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Estimating the maximum of smoothed highest 3-hourly aa index in 3 days by the preceding minimum for the solar cycle

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Abstract

Predicting the maximum intensity of geomagnetic activity for an upcoming solar cycle is important in space weather service and for planning future space missions. This study analyzed the highest/lowest 3-hourly aa index ($aa_{\mathsf{H}}/aa_{\mathsf{L}}$) in 3-day-interval, smoothed by 363 days to analyze their variation with the 11-year solar cycle. It is found that the maximum of aa_{H} (aa_{Hmax}) is well correlated with the preceding minimum of either aa_{H} (aa_{Hmin}, $r = 0.85$) or aa_{L} (aa_{Lmin}, $r = 0.89$) for the solar cycle. Based on these relationships, the intensity of aa_{Hmax} for solar cycle 25 is estimated to be $aa_{\mathsf{Hmax}}(25) = 83.7 \pm 6.9$ (nT), about 29% stronger than that of solar cycle 24. This value is equivalent to the Ap index of $Ap_{\mathsf{max}}(25) = 47.4 \pm 4.4$ (nT) if employing the high correlation between Ap and aa ($r = 0.93$). The maximum of aa_{L} (aa_{Lmax}) is also well correlated with the preceding aa_{Hmin} ($r = 0.80$). The maximum amplitude of the sunspot cycle (R_{m}) is much better correlated with the high geomagnetic activity (aa_{Hmax}, $r = 0.79$) than with the low one (aa_{Lmax}, $r = 0.37$). The rise time from aa_{Hmin} to aa_{Hmax} is weakly anti-correlated to the following aa_{Hmax} ($r = -0.42$). Similar correlations are also found for the 13-month smoothed monthly mean aa index. These results are expected to be useful in understanding the geomagnetic activity intensity of solar cycle 25.

1 Introduction

Studying and predicting geomagnetic activity are important in both geophysics and space weather. Severe geomagnetic activity may cause intense geomagnetic storms (Gonzalez et al., 1989, 1994; Chen et al., 2019), leading to disruptions in communication and deviations of orbital motions of satellites (Yoshida and Yamagishi, 2010; Petrovay, 2020). Now, the old solar cycle 24 being over, satellite and spacecraft-related departments want to know the maximum intensity of both solar and geomagnetic activity in the new solar cycle 25 for planning future space missions.

Among various indices to quantitatively describe geomagnetic activity, the aa index

(Mayaud, 1972), derived from the 3-hourly K index at two near-antipodal midlatitude stations in England and Australia, is the longest time series (since 1868) and has been widely used to analyze the long-term trend of global geomagnetic activity (Russell and Mulligan, 1995; Marat et al., 2017; Du, 2011b; El-Borie et al., 2019) and its correlation with both climate change (Cliver et al., 1998; Dobrica et al., 2009; Gavrilyeva et al., 2017) and solar activity (Echer et al., 2004; Prestes et al., 2006; Lukianova et al., 2009; Du, 2011a,c; Du and Wang, 2012; Singh and et al., 2019). The minimum aa index (aa_{min}), at or near the minimum of the solar cycle, has been widely used to predict the maximum amplitude of the sunspot cycle (R_{m}), the so-called Ohl's precursor method (Brown and Williams, 1969; Ohl and Ohl, 1979; Du et al., 2009). But it is seldom used to directly predict the maximum aa index (aa_{max}) of an ensuing solar cycle.

The planetary geomagnetic index Ap (available since 1932, Bartels, 1963), derived from the average of the measurements at 13 observatories around the globe, is a daily measure of the response of geomagnetic field to variations in the interplanetary magnetic field (IMF) and the solar wind (Li, 1997; McPherron, 1999; Tsurutani et al., 2006). It is the main global magnetic index forecasted by government agencies (McPherron, 1999). Most works on forecasting the geomagnetic activity have been over short intervals, on the order of hours or days (McPherron, 1999; Abunina et al., 2013). In the earlier years, Kane (1988) pointed out that it is impossible to forecast the long-term geomagnetic activity through analyzing the daily, monthly and annual Ap and aa indices. Gordon (2015) demonstrated that the long-term geomagnetic activity can only be predicted to within a limited threshold of accuracy due to the irregular trends and cycles in the annual data and nonlinear variability in the monthly series, through analyzing the aa index.

In this study, we analyze the relationship between the maximum aa index and its preceding minimum for the 11-year solar cycle. The data and parameters used in this study are described in Sect. 2. We find out the highest/lowest 3-hourly aa index ($aa_{\mathsf{H}}/aa_{\mathsf{L}}$) in each 3 days' interval, smoothed by 363 days (121 points) to analyze their variation with the solar cycle. In Sect. 3, it is found that the maximum of aa_{H} (aa_{Hmax})

is well correlated with the preceding minimum of either aa_{H} (aa_{Hmin}) or aa_{L} (aa_{Lmin}) for the solar cycle (Sect. 3.1), which can be used to estimate the intensity of aa_{Hmax} for solar cycle 25. The maximum of aa_{L} (aa_{Lmax}) is also found to be well correlated with the preceding aa_{Hmin} (Sect. 3.2). The rise time from aa_{Hmin} to aa_{Hmax} is only weakly anti-correlated to the following aa_{Hmax} (Sect. 3.3). Using the relationship between Ap and aa, the maximum intensity of the Ap index for solar cycle 25 is estimated (Sect. 3.4). Similar correlations are analyzed using the 13-month smoothed (with half weight at the two ends) monthly mean aa index (Sect. 4). Some conclusions are discussed and summarized in Sect. 5.

2 Data

We use the 3-hourly aa index (in unit of nT) since 1 January 1868 (updated to 10 October 2020) from the International Service of Geomagnetic Indices (ISGI, isgi.unistra.fr/). We find out the highest/lowest aa index ($aa_{\mathsf{H}}/aa_{\mathsf{L}}$) from 24 values of the 3-hourly aa index in each 3 days' interval, as shown in Fig. 1. The values of aa_{H} are in the range [5, 715] nT and those of aa_{L} are in the range [2, 45] nT.

It is seen in the figure that both aa_{H} and aa_{L} vary dramatically: the average absolute differences of the adjacent values are 36.7 nT and 2.5 nT, respectively. It is hard to see the variation in aa_{H} or aa_{L} with the solar cycle. Especially, most (63%) values of aa_{L} are the minimum (2 nT). In order to analyze the long-term trend of aa with the solar cycle conventionally represented by the 13-month smoothed monthly mean sunspot number, both aa_{H} and aa_{L} are smoothed by $w = 121$ points (363 days) using the following running smoothing technique,

$$\overline{x}(i) = \frac{1}{w} \sum_{j=-(w-1)/2}^{(w-1)/2} x(i+j),\ i = (w-1)/2, ..., N_0 - (w-1)/2, \tag{1}$$

for $x = aa_{\mathsf{H}}$ and aa_{L} ($N_0 = 18600$). There are $N = N_0 - (w-1) = 18480$ data points left after removing the ($w-1$) incomplete-smooth points at the two ends of the series, as

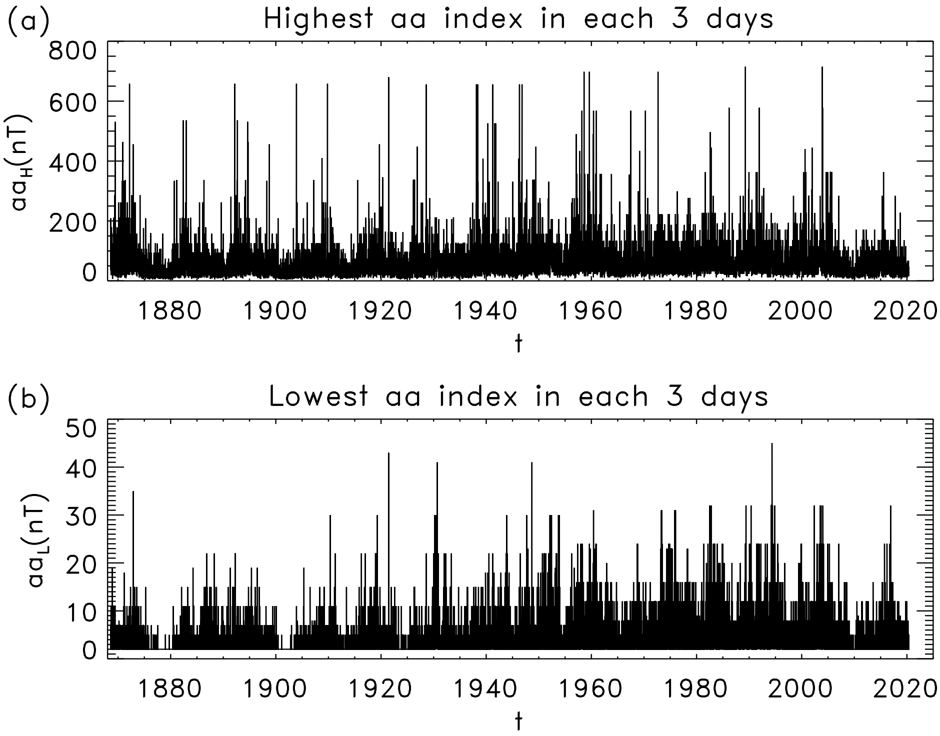

Fig. 1. (a) The highest ($aa_{\rm H}$) and (b) lowest ($aa_{\rm L}$) 3-hourly aa index in each 3 days.

shown in Fig. 2 (solid). The smoothed monthly mean ($w = 13$ months with half weight

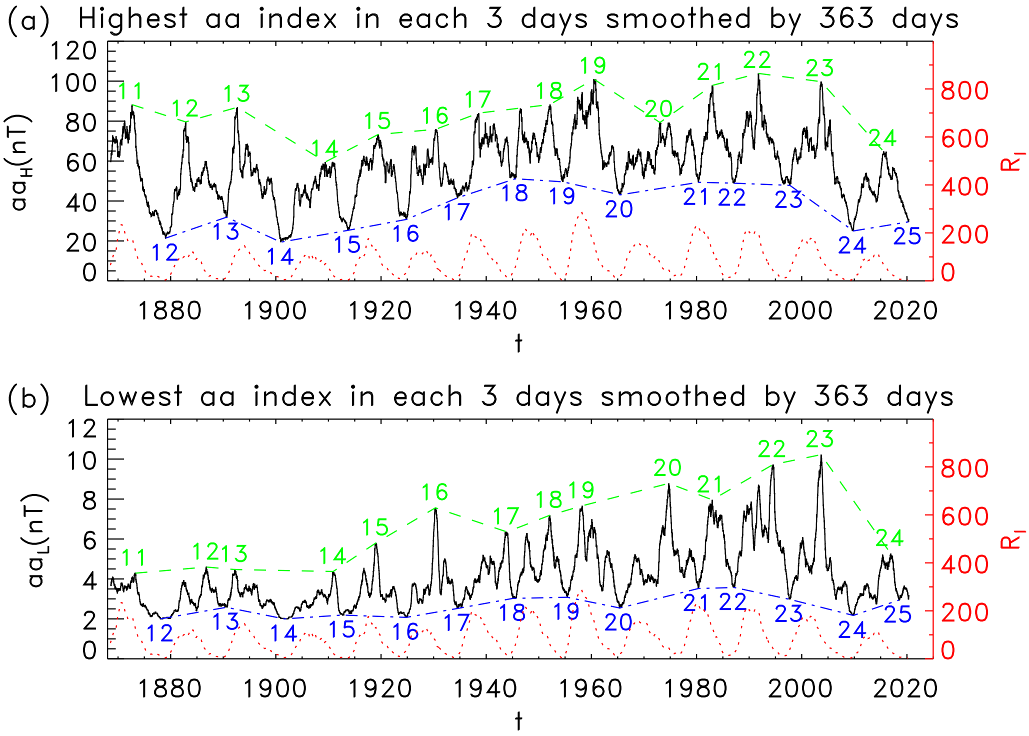

Fig. 2. (a) The highest (aa_{H}) and (b) lowest (aa_{L}) 3-hourly aa index in each 3 days (black solid), smoothed by 121 points (363 days). The numbers in the figure represent the 11-year solar cycles. The upper dashed and lower dash-dotted lines indicate the maxima and minima, respectively, for the solar cycle. The red dotted line shows the 13-month smoothed monthly mean sunspot number (R_{I}) for comparison.

at the two ends) International sunspot number series (R_{I}, Clette et al., 2016),

$$R_{\mathrm{I}}(i) = \frac{1}{12}\left(\frac{R(i-6)}{2} + \sum_{j=-5}^{5} R(i+j) + \frac{R(i+6)}{2}\right), \tag{2}$$

of the second version (www.sidc.be/silso/datafiles) is used for comparison (dotted), where R represents the (non-smoothed) monthly value (Petrovay, 2020). The smooth-width is selected to be $w = 121$ points (363 days) as it is close to that (1 year = 12 months) used in the smoothing of R_{I}.

In the upper panel of the figure, the dashed (dash-dotted) line indicates the maximum (minimum) of aa_{H}, aa_{Hmax} (aa_{Hmin}). In the lower panel, the dashed (dash-dotted) line indicates the maximum (minimum) of aa_{L}, aa_{Lmax} (aa_{Lmin}). The value of aa_{Hmax} (aa_{Lmax}) is the maximum of aa_{H} (aa_{L}) during the time period between two adjacent solar cycle minima (R_{min}) determined by the smoothed monthly mean R_{I}. The value of aa_{Hmin} (aa_{Lmin}) is the minimum of aa_{H} (aa_{L}) during the time period between two adjacent aa_{Hmax} (aa_{Lmax}). These parameters are displayed in Table 1, in which T_{r} is the rise time from aa_{Hmin} to aa_{Hmax} and R_{m} the maximum of R_{I} for the 11-year solar cycle. The last row denotes the averages of the parameters.

3 Result

The correlation coefficients between the parameters in Table 1 are listed in Table 2 for comparison. It is seen in Table 2 that R_{m} is well correlated with aa_{Hmin} ($r = 0.84$), aa_{Hmax} ($r = 0.79$), aa_{Lmin} ($r = 0.81$), and positively correlated with aa_{Lmax} ($r = 0.37$). This implies that the stronger the solar activity (R_{I}), the higher the geomagnetic activity (aa). But the maximum amplitude of sunspot cycle (R_{m}) is much better correlated with the high geomagnetic activity (aa_{Hmax}, $r = 0.79$) than with the low one (aa_{Lmax}, $r = 0.37$), implying that the low geomagnetic activity depends less on the solar activity than the high one does. The correlation between R_{m} and aa_{Hmin} (aa_{Lmin}), $r = 0.84$ (0.81), is

Table 1. The minimum (aa_{Hmin}) and maximum (aa_{Hmax}) of 363-day smoothed highest 3-hourly aa index (aa_{H}) in each 3 days, the rise time from aa_{Hmin} to aa_{Hmax} (T_{r}), the minimum (aa_{Lmin}) and maximum (aa_{Lmax}) of 363-day smoothed lowest 3-hourly aa index (aa_{L}) in each 3 days, and the maximum (R_{m}) of 13-month smoothed monthly mean R_{I} for solar cycles 11-25.

n (unit)	aa_{Hmin} (nT)	aa_{Hmax} (nT)	T_{r} (year)	aa_{Lmin} (nT)	aa_{Lmax} (nT)	R_{m}
11		88.24			4.29	234.0
12	21.41	79.60	3.81	2.00	4.59	124.4
13	31.88	86.92	2.08	2.58	4.47	146.5
14	19.41	59.54	8.47	2.00	4.37	107.1
15	25.31	73.28	5.72	2.19	5.80	175.7
16	30.80	75.90	5.56	2.07	7.56	130.2
17	41.60	84.01	4.01	2.52	6.36	198.6
18	51.13	88.31	6.57	3.03	7.18	218.7
19	49.60	100.99	5.89	3.08	7.66	285.0
20	43.08	79.31	7.54	2.55	8.78	156.6
21	49.35	97.87	2.63	3.52	7.96	232.9
22	48.93	103.89	4.78	3.57	9.73	212.5
23	47.84	99.75	5.92	2.97	10.22	180.3
24	25.02	64.81	5.73	2.17	5.27	116.4
25	29.57			2.94		
Av.	36.78	84.46	5.29	2.66	6.73	179.9

related to the Ohl's precursor method (Ohl and Ohl, 1979) for predicting R_{m}. Some other correlations are analyzed to estimate aa_{Hmax} (Sect. 3.1), aa_{Lmax} (Sect. 3.2), and T_{r} (Sect. 3.3).

Table 2. Correlation coefficients between the parameters in Table 1.

r	aa_{Hmin}	aa_{Hmax}	T_{r}	aa_{Lmin}	aa_{Lmax}	R_{m}
aa_{Hmin}	1.00	0.85	−0.10	0.84	0.80	0.84
aa_{Hmax}	0.85	1.00	−0.42	0.89	0.63	0.79
T_{r}	−0.10	−0.42	1.00	−0.28	0.13	−0.18
aa_{Lmin}	0.84	0.89	−0.28	1.00	0.70	0.81
aa_{Lmax}	0.80	0.63	0.13	0.70	1.00	0.37

3.1 Relationship between aa_{Hmax} and its preceding $aa_{\mathrm{Hmin}}/aa_{\mathrm{Lmin}}$

One can see in Table 2 that aa_{Hmax} is well correlated with its preceding aa_{Hmin} $(r = 0.85)$ and aa_{Lmin} $(r = 0.89)$, as shown in Fig. 3 for the scatter plots of aa_{Hmax} against aa_{Hmin} (a) and aa_{Lmin} (b). The solid lines represent the linear fits of aa_{Hmax} to aa_{Hmin} and aa_{Lmin} with the least-squares-fit regression equations given by

$$\begin{aligned} aa_{\mathrm{Hmax}} &= 47.1 \pm 7.1 + (0.99 \pm 0.18) aa_{\mathrm{Hmin}},\ \sigma = 7.3, \\ aa_{\mathrm{Hmax}} &= 25.3 \pm 9.5 + (22.3 \pm 3.5) aa_{\mathrm{Lmin}},\ \sigma = 6.5, \end{aligned} \tag{3}$$

where $\pm$ indicates the 1-σ deviation of the fitting coefficient and σ is the standard deviation of the regression.

Based on the above relationships, the minimum aa index (around the solar minimum) can be used as an indicator to estimate the following maximum. One can estimate aa_{Hmax} for solar cycle 25 by substituting the values of aa_{Hmin} (29.57 nT) and aa_{Lmin} (2.94 nT) into the above equations,

$$\begin{aligned} aa_{\mathrm{Hmax1}}(25) &= 76.4 \pm 7.3 (\mathrm{nT}),\ \text{from } aa_{\mathrm{Hmin}}, \\ aa_{\mathrm{Hmax2}}(25) &= 91.0 \pm 6.5 (\mathrm{nT}),\ \text{from } aa_{\mathrm{Lmin}}, \end{aligned} \tag{4}$$

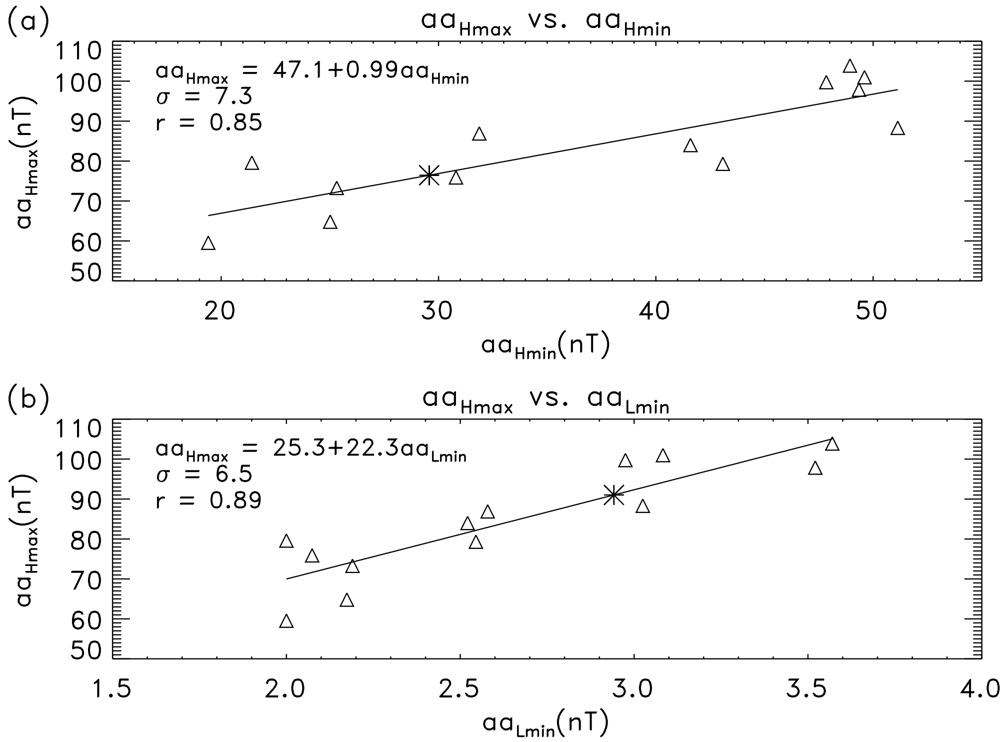

Fig. 3. Scatter plots of aa_{Hmax} against aa_{Hmin} (a) and aa_{Lmin} (b).

(labelled by asterisks). Since these values are derived by the fitting equations (Eq. (3)) with similar correlation coefficients (0.85 and 0.89), we take their average,

$$
\begin{aligned}
aa_{\mathrm{Hmax}}(25) &= \frac{1}{2}[aa_{\mathrm{Hmax1}}(25) + aa_{\mathrm{Hmax2}}(25)] \\
&= 83.7 \pm 6.9 (\mathrm{nT}),
\end{aligned} \tag{5}
$$

as an estimate of $aa_{\mathsf{Hmax}}(25)$. It implies that the 363-day smoothed highest 3-hourly aa index in 3-day-interval during the maximum period of solar cycle 25 is estimated to be close to the average (84.46 nT) over the past cycles (Table 1), but higher than that (64.81 nT) of solar cycle 24 by about 29.2%.

It should be pointed out that the above estimate may be an upper limit of $aa_{\mathsf{Hmax}}(25)$ as the values of $aa_{\mathsf{Hmin}}(25)$ and $aa_{\mathsf{Lmin}}(25)$ may not be finally determined (refer to Discussion in Sect. 5). Although we are not quite sure if the current aa_{H} (aa_{L}), 30.69 (3.13) in April 2020, would decrease to a smaller value than that, 29.57 (2.94), used in the current work, there would not be significant variations in aa_{Hmin}, aa_{Lmiin}, and the above estimate, because solar cycle 25 (in term of the smoothed monthly mean R_{I}) has already started (December 2019) a few months.

3.2 Relationship between aa_{Lmax} and the preceding aa_{Hmin}

One can also see in Table 2 that aa_{Lmax} is well correlated with the preceding aa_{Hmin} $(r = 0.80)$ or aa_{Lmin} $(r = 0.70)$. Fig. 4(a) shows the scatter plot of aa_{Lmax} against aa_{Hmin}. The linear fitting equation of aa_{Lmax} to aa_{Hmin} (solid) is

$$aa_{\mathsf{Lmax}} = 2.0 \pm 1.2 + (0.131 \pm 0.030) aa_{\mathsf{Hmin}}, \; \sigma = 1.2. \tag{6}$$

Substituting $aa_{\mathsf{Hmin}}(25) = 29.57$ (nT) into this equation, one can estimate the 363-day smoothed *lowest* 3-hourly aa index in 3-day-interval during the maximum period of solar cycle 25, $aa_{\mathsf{Lmax}} = 5.9 \pm 1.2$ (nT). This value is slightly lower than the average (6.73 nT) over the past cycles, but higher than that (5.27 nT) of solar cycle 24 by 12.0%.

3.3 Relationship between the rise time and the following maximum

Now, we analyze if the rise time of the aa geomagnetic index for the solar cycle is correlated with the following maximum so that it can be used to estimate the rise time, as the case often used in the solar (sunspot) cycle (Waldmeier, 1939).

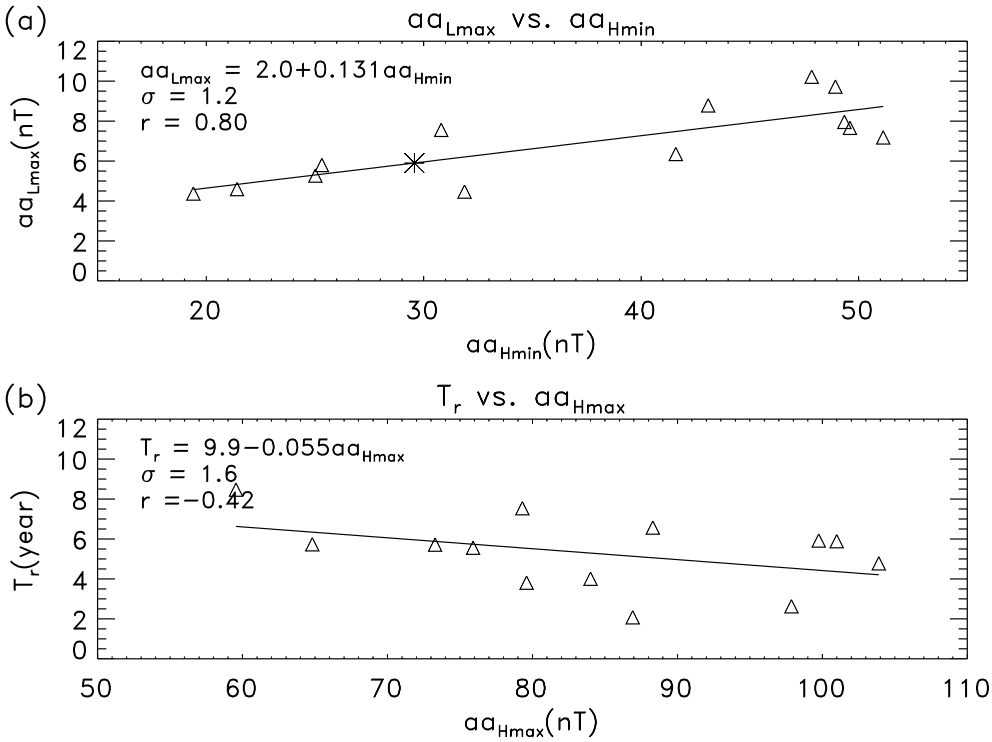

Fig. 4. Scatter plots of aa_{Lmax} against aa_{Hmin} (a) and T_{r} against aa_{Hmax} (b).

Figure 4(b) shows the scatter plot of the rise time (T_{r}) from aa_{Hmin} to aa_{Hmax} for the solar cycle against its following maximum (aa_{Hmax}). The solid line indicates the linear fit of T_r to aa_Hmax by

$$T_\mathsf{r} = 9.9 \pm 3.0 - (0.055 \pm 0.036) aa_\mathsf{Hmax},\ \sigma = 1.6\ (\text{years}). \quad (7)$$

The anti-correlation coefficient between T_r and aa_Hmax, $r = -0.42$ (at a confidence level of about 84%), is so weak that it can hardly be used to estimate the rise time (T_r). If the rise time is computed from the minimum (R_min) of the sunspot cycle to aa_Hmax, the correlation is even weaker, $r = -0.14$.

3.4 Relationship between Ap and aa

Another important index often used to evaluate geomagnetic activity is the Ap index (www.gfz-potsdam.de/en/kp-index). Similar correlations as those for the aa index can also be obtained. But the Ap index is available only from 1932 to April 2018. In this section, we employ the relationship between Ap and aa to estimate the maximum intensity of the Ap index for solar cycle 25 from the previous result. Figure 5(a) shows the scatter plot of the 363-day smoothed 3-hourly Ap against aa (dots). The solid line represents the linear fit of Ap to aa by

$$Ap = 0.12 \pm 0.01 + (0.5647 \pm 0.0005) aa,\ \sigma = 2.1. \quad (8)$$

It is obvious that Ap is highly correlated with aa, $r = 0.93$, as they are based on the same observations.

According to this equation, the estimated $aa_\mathsf{Hmax}(25) = 83.7 \pm 6.9$ (nT) in Sect. 3.1 is equivalent to $Ap_\mathsf{max}(25) = 47.4 \pm 3.9 \pm 2.1 = 47.4 \pm 4.4$ (nT), in which ± 3.9 is derived from the uncertainty (± 6.9) of $aa_\mathsf{Hmax}(25)$, ± 2.1 is the standard deviation of the regression, and $\sqrt{3.9^2 + 2.1^2} = 4.4$.

4 Result for the 13-month smoothed monthly mean aa index

At last in this section, we simply analyze the previous correlations using the 13-month smoothed monthly mean aa index, as shown in Fig. 5(b). The upper dashed and

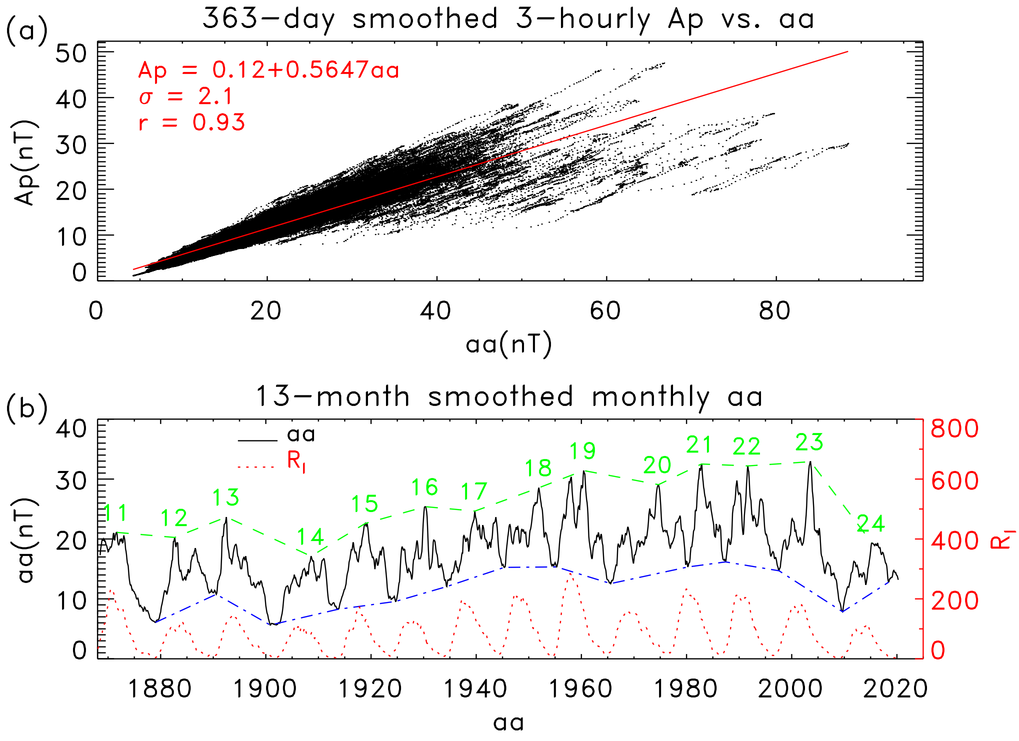

Fig. 5. (a) Scatter plot of the 363-day smoothed 3-hourly Ap against aa (dots) and the linear fit (solid). (b) The 13-month smoothed monthly mean aa (solid) and R_{I} (dotted). The numbers in the figure indicate the solar cycles. The upper dashed and lower dash-dotted lines indicate the maximum (aa_{max}) and minimum (aa_{min}) of aa, respectively, for the solar cycle.

Table 3. Parameters of the 13-month smoothed monthly mean aa for the solar cycle.

n	aa_{min}(nT)	aa_{max}(nT)	T_{r}(month)
11		21.10	
12	6.07	20.25	44
13	10.77	23.66	23
14	5.64	17.12	93
15	8.26	22.60	63
16	9.57	25.39	68
17	12.06	24.66	64
18	15.26	28.56	79
19	15.34	31.42	62
20	12.56	29.07	109
21	15.33	32.51	31
22	16.18	32.20	51
23	14.69	32.90	72
24	7.85	19.33	67
25	12.78		
Av.	11.60	25.77	63.5

lower dash-dotted lines indicate the maximum (aa_{max}) and minimum (aa_{min}) of the aa index, respectively, for the solar cycle. The value of aa_{max} is the maximum of aa during the time period between two adjacent solar cycle minima (R_{min}) determined by the smoothed monthly mean R_{I} (dotted). The value of aa_{min} is the minimum of aa during the time period between two adjacent aa_{max}. The parameters are listed in Table 3, in which T_{r} is the rise time from aa_{min} to aa_{max}.

Figure 6(a) shows the scatter plot of aa_{max} against aa_{min} (triangles). The solid line

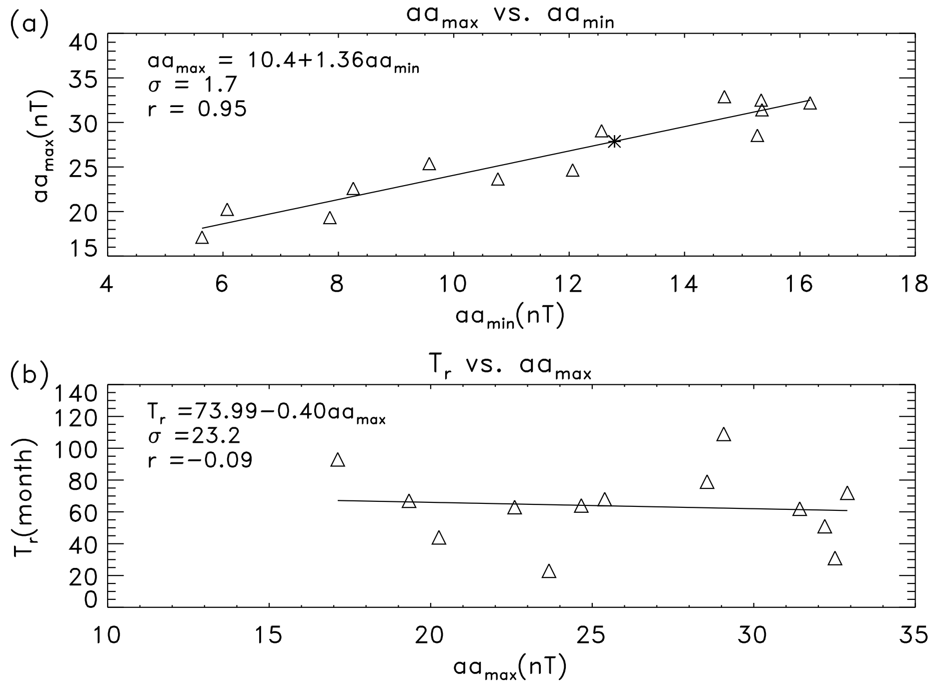

Fig. 6. Scatter plots of aa_{max} against aa_{min} (a) and T_{r} against aa_{max} (b).

indicates the linear fit of aa_{max} to aa_{min} by

$$aa_{\mathrm{max}} = 10.4 \pm 1.7 + (1.36 \pm 0.14) aa_{\mathrm{min}}, \ \sigma = 1.7. \tag{9}$$

The correlation coefficient between aa_{max} and aa_{min} is $r = 0.95$ (at a confidence level greater than 99%), slightly higher than that, $r = 0.85$ (0.89), for the correlation between

aa_{Hmax} and aa_{Hmin} (aa_{Lmin}) in Fig. 3 using the 363-day smoothed highest (lowest) 3-hourly aa index in 3-day-interval.

Substituting $aa_{\mathrm{min}}(25) = 12.78$ into this equation, one can estimate $aa_{\mathrm{max}}(25) = 27.9 \pm 1.7$ (asterisk), about 44% higher than that (19.33) of solar cycle 24. This estimate is similar to the case in Sect. 3.1 that the estimate (91.0) of $aa_{\mathrm{Hmax}}(25)$ from aa_{Lmin} in Eq. (4) is about 40% higher than that (64.81 nT) of solar cycle 24 using the minimum of 363-day smoothed lowest 3-hourly aa index in 3-day-interval.

Figure 6(b) shows the scatter plot of the rise time (T_{r}) from aa_{min} to aa_{max} against the maximum (aa_{max}). The data points are much scattered and T_{r} is nearly uncorrelated to the following aa_{max}, $r = -0.09$. Therefore, this correlation is unable to be used to estimate the rise time of aa_{max}.

5 Discussion and Conclusions

It is well known that the aa index is positively correlated with the solar activity (as represented by R_{I}), since the latter is the main source of the former (Legrand and Simon, 1981; Feynman, 1982; Echer et al., 2004). In general, the stronger the solar activity, the higher the (aa) geomagnetic activity. However the relationship between aa and R_{I} is not a simple linear one (Borello-Filisetti et al., 1992; Mussino et al., 1994; Kishcha et al., 1999; Lockwood et al., 1999; Echer et al., 2004; Tsurutani et al., 2006; Du, 2011a,c, 2020). The aa index tends to lag behind R_{I} about 2–3 years around a solar cycle maximum (Wang et al., 2000; Echer et al., 2004), and (only) about 1 year around a solar cycle minimum (Legrand and Simon, 1981; Wang and Sheeley, 2009; Du, 2011b), as indicated in Fig. 7 for the time difference (ΔT_{max}) of the 13-month smoothed monthly mean aa_{max} to R_{m} (a) and that (ΔT_{min}) of aa_{min} to R_{min} (b). The intensity of geomagnetic activity can only be roughly evaluated from that of solar (sunspot) activity, as the linear correlation coefficient between the smoothed monthly mean aa and R_{I} is only 0.61 (Du, 2011c) or even lower (0.43) if using the non-smoothed series (Du, 2011b). In addition, the future solar activity is also unknown at the current

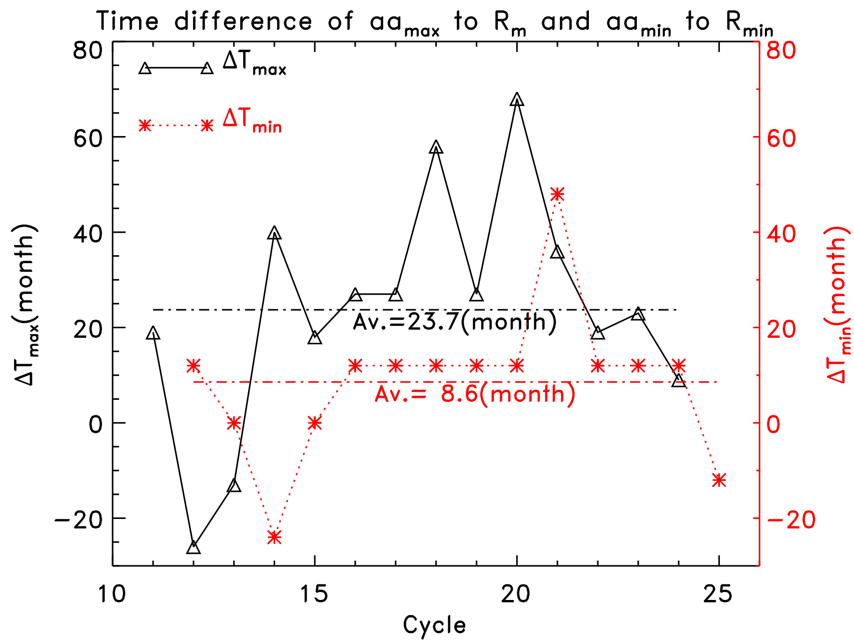

Fig. 7. The time difference between aa_{max} and R_{m} (a) and that between aa_{min} and R_{min} (b).

time and so it can not be directly used to estimate the future geomagnetic activity.

There are many methods that can be used to predict the maximum amplitude of the sunspot cycle (R_{m}), such as (i) statistical methods, employing the relationship between the inter-cycle parameters (Thompson, 1988; Hathaway et al., 1994) or the early rising rate (Thompson, 1988; Cameron and Schüssler, 2008; Du and Wang, 2012); (ii) the functional methods, using mathematical functions of a few parameters (Hathaway et al., 1994; Du, 2011d) for extrapolating the following monthly values; (iii) the geomagnetic precursor methods (Brown and Williams, 1969; Ohl and Ohl, 1979; Du et al., 2009), using the geomagnetic activity near the solar minimum; and (iv) the solar precursor ones (Schatten et al., 1978; Pesnell and Schatten, 2018), using the previous cycle's polar field.

In contrast, there are less methods found to predict the maximum amplitude of geomagnetic activity index for the 11-year solar cycle. Geomagnetic activity forecast has been over the order of hours or days (McPherron, 1999; Abunina et al., 2013). The annual or monthly prediction on the geomagnetic activity is within a limited accuracy (over 20%) due to the irregular variation in the time series (McPherron, 1999; Gordon, 2015). In the earlier years, Kane (1988) even pointed out that it is impossible to forecast the long-term geomagnetic activity through analyzing the time series of the Ap and aa index (refer also to Gordon, 2015). The geomagnetic activity near the solar minimum or at the decreasing phase of the solar cycle has been widely used to predict the maximum amplitude of the sunspot cycle, but was seldom used to predict the maximum amplitude of the geomagnetic activity itself.

In the current work, we analyzed the highest (aa_{H}) and lowest (aa_{L}) 3-hourly aa index in each 3 days' interval, smoothed by 363 days (121 points) to analyze their variation with the solar cycle represented by the 13-month smoothed monthly mean R_{I}. It is found that the maximum of aa_{H} (aa_{Hmax}) is well correlated with the preceding minimum of either aa_{H} (aa_{Hmin}, $r = 0.85$) or aa_{L} (aa_{Lmin}, $r = 0.89$) for the 11-year solar cycle. So, these relationships can be used to estimate the maximum intensity of geomagnetic activity for the solar cycle by employing the time series itself, $aa_{\mathsf{Hmax}}(25) = 83.7 \pm 6.9$ (nT). It implies that the maximum intensity of geomagnetic activity for solar cycle 25 would be similar to the average over the past cycles, but higher than that of solar cycle 24 by about 29.2%. Certainly, this estimate may be an upper limit, as aa_{Hmin} and aa_{Lmin} may be finally determined a few months after the solar minimum (Fig. 7).

Similar results can also be obtained if using the Ap index. However, the Ap index is available only up to April 2018. So, we employed the relationship between Ap and aa to estimate the maximum Ap index for solar cycle 25: $Ap_{\mathsf{max}}(25) = 47.4 \pm 4.4$ (nT). For the 13-month smoothed monthly mean aa index, the maximum aa index (aa_{max}) of the solar cycle is also well correlated to the preceding minimum (aa_{min}), with a correlation coefficient of $r = 0.95$.

The well known 'Waldmeier effect' (Waldmeier, 1939) that the rise time of a solar cycle is well anti-correlated with the following maximum amplitude has been widely used to estimate the rise or peak time of a solar cycle if the amplitude has been predicted. However, such a correlation is very weak for the geomagnetic activity index. The rise time (T_{r}) from aa_{Hmin} to aa_{Hmax} for the solar cycle is found to be only weakly anti-correlated to the following maximum (aa_{Hmax}), $r = -0.42$. This weak correlation may be related to the fact that the geomagnetic activity maximum (minimum) is not aligned to the solar (sunspot) activity maximum (minimum) in time (Fig. 7). In most cases, aa_{max} (aa_{min}) lags behind R_{m} (R_{min}). But in some other cases, aa_{max} (aa_{min}) precedes R_{m} (R_{min}). The weak correlation between the rise time and the following maximum of geomagnetic activity for the solar cycle can hardly be used to estimate the former.

According to the analysis above, the following conclusions may be summarized.

1. The 363-day smoothed highest (aa_{H}) and lowest (aa_{L}) 3-hourly aa index in 3-day-interval is analyzed, finding that the maximum of aa_{H} (aa_{Hmax}) is well correlated with the preceding minimum of either aa_{H} (aa_{Hmin}, $r = 0.85$) or aa_{L} (aa_{Lmin}, $r = 0.89$) for the 11-year solar cycle. As a result, the maximum aa index for the current solar cycle 25 is estimated to be $aa_{\mathrm{Hmax}}(25) = 83.7 \pm 6.9$ (nT), about 29% higher than that of solar cycle 24. This value is equivalent to the Ap index of $Ap_{\mathrm{max}}(25) = 47.4 \pm 4.4$ (nT) if using the relationship between Ap and aa (Eq. (8)).

2. The maximum (aa_{Lmax}) of aa_{L} is also found to be well correlated with the preceding aa_{Hmin}, $r = 0.80$. Based on this correlation, $aa_{\mathrm{Lmax}}(25)$ is estimated to be 5.9 ± 1.2 (nT), about 12% higher than that of solar cycle 24.

3. The maximum amplitude of the sunspot cycle (R_{m}) is much better correlated with the high geomagnetic activity (aa_{Hmax}, $r = 0.79$) than with the low one (aa_{Lmax}, $r = 0.37$).

4. The rise time (T_r) from aa_{Hmin} to aa_{Hmax} is found to be weakly anti-correlated to the following maximum (aa_{Hmax}) for the solar cycle, $r=-0.42$ at the 84% confidence level.

5. For the 13-month smoothed monthly mean aa index, the maximum aa index (aa_{max}) of the solar cycle is well correlated with the preceding minimum (aa_{min}, $r=0.95$). The rise time from aa_{min} to aa_{max} is nearly uncorrelated to the following maximum ($r=-0.09$).

Author contribution

This work is completed by Z. L. Du.

Competing interests

The author declares that he has no conflicts of interest.

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