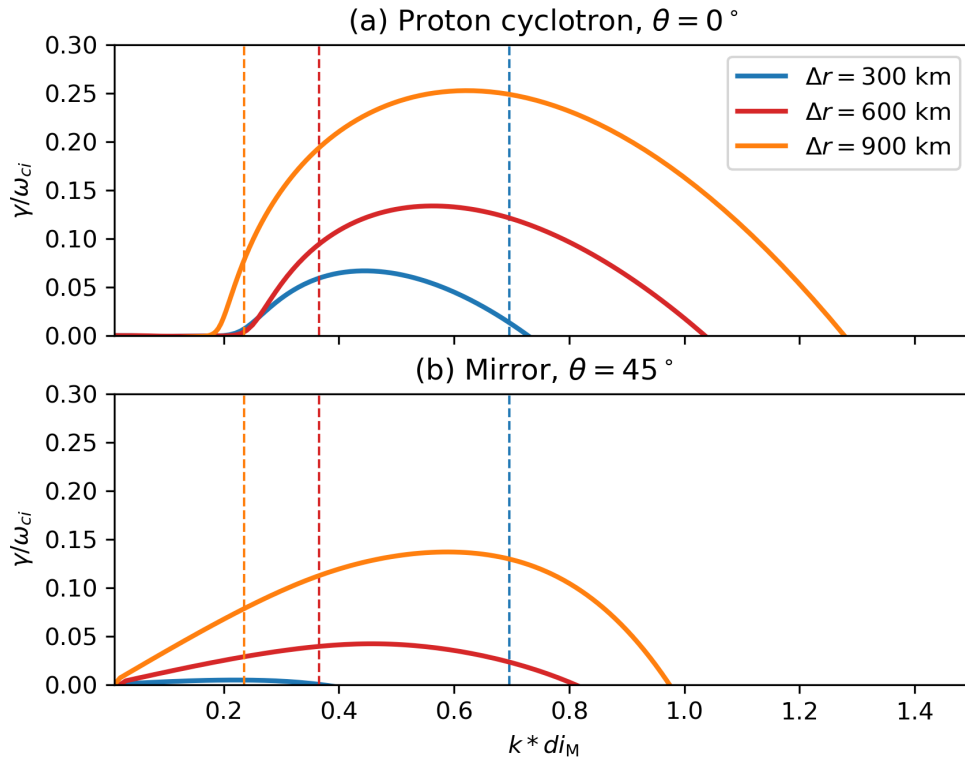


Figure 6. Growth rates γ of the proton cyclotron (panel (a)) and Mirror (panel (b)) instabilities calculated by HYDROS for the three different spatial resolutions: $\Delta r = 300$ km (blue), $\Delta r = 600$ km (red) and $\Delta r = 900$ km (orange). The dashed lines represent the maximum wave number $k_{\max} = \pi/2\Delta r$ which can be resolved by the model to get a proper description of the signal at each resolution.

(a) $t = 1000.0$ s, $\Delta r = 300$ km (b) $t = 1000.0$ s, $\Delta r = 600$ km (c) $t = 1000.0$ s, $\Delta r = 900$ km

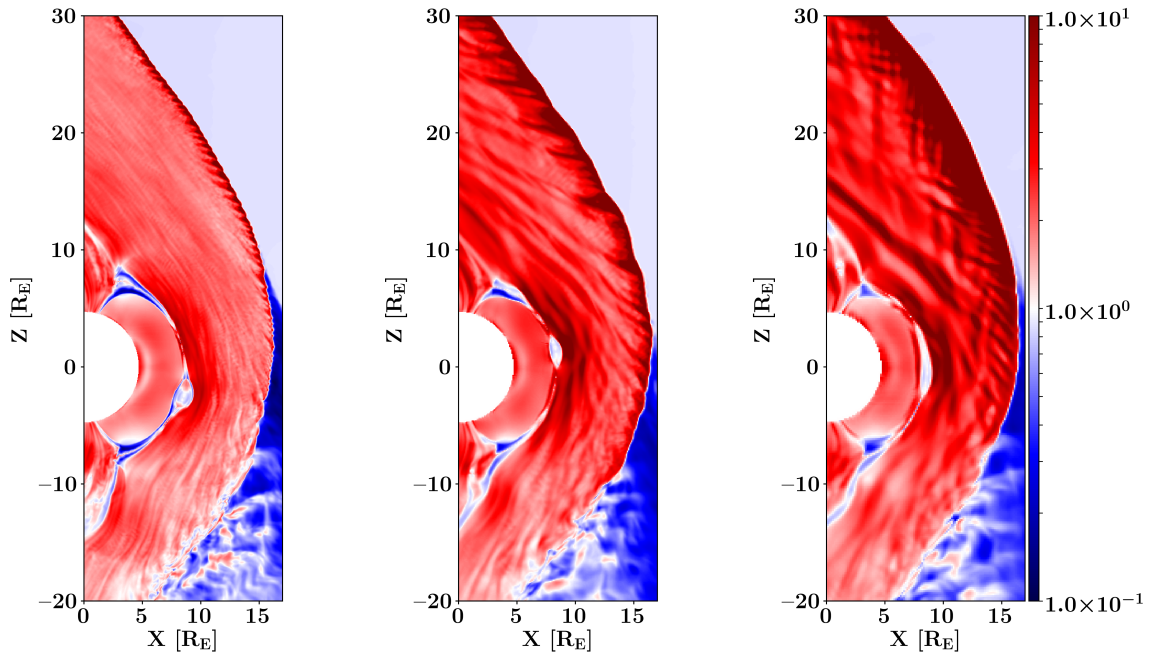


Figure 7. Temperature anisotropy for the resolution: (a) $\Delta r = 300$ km, (b) $\Delta r = 600$ km, (c) $\Delta r = 900$ km.

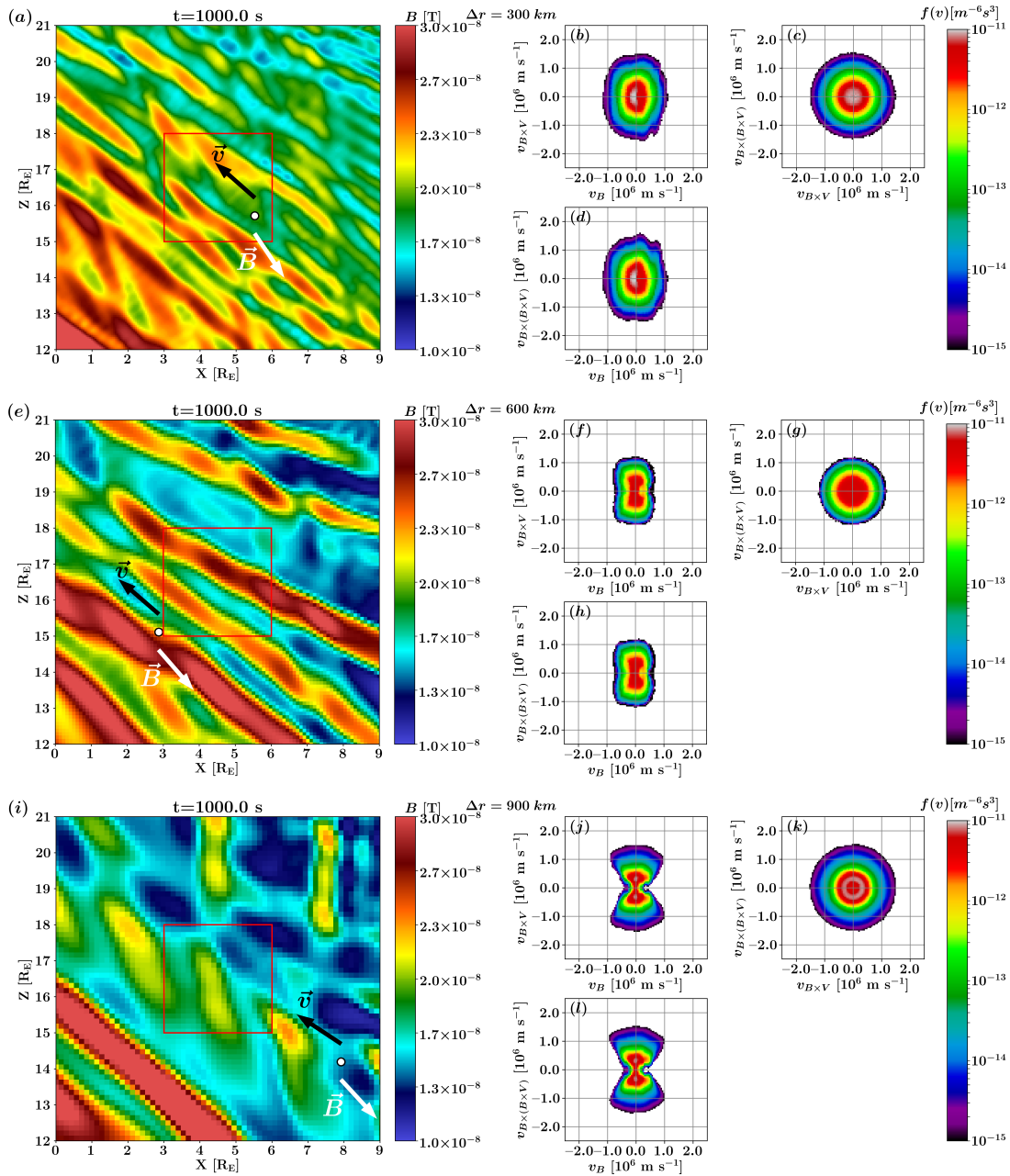


Figure 8. Colormap of the magnetic field (left) and velocity distribution functions in the three directions (right) located at the black and white circle. Panels (a)-(d) display the run with resolution $\Delta r = 300$ km, panels (e)-(h) the run with resolution $\Delta r = 600$ km and panels (i)-(l) the run with resolution $\Delta r = 900$ km. The black arrow displays the plasma bulk velocity and the white arrow displays the magnetic field direction, both taken at the location indicated by the black and white circle. The red square displays the area where the 2D-FFT is taken (Fig. 2).

Table 1. Summary table for each run with spatial resolutions, the inertial length $d_{i,M}$ in the solar wind, the maximum wave vector allowed by the simulation $k_{\max} = \pi/\Delta r$, the wave vector at which the proton cyclotron instability's growth rate is maximum $k_{\gamma_{\max}}$, status of the proton cyclotron and mirror instabilities in the simulation, VDF shapes in the $(\mathbf{v}_B, \mathbf{v}_B \times \mathbf{v})$ plane (Fig. 8b, f and j), temperature anisotropy, and plasma beta taken at the location indicated in Fig. 1 averaged over the time range used in this study.

Δr (km)	$d_{i,SW}$ (km)	$d_{i,M}$ (km)	$k_{\max}d_{i,M}$	$k_{\gamma_{\max}}d_{i,M}$	Proton cyclotron	Mirror	VDF $(\mathbf{v}_B, \mathbf{v}_B \times \mathbf{v})$	T_{\perp}/T_{\parallel}	β
300	228	135	0.68	0.44	yes	yes	Bi-Maxwellian	1.97	2.58
600	228	135	0.35	0.53	partially	yes	Nearly bi-Maxwellian beginning of loss-cone	2.56	2.72
900	228	135	0.23	0.63	no	yes	Loss-cone	3.41	4.18

Code and data availability. Vlasiator (<http://www.physics.helsinki.fi/vlasiator/>, (Palmroth et al., 2018) is distributed under the GPL-2 open source license at <https://github.com/fmihpc/vlasiator/> (Palmroth & the Vlasiator team, 2019). Vlasiator uses a data structure developed in-house (<https://github.com/fmihpc/vlsv/>, Sandroos, 2018), which is compatible with the VisIt visualization software (Childs et al., 2012) using a plugin available at the VLSV repository. The Analysator software (<https://github.com/fmihpc/analysator/>, Hannuksela & the Vlasiator team, 2018) was used to produce the presented figures. The runs described here take several terabytes of disk space and are kept in storage maintained within the CSC – IT Center for Science. Data presented in this paper can be accessed by following the data policy on the Vlasiator website.

Video supplement. TEXT

Author contributions. MD carried out most of the study and the writing of the paper. UG and YP-K participated in running the different simulations and the development of the analysis methods. MB participated in the development of the analysis methods. AO, MG, AJ and LT contributed in the data analysis and discussions. MP is the PI of the Vlasiator model and gave input on the interpretation of the simulation results. All co-authors participated in the discussion of the results and contributed in improving the manuscript.

Competing interests. The authors declare they have no conflicts of interest.

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