Reply to two referees

Overall modifications

The manuscript has been thoroughly revised based on two referees.

To clearly describe the data used, the title is changed to 'Estimating the maximum of 363-day-smoothing highest 3-hourly aa index in 3-day-interval by the preceding minimum of highest/lowest aa value for the 11-year solar cycle'.

In the current version, we use the 3-hourly aa index since 1868 in Sect 2. For each 3 days' interval, we find out the highest/lowest aa index (aa_H/aa_L) from 24 values of the 3-hourly aa indices. In order to reduce accidental events in the data, both aa_H and aa_L are smoothed by 363 days (121 points) to mimic the 13-month smoothing, as suggested by Referee 1. The maximum of aa_H (aa_{Hmax}) is found to be well correlated to the preceding minimum of either aa_H (aa_{Hmin}, r=0.85) or aa_L (aa_{Lmin}, r=0.89) for the 11-year solar cycle. Based on these correlations, the strength of geomagnetic activity for cycle 25 is estimated to be aa_{Hmax} (25)= 84.5±6.9 (nT), similar to the average over the past cycles, but about 30% higher than that of cycle 24. The rise time (T_{Hr}) from aa_{Hmin} to aa_{Hmax} is found to be only weakly anti-correlated to the following aa_{Hmax}, r=-0.42. Such a weak

correlation is no longer used to estimate T_{Hr} as suggested by Referee 2.

Similar result can also be obtained if using the 363-day-smoothing highest/lowest 3-hourly Ap index in 3-day-interval (Ap_H/Ap_L), shown in Sect. 3. The maximum of Ap_H (Ap_{Hmax}) is well correlated to the preceding minimum of Ap_H (Ap_{Hmin}, r=0.96) or Ap_L (Ap_{Lmin}, r=0.79) for the 11-year solar cycle. The rise time (T_{Ha}) from Ap_{Hmin} to Ap_{Hmax} is weakly anti-correlated to the following maximum ($Ap_{Hmax,}$ r=-0.33), but reversely correlated to the preceding minimum of Ap_L (Ap_{Lmin}, r=-0.72).

For the 13-month smoothed monthly mean aa (Ap) index, the result is moved down to Sect. 4, retained as a comparison, as suggested by Referee 2, but using only the aa index since 1868. The maximum aa(Ap) index, aa_{max} (Ap_{max}), of the solar cycle is also well correlated to the preceding minimum, aa_{min} (Ap_{min}), with a correlation coefficient of r= 0.95(0.86).

'Predict' is changed to 'estimate' as suggested by Referee 2.

The sizes of figures are increased appropriately.

Reply to Referee # 1

The author uses smoothed monthly aa/Ap index to study the relation between the minimum and maximum aa/Ap values in order to predict the maximum value of the aa/Ap index. Usually long-term smoothing is used to study the solar cycle; to show the correlation between the solar cycle and the index variations. Due to the small number of high amplitude values, smoothing removes all the high amplitude maxima and move the data towards the minimum. The author states that “the maximum aa index for the ensuing cycle 25 is predicted to be aamax(25) = 26.9 ± 2.6.” This is very small value and it could be mistakenly understood that this solar cycle will be very quiet. The values listed in Table 1 under the aamax are much smaller than those observed in any disturbed day. These values can’t represent the maximum aa index or the strength of the geomagnetic activities. As it could be seen from Fig. 1 the aa index has arrive to a peak value of about 67 nT in 19 March 2020 and the Kp value for this time is 4+. Also the paper is based on the data listed in Table 1. Which have been retrieved from smoothed aa index data. The smoothing could be done in many different ways each will produce different data sets. However, when considering the geomagnetic activities, we are usually interested to know how sever it will be and for how long it will last.

1) Therefore, I suggest the following. It should be stated clearly that these max values are for smoothed aa index and it should be given a special note. The paper title should also indicate this.

R: Yes. Thank you.

The title is changed to 'Estimating the maximum of 363-day-smoothing highest 3-hourly aa index in 3-day-interval by the preceding minimum of highest/lowest aa value for the 11-year solar cycle'.

2) The author could try to compare the expect strength of the 25 cycle with the previous cycles. So, we could understand is it will be more active or less active.

R: Yes. We do.

The strength of aa_{Hmax} for cycle 25 is estimated to be $aa_{Hmax}(25)=84.5\pm6.9$ (nT), about 30% stronger than that of cycle 24.

3) The author could try to predict a more reliable maximum of the aa index for the 25th cycle. To do so I could suggest to construct two data sets of the observed aa index minimum and maximum values for each 3 days or more. These two sets could be smoothed for 13 months. The correlation between these two data sets (for 3 days min

and max values) are about 0.79. From these two data sets the author could peak the maximum and minimum aa index for each solar cycle and replace these values with those in Table 1.

R: Yes. Thank you.

In the revised manuscript, we used the 3-hourly aa index of ISGI since 1868. For each 3-day-interval, we find out the highest aa index (aa_H) and the lowest aa index (aa_L) from 24 values of the 3-hourly aa indices. Then, both aa_H and aa_L are smoothed by 363 days (121 points) to mimic the 13-month smoothing. The results are similar to those using 13-month smoothed monthly mean values, apart from that the maximum is estimated to be around 84.5 for the highest value.

The results using 13-month smoothed monthly mean values are now retained and changed to Section 4, as suggested by Referee # 2.

4) Finally, the units of the indices (nT) should be written in text and on the Figures.

R: Yes. We do.

Reply to Referee # 2

The author identified maximum and minimums of smoothed aa and Ap indices through several solar cycles and calculate correlations between minimum and maximum values and between min/max values with respect to the preceding-following cycle. The relations that are found through the means of linear regression are then use to predict estimated aa/Ap minimum/maximum values for solar cycle 25.

Main comments

1. My main concern is with the selection of the dataset. It is not clear to me why the author chooses to work with the 13-month smoothed aa index instead of the highest resolution available. Smoothing everything will naturally result in predictions that converge to the mean values and therefore fail to capture the spiky behavior of storm indices. This is particularly relevant in the case of Ap index. As shown in Table 1, the Ap smoothed monthly means corresponds to period of at most minor geomagnetic activity. Therefore, all storm activity is lost. I suggest the author repeat the calculations using the highest available temporal resolution of the indices and compare them with the current results of the manuscript.

R: Yes. We did. This suggestion is similar to that by Referee # 1: "To do

so I could suggest to construct two data sets of the observed aa index minimum and maximum values for each 3 days or more. These two sets could be smoothed for 13 months."

In the revised manuscript, we used the 3-hourly aa index of ISGI since 1868 (highest resolution). For each 3-day-interval, we find out the highest aa index (aa_H) and the lowest aa index (aa_L) from 24 values of the 3-hourly aa indices. Then, both aa_H and aa_L are smoothed by 363 days (121 points) to mimic the 13-month smoothing, as suggested by Referee # 1. The results are similar to those using 13-month smoothed monthly mean values, apart from that the maximum is estimated to be around 84.5 for the highest value.

The results using 13-month smoothed monthly mean values are now retained and changed to Section 4.

2 Page 2 L19-21

These results are hardly relevant. Simpler methods will estimate the duration of solar cycle phases with significantly better accuracy (For example, NOAA predicts a rising duration with an error of $\sim$8 months). Estimating the duration of half a cycle with an uncertainty of almost half the solar cycle results in a disconnection between the mathematical results and the known repetitiveness of the studies phenomena. There's a reason it is called the 11-year cycle. I suggest the author to revise the calculations and to interpret them in the context of what could be a reasonable assumption of the duration of the phases of SC25.

R: As the anti-correlation coefficient between the rise time and the following maximum is very weak (-0.42 for aa or -0.33 for Ap), we do no longer use it to estimate the rise time.

The rise time of aa index is defined as the time duration from the minimum to the following maximum of aa/Ap index. The weak correlation between the rise time and the following maximum is related to the fact that the geomagnetic activity minimum (maximum) is not aligned to the solar (sunspot) activity minimum (maximum) in time, as shown in Fig.10 for the time difference of aa_{max} to R_{max}, ΔT_{max}(a), and that of aa_{min} to R_{min}, ΔT_{min} (b). In most cases, aa_{max} (aa_{min}) lags behind R_{max} (R_{min}). But in some other cases, aa_{max} (aa_{min}) precedes R_{max} (R_{min}). If the rise time is computed from the minimum of sunspot activity(R_{min}) to aa_{Hmax}, the correlation is even weaker, r=-0.14.

3. The main results of the paper (shown in Figures 1-4) are heavily influenced by the decision of using smoothed indices. While they may be correct in that particular context, the author should consider if the methodology utilized is the appropriate for this particular problem. Going back to point 1, if a different dataset is utilized, all figures need to

be remade. On a note regarding presentation of the figures, adding colors to the different lines and making the figures of the appropriate size will significantly improve the readability.

R: Yes. The dataset is replaced by the 3-hourly aa index of ISGI since 1868 as stated in 1.

The rise time is defined as in 2.

The lines of figures are shown in colors and whose sizes are enlarged.

Specific comments

1) Title: I suggest replacing "minimum" with "solar minimum" to explicitly refer to solar cycle. Note that currently the title is misleading, as the prediction is regarding the smoothed data. Please adjust accordingly.

R: Yes. We did.

To clearly describe the data used, the title is changed to 'Estimating the maximum of 363-day-smoothing highest 3-hourly aa index in 3-day-interval by the preceding minimum of highest/lowest aa value for the 11-year solar cycle'.

2) Page 2

L10 - What is the meaning of a double plus-minus. Is it referring to different error sources? In that case please specify. L18 – Do you mean anti-correlated? L26 What do you mean by deviations? Please provide relevant references.

R: L10: **the double plus-minus refer to different error sources, 'where ±3.9 and ±2.1 are derived from the uncertainty of aa_{Hmax} (25) and the standard deviation of the fitting of Ap to aa, respectively.'**

L18: **yes. Thank you. It means anti-correlated.**

L26: i**t changed to 'deviations of orbital motions of Satellites' (Yoshida and Yamagishi, 2010; Petrovay, 2020).**

3) Page 4

L2-3 Predicted or estimated? A prediction is a statement about the future. A correlation between two variables at most indicates the ability to estimate one when the other is available, which appear to be the case.

R: Yes. We changed 'estimated' to 'estimated'.

4) Page 7

L2-4 This extremely high correlation is clearly affected by the process of smoothing the data. Similar with other figures and equations, please correct based on major comments.

R: Yes. We do.

This figure is replaced by Fig.7(a) for the scatter plot of the 363-day-smoothing 3-hourly Ap against aa indices since 1932 (dots). The correlation coefficient between them is r=0.93 (or 0.75 if using the non-smoothed series).

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Date: 4 August 2020

Estimating the maximum of 363-day-smoothing highest 3-hourly aa index in 3-day-interval by the preceding minimum of highest/lowest aa value for the 11-year solar cycle

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Abstract

Predicting the strength of geomagnetic activity for an upcoming cycle is important in space weather service for planning future space missions. This study analyzed the **highest/lowest 3-hourly aa index ($aa_{\mathsf{H}}/aa_{\mathsf{L}}$) in 3-day-interval, smoothed by 363 days to mimic the 13-month smoothing. It is found that the maximum of aa_{H} (aa_{Hmax}) is well correlated to the preceding minimum of either aa_{H} ($aa_{\mathsf{Hmin}}, r = 0.85$) or aa_{L} ($aa_{\mathsf{Lmin}}, r = 0.89$) for the 11-year solar cycle. Based on these relationships, the strength of aa_{Hmax} for cycle 25 is estimated to be $aa_{\mathsf{Hmax}}(25) = 84.5 \pm 6.9$ (nT), about 30% stronger than that of cycle 24. This value is equivalent to the Ap index of $Ap_{\mathsf{Hmax}}(25) = 47.8 \pm 3.9 \pm 2.1$ (nT) if employing the high correlation between Ap and aa ($r = 0.93$), here ± 3.9 and ± 2.1 are derived from the uncertainty of $aa_{\mathsf{Hmax}}(25)$ and the standard deviation of the fitting of Ap to aa, respectively. The maximum of aa_{L} (aa_{Lmax}) is also well correlated to the preceding $aa_{\mathsf{Hmin}}(r = 0.80)$. The maximum of sunspot cycle (R_{m}) is much better correlated to the high geomagnetic activity ($aa_{\mathsf{Hmax}}, r = 0.79$) than to the low one ($aa_{\mathsf{Lmax}}, r = 0.37$).** The rise time from aa_{Hmin} to aa_{Hmax} is weakly anti-correlated to the following aa_{Hmax}, $r = -0.42$. **Similar correlations are also found both for the 363-day-smoothing highest/lowest 3-hourly Ap index in 3-day-interval and for the 13-month smoothed monthly mean aa/Ap index. These results are expected to be useful in understanding the geomagnetic activity strength of solar cycle 25.**

1 Introduction

Studying and predicting geomagnetic activities are important in both geophysics and space weather. Severe geomagnetic activities may cause intense geomagnetic storms (Gonzalez et al., 1989, 1994; Chen et al., 2019), leading to disruptions in communication and deviations of **orbital motions of satellites(Yoshida and Yamagishi, 2010; Petrovay, 2020)**. With the current solar cycle 24 approaching its end, satellite and spacecraft-related departments want to know the strengths of both solar and geomagnetic activities in the ensuing cycle 25 for planning future space missions.

Among various indices to quantitatively describe geomagnetic activities, the aa index (Mayaud, 1972), derived from the 3-hourly K indices at two near-antipodal midlatitude stations in England and Australia, is the longest time series (since 1868) and has been widely used for analyzing long-term trends in the global geomagnetic activity (Russell and Mulligan, 1995; Marat et al., 2017; Du, 2011b; El-Borie et al., 2019) and for analyzing its correlation with both climate change (Cliver et al., 1998; Dobrica et al., 2009; Gavrilyeva et al., 2017) and solar activity (Echer et al., 2004; Prestes et al., 2006; Lukianova et al., 2009; Du, 2011a,c; Du and Wang, 2012; Singh and et al., 2019). The minimum aa index (aa_{min}), at or near the minimum of the solar cycle, has been widely used in predicting the maximum amplitude of the sunspot cycle (R_{m}), the so-called Ohl's precursor method (Ohl, 1979; Brown and Williams, 1969; Du et al., 2009). But it is seldom used to directly predict the maximum aa index (aa_{max}) of an ensuing cycle.

The planetary geomagnetic index Ap (Bartels, 1963) available since 1932, derived from the average of the measurements at 13 observatories around the globe, is a daily measure of the response of geomagnetic field to variations in the interplanetary magnetic field (IMF) and the solar wind (Li, 1997; McPherron, 1999; Tsurutani et al., 2006). It is the main global magnetic index forecasted by government agencies (McPherron, 1999). Most works on forecasting geomagnetic activity have been over short intervals, on the order of hours or days (McPherron, 1999; Abunina et al., 2013). In the earlier years, Kane (1988) pointed out that it is impossible to forecast the long-term geomagnetic activity through analyzing the daily, monthly and annual values of Ap and aa indices. Gordon (2015) demonstrated that long-term geomagnetic activity can only be predicted to within a limited threshold of accuracy due to the irregular trends and cycles in the annual data and nonlinear variability in the monthly series, through analyzing the aa index.

In this study, we analyze the relationship between the maximum aa/Ap index and its preceding minimum for the 11-year solar cycle. First, in Sect. 2, we use the highest/lowest 3-hourly aa index ($aa_{\mathsf{H}}/aa_{\mathsf{L}}$) in each 3 days' interval, smoothed by 363 days (121 points) to

mimic the 13-month smoothing (Sect. 2.1). It is found that the maximum of aa_{H} (aa_{Hmax}) is well correlated to the preceding minimum of either aa_{H} (aa_{Hmin}) or aa_{L} (aa_{Lmin}) for the 11-year solar cycle (Sect. 2.2), which can be used to estimate aa_{Hmax} of the ensuing cycle. The maximum of aa_{L} (aa_{Lmax}) is also found to be well correlated to the preceding aa_{Hmin} (Sect. 2.3). The rise time of aa_{H} from aa_{Hmin} to aa_{Hmax} is only weakly anti-correlated to the following aa_{Hmax} (Sect. 2.4). Similar results are analyzed by using the 363-day-smoothing highest and lowest 3-hourly Ap indices in each 3 days' interval (Sect. 3) and by using the 13-month smoothed (with half weight at the two ends) monthly mean aa and Ap indices (Sect. 4). Some conclusions are discussed and summarized in Sect. 5.

2 Result for 363-day-smoothing highest/lowest 3-hourly aa index in 3-day-interval

2.1 Data

First in this section, we use the **3-hourly** aa index since 1868 (to 2020 **August 1st**) from the International Service of Geomagnetic Indices (ISGI)[1]. **We find out the highest/lowest aa index ($aa_{\mathsf{H}}/aa_{\mathsf{L}}$) from 24 values of the 3-hourly aa indices in each 3 days' interval. In order to reduce accidental events in the data, both aa_{H} and aa_{L} are smoothed by 363 days (121 points) to mimic the 13-month smoothing, as shown in Fig. 1 (solid). The 13-month smoothed monthly mean International sunspot number series (R_{I}, Clette et al., 2016) of the second version[2] is used for comparison (dotted).**

In the upper panel of the figure, the dashed (dash-dotted) line indicates the maximum (minimum) of aa_{H}, aa_{Hmax} (aa_{Hmin}). While in the lower panel, the dashed (dash-dotted) line indicates the maximum (minimum) of aa_{L}, aa_{Lmax} (aa_{Lmin}). These parameters are displayed in Table 1, in which T_{Hr} is the rise time of aa_{H} from aa_{Hmin} to aa_{Hmax}, and R_{m} the maximum of R_{I} for the 11-year solar cycle. The last row denotes the averages of the parameters.

[1] http://isgi.unistra.fr/
[2] http://www.sidc.be/silso/datafiles

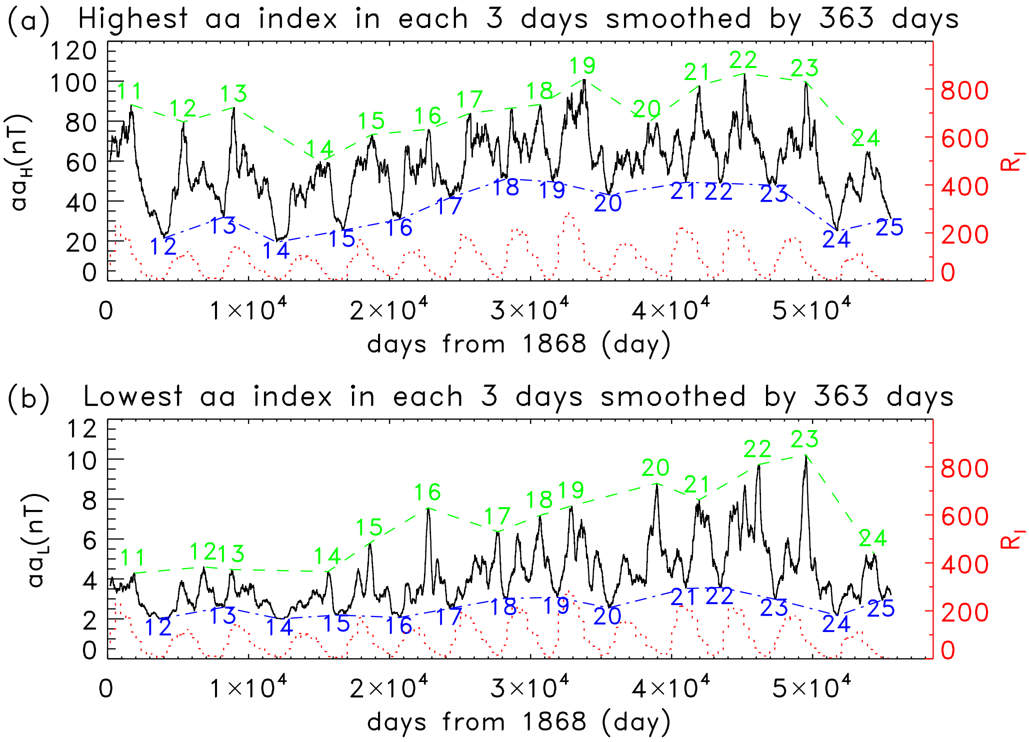

Fig. 1. (a) The highest (aa_H) and (b) the lowest (aa_L) 3-hourly aa index in each 3 days (black solid), smoothed by 121 points (363 days). The numbers in the figure indicate the 11-year solar cycles. The upper dashed and lower dash-dotted lines indicate the maxima and minima, respectively, for the 11-year solar cycle. The red dotted line represents the 13-month smoothed monthly mean sunspot number (R_I) for comparison.

Table 1. The minimum (aa_{Hmin}) and maximum (aa_{Hmax}) of 363-day-smoothing highest 3-hourly aa index (aa_{H}) in each 3 days, the rise time of aa_{Hmax} (T_{Hr}), the minimum (aa_{Lmin}) and maximum (aa_{Lmax}) of 363-day-smoothing lowest 3-hourly aa index (aa_{L}) in each 3 days, and the maximum (R_{m}) of 13-month smoothed monthly mean sunspot number for solar cycles 11-25.

n	aa_{Hmin}(nT)	aa_{Hmax}(nT)	T_{Hr}(yr)	aa_{Lmin}(nT)	aa_{Lmax}(nT)	R_{m}
11		88.24			4.29	234.0
12	21.41	79.60	3.81	2.00	4.60	124.4
13	31.88	86.92	2.08	2.58	4.47	146.5
14	19.40	59.55	8.47	2.00	4.37	107.1
15	25.31	73.28	5.72	2.19	5.80	175.7
16	30.80	75.90	5.56	2.07	7.56	130.2
17	41.60	84.01	4.01	2.52	6.36	198.6
18	51.13	88.31	6.57	3.02	7.18	218.7
19	49.60	100.99	5.89	3.08	7.66	285.0
20	43.08	79.31	7.54	2.55	8.79	156.6
21	49.35	97.87	2.63	3.52	7.96	232.9
22	48.93	103.89	4.78	3.57	9.73	212.5
23	47.84	99.75	5.92	2.98	10.22	180.3
24	25.02	64.81	5.73	2.17	5.27	116.4
25	31.05			2.94		
Av.	36.89	84.46	5.29	2.66	6.73	179.9

The correlation coefficients between the parameters in Table 1 are listed in Table 2 for comparison. It is seen in Table 2 that R_{m} is well correlated to $aa_{\mathrm{Hmin}}(r=0.84)$, $aa_{\mathrm{Hmax}}(r=0.79)$, $aa_{\mathrm{Lmin}}(r=0.81)$, and positively correlated to $aa_{\mathrm{Lmax}}(r=0.37)$. It implies that the stronger the solar activity (R_{I}), the higher the geomagnetic activity (aa). But the maximum of sunspot cycle (R_{m}) is much better correlated to the high geomagnetic activity ($aa_{\mathrm{Hmax}}, r=0.79$) than to the low one ($aa_{\mathrm{Lmax}}, r=0.37$), implying that the low geo-

Table 2. Correlation coefficients between parameters in Table 1.

x/ y	aa_{Hmin}	aa_{Hmax}	T_{Hr}	aa_{Lmin}	aa_{Lmax}	R_{m}
aa_{Hmin}	1.00	0.85	−0.10	0.85	0.80	0.84
aa_{Hmax}	0.85	1.00	−0.42	0.89	0.63	0.79
T_{Hr}	−0.10	−0.42	1.00	−0.28	0.13	−0.18
aa_{Lmin}	0.85	0.89	−0.28	1.00	0.70	0.81
aa_{Lmax}	0.80	0.63	0.13	0.70	1.00	0.37

magnetic activity depends less on the solar activity than the high one does. The correlation between R_{m} and aa_{Hmin} (or aa_{Lmin}) is related to the Ohl's precursor method (Ohl, 1979) for predicting R_{m}. Some other correlations are analyzed to estimate aa_{Hmax} (Sect. 2.2), aa_{Lmax} (Sect. 2.3), and T_{Hr} (Sect. 2.4) in this section.

2.2 Relationship between aa_{Hmax} and its preceding $aa_{\mathrm{Hmin}}/aa_{\mathrm{Lmin}}$

One can see in Table 2 that aa_{Hmax} is well correlated to its preceding $aa_{\mathrm{Hmin}}(r=0.85)$ and $aa_{\mathrm{Lmin}}(r=0.89)$, as shown in Fig. 2 for the scatter plots of aa_{Hmax} against aa_{Hmin}(a) and aa_{Lmin}(b). The solid line represents the linear fit of aa_{Hmax} to aa_{Hmin} (aa_{Lmin}) with the least-squares-fit regression equation given by

$$\begin{cases} aa_{\mathrm{Hmax}} = 47.1 \pm 7.1 + (0.99 \pm 0.18) aa_{\mathrm{Hmin}}, \ \sigma = 7.3, \\ aa_{\mathrm{Hmax}} = 25.3 \pm 9.4 + (22.3 \pm \ \ 3.5) aa_{\mathrm{Lmin}}, \ \sigma = 6.5, \end{cases} \quad (1)$$

where $\pm$ indicates the 1-σ deviation of the fitting coefficient and σ the standard deviation of the fitting.

Based on the above relationships, the aa index at the minimum can be used as an indicator to estimate the following maximum. One can estimate aa_{Hmax} for cycle $n=25$ by

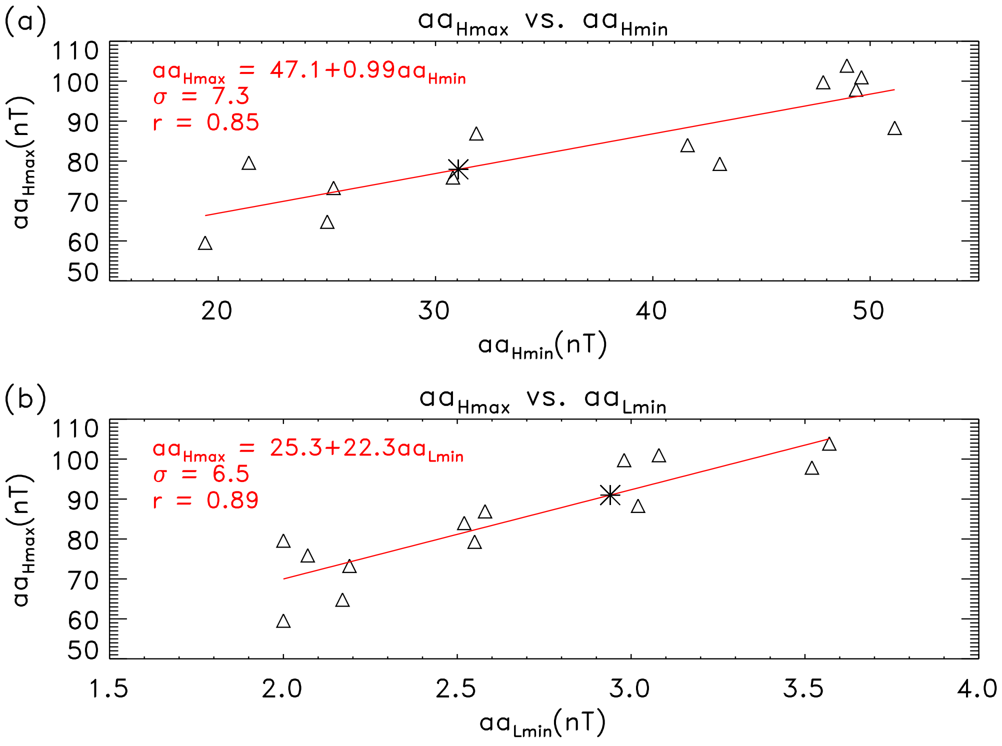

Fig. 2. (a) Scatter plot of aa_{Hmax} against aa_{Hmin} (triangles) and the linear fit (solid). (b) Scatter plot of aa_{Hmax} against aa_{Lmin}(triangles) and the linear fit (solid).

substituting the values of aa_{Hmin}(31.05 nT) and aa_{Lmin}(2.94 nT) into the above equations,

$$\begin{cases} aa_{\mathrm{Hmax1}}(25) = 77.9 \pm 7.3(\mathrm{nT}), \text{ from } aa_{\mathrm{Hmin}}, \\ aa_{\mathrm{Hmax2}}(25) = 91.0 \pm 6.5(\mathrm{nT}), \text{ from } aa_{\mathrm{Lmin}}, \end{cases} \tag{2}$$

(labelled by asterisk). As these values are derived by the fitting equations (Eq.(1)) with similar correlation coefficients (0.85 and 0.89), we take their average,

$$\begin{aligned} aa_{\mathsf{Hmax}}(25) &= \frac{1}{2}[aa_{\mathsf{Hmax1}}(25) + aa_{\mathsf{Hmax2}}(25)] \\ &= 84.5 \pm 6.9(\mathsf{nT}), \end{aligned} \tag{3}$$

as an estimate of $aa_{\mathsf{Hmax}}(25)$. It implies that the 363-day-smoothing highest 3-hourly aa index in 3-day-interval during the maximum period of cycle 25 is estimated to be close to the average (84.46 nT) over the past cycles (Table 1), but higher than that (64.81 nT) of cycle 24 by about 30.3%.

It should be pointed out that the above estimate may be an upper limit of $aa_{\mathsf{Hmax}}(25)$ as the values of $aa_{\mathsf{Hmin}}(25)$ and $aa_{\mathsf{Lmin}}(25)$ may not be finally determined (usually about one year after the minimum of a solar cycle (24)). Although we are not quite sure if the current aa_{H} (aa_{L}), 31.05 (3.19) in January 2020, would decrease to a smaller value than that, 31.05 (2.94), used in the current work, there would not be significant variations in aa_{Hmin}, aa_{Lmiin} and the above estimate, because the sunspot number (R_{I}) shows a sign to stop decreasing and to oscillate around the minimum during the recent few months.

2.3 Relationship between aa_{Lmax} and the preceding aa_{Hmin}

One can also see in Table 2 that aa_{Lmax} is well correlated to the preceding $aa_{\mathsf{Hmin}}(r = 0.80)$ or $aa_{\mathsf{Lmin}}(r = 0.70)$. Fig. 3(a) shows the scatter plot of aa_{Lmax} against aa_{Hmin}. The linear fitting equation of aa_{Lmax} to aa_{Hmin} (solid) is

$$aa_{\mathsf{Lmax}} = 2.0 \pm 1.2 + (0.131 \pm 0.030) aa_{\mathsf{Hmin}},\ \sigma = 1.2. \tag{4}$$

Substituting $aa_{\mathsf{Hmin}}(25) = 31.05$ (nT) into this equation, one can estimate the 363-day-smoothing *lowest* 3-hourly aa index in 3-day-interval during the maximum period of cycle 25, $aa_{\mathsf{Lmax}} = 6.1 \pm 1.2$ (nT). This value is slightly lower than the average (6.73 nT) over the past cycles, but higher than that (5.27 nT) of cycle 24 by 15.7%.

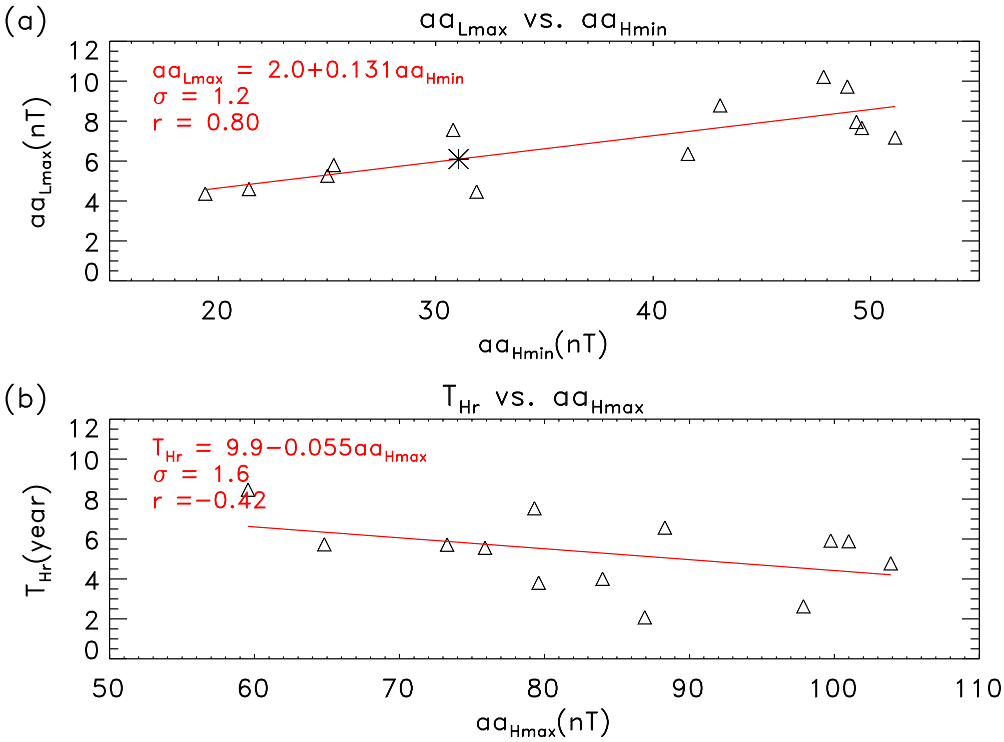

Fig. 3. (a) Scatter plot of aa_{Lmax} against aa_{Hmin} (triangles) and the linear fit (solid). (b) Scatter plot of T_{Hr} against aa_{Hmax} (triangles) and the linear fit (solid).

2.4 Relationship between the rise time and following maximum

Now, we analyze if the rise time of the aa geomagnetic index for the 11-year solar cycle **is correlated to** the following maximum **so that it can be used to estimate the rise time**, as the case often used in the solar (sunspot) cycle (Waldmeier, 1939).

Figure 3(b) shows the scatter plot of the rise time (T_{Hr}) from aa_{Hmin} to aa_{Hmax} for the 11-year solar cycle against **its following** maximum (aa_{Hmax}). **The solid line indicates the linear fit of T_{Hr} to aa_{Hmax} by the following fitting equation,**

$$T_{\mathrm{Hr}} = 9.9 \pm 3.0 - (0.055 \pm 0.036) aa_{\mathrm{Hmax}}, \ \sigma = 1.6(\text{years}). \quad (5)$$

The anti-correlation coefficient between T_{Hr} and aa_{Hmax}, $r = -0.42$ (at a confidence level of about 84%), is so weak that it can hardly be used to estimate the rise time (T_{Hr}). If the rise time is computed from the minimum of sunspot activity (R_{I}) to aa_{Hmax}, the correlation is even weaker, $r = -0.14$.

3 Result for 363-day-smoothing highest/lowest 3-hourly Ap index in 3-day-interval

Then in this section, we analyze the previous result using the 3-hourly Ap index since 1932[3] (available to 2018 April). Similar to the aa index, we find out the highest/lowest 3-hourly Ap index ($Ap_{\mathrm{H}}/Ap_{\mathrm{L}}$) in each 3 days' interval. Both Ap_{H} and Ap_{L} are smoothed by 363 days (121 points), as shown in Fig. 4 (solid). Table 3 displays the maximum/minimum of Ap_{H} ($Ap_{\mathrm{Hmax}}/Ap_{\mathrm{Hmin}}$), the rise time of Ap_{H} from Ap_{Hmin} to Ap_{Hmax} (T_{Ha}), and the maximum/minimum of Ap_{L} ($Ap_{\mathrm{Lmax}}/Ap_{\mathrm{Lmin}}$).

The correlation coefficients between the parameters in Table 3 are listed in Table 4. One can see in Table 4 that R_{m} is well correlated to $Ap_{\mathrm{Hmin}}(r = 0.88)$, $Ap_{\mathrm{Hmax}}(r = 0.73)$, $Ap_{\mathrm{Lmin}}(r = 0.82)$, and positively correlated to $Ap_{\mathrm{Lmax}}(r = 0.35)$, similar to the case for the aa index. Some other significant correlations are analyzed in this section.

3.1 Relationship between Ap_{Hmax} and the preceding $Ap_{\mathrm{Hmin}}/Ap_{\mathrm{Lmin}}$

It is seen in Table 4 that Ap_{Hmax} is well correlated to the preceding $Ap_{\mathrm{Hmin}}(r = 0.96)$ and $Ap_{\mathrm{Lmin}}(r = 0.79)$, as shown in Fig. 5 for the scatter plots of Ap_{Hmax} against Ap_{Hmin}(a)

[3] http://www.gfz-potsdam.de/en/kp-index

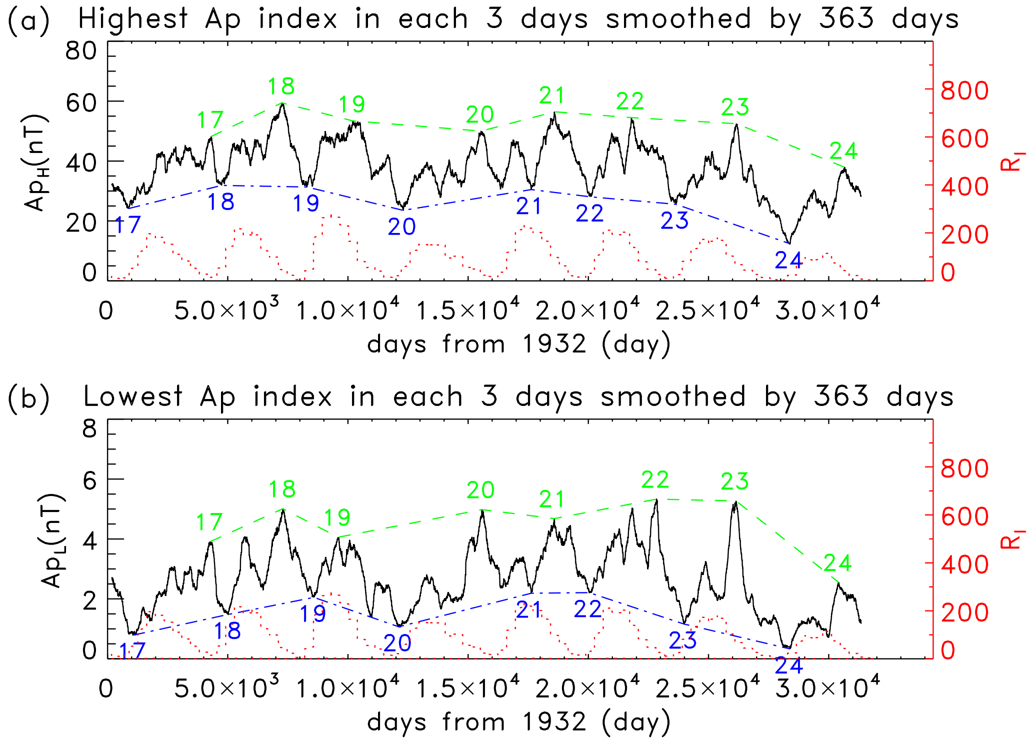

Fig. 4. Similar to Fig. 1 for the Ap index: (a) the highest Ap index (Ap_{H}) and (b) the lowest Ap index (Ap_{L}) in each 3 days (black solid), smoothed by 121 points (363 days).

and Ap_{Lmin}(b). The linear fitting equation between Ap_{Hmax} and Ap_{Hmin} (Ap_{Lmin}) is

$$\begin{cases} Ap_{\mathrm{Hmax}} = 25.9 \pm 3.0 + (0.99 \pm 0.11) Ap_{\mathrm{Hmin}}, \ \sigma = 1.8, \\ Ap_{\mathrm{Hmax}} = 41.1 \pm 3.7 + \ (7.4 \pm \ 2.4) Ap_{\mathrm{Lmin}}, \ \sigma = 4.0. \end{cases} \quad (6)$$

Table 3. The minimum (Ap_{Hmin}) and maximum (Ap_{Hmax}) of 363-day-smoothing highest 3-hourly Ap index (Ap_{H}) in each 3 days, the rise time of Ap_{Hmax} (T_{Ha}), the minimum (Ap_{Lmin}) and maximum (Ap_{Lmax}) of 363-day-smoothing lowest 3-hourly Ap index (Ap_{L}) in each 3 days, and the maximum (R_{m}) of 13-month smoothed sunspot number for solar cycles 17-24.

n	Ap_{Hmin}(nT)	Ap_{Hmax}(nT)	T_{Ha}(yr)	Ap_{Lmin}(nT)	Ap_{Lmax}(nT)	R_{m}
17	24.15	48.19	9.40	0.79	3.91	198.6
18	31.86	59.43	6.86	1.48	5.01	218.7
19	31.34	53.40	5.22	2.05	4.06	285.0
20	23.55	50.01	8.89	1.07	4.98	156.6
21	30.47	56.45	2.62	2.19	4.68	232.9
22	28.03	54.50	4.60	2.21	5.33	212.5
23	25.58	52.48	6.89	1.16	5.27	180.3
24	12.39	37.92	6.23	0.33	2.55	116.4
Av.	25.92	51.55	6.34	1.41	4.47	200.1

Table 4. Correlation coefficients between parameters in Table 3.

x/ y	Ap_{Hmin}	Ap_{Hmax}	T_{Ha}	Ap_{Lmin}	Ap_{Lmax}	R_{m}
Ap_{Hmin}	1.00	0.96	−0.34	0.83	0.70	0.88
Ap_{Hmax}	0.96	1.00	−0.33	0.79	0.82	0.73
T_{Ha}	−0.34	−0.33	1.00	−0.72	−0.09	−0.44
Ap_{Lmin}	0.83	0.79	−0.72	1.00	0.59	0.82
Ap_{Lmax}	0.70	0.82	−0.09	0.59	1.00	0.35

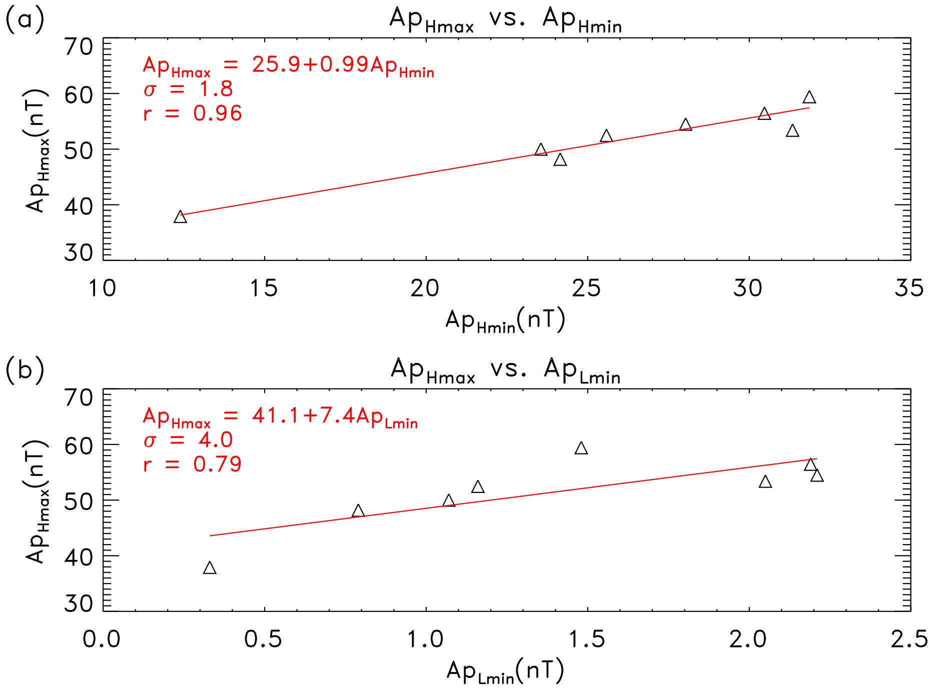

Fig. 5. (a) Scatter plot of Ap_{Hmax} against Ap_{Hmin} (triangles) and the linear fit (solid). (b) Scatter plot of Ap_{Hmax} against Ap_{Lmin}(triangles) and the linear fit (solid).

If the values of Ap_{Hmin} and Ap_{Lmin} are obtained in advance, the value of Ap_{Hmax} can be estimated from the above equations. However, these values are unknown at present as the Ap index is available only to 2018 April.

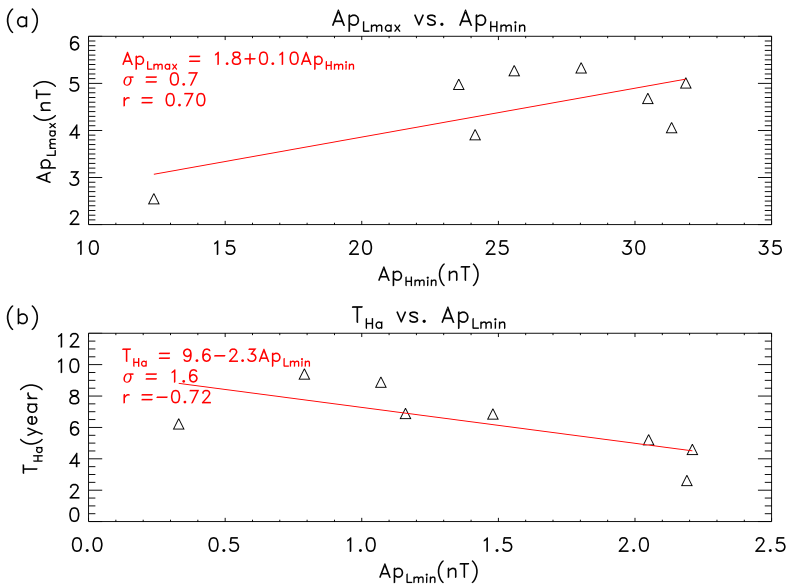

Fig. 6. (a) Scatter plot of Ap_{Lmax} against Ap_{Hmin} (triangles) and the linear fit (solid). (b) Scatter plot of T_{Ha} against Ap_{Lmin} (triangles) and the linear fit (solid).

3.2 Relationship between Ap_{Lmax} and the preceding Ap_{Hmin}

It is also seen in Table 4 that Ap_{Lmax} is well correlated to the preceding $Ap_{\mathrm{Hmin}} (r = 0.70)$, as shown in Fig. 6(a). The linear fitting equation of Ap_{Lmax} to Ap_{Hmin} (solid) is

$$Ap_{\mathrm{Lmax}} = 1.8 \pm 1.1 + (0.10 \pm 0.04) Ap_{\mathrm{Hmin}}, \; \sigma = 0.7. \tag{7}$$

If Ap_{Hmin} is known, Ap_{Lmax} can be estimated from this equation.

3.3 Relationship between the rise time and preceding minimum

One may note in Table 4 that the anti-correlation between the rise time (T_{Ha}) from Ap_{Hmin} to Ap_{Hmax} and the following maximum (Ap_{Hmax}) is very weak, $r = -0.33$. While the anti-correlation between T_{Ha} and the preceding minimum (Ap_{Lmin}) is strong, $r = -0.72$ (at the 95% confidence level). Figure 6(b) shows the scatter plot of T_{Ha} against Ap_{Lmin}, fitted by the following linear equation,

$$T_{\mathrm{Ha}} = 9.6 \pm 1.4 - (2.3 \pm 0.9) Ap_{\mathrm{Lmin}},\ \sigma = 1.6(\mathrm{years}). \quad (8)$$

Similarly, if $Ap_{\mathrm{Lmin}}(25)$ is known, T_{Ha} for cycle 25 can be estimated from this equation.

3.4 Relationship between Ap and aa

Now, we analyze the relationship between the Ap and aa indices, as shown in Fig. 7(a) for the scatter plot of the 363-day-smoothing 3-hourly Ap against aa indices since 1932 (dots). The solid line represents the linear fit of Ap to aa with the least-squares-fit regression equation given by

$$Ap = 0.12 \pm 0.01 + (0.5647 \pm 0.0005) aa,\ \sigma = 2.1. \quad (9)$$

The correlation coefficient between the fitted and observed values is $r = 0.93$ (or 0.75 if using the non-smoothed series) at a confidence level greater than 99%. It is obvious that Ap is highly correlated with aa, as they are based on the same observations.

According to this equation, the maximum of 363-day smoothing highest Ap value for cycle 25 can be estimated by substituting the estimated $aa_{\mathrm{Hmax}}(25) = 84.5 \pm 6.9$ (nT) in Sect. 2.2 into this equation, $Ap_{\mathrm{Hmax}}(25) = 47.8 \pm 3.9 \pm 2.1$ (nT), here ± 3.9 is derived from the uncertainty (± 6.9) of $aa_{\mathrm{Hmax}}(25)$ and ± 2.1 is the standard deviation of the fitting of Ap to aa.

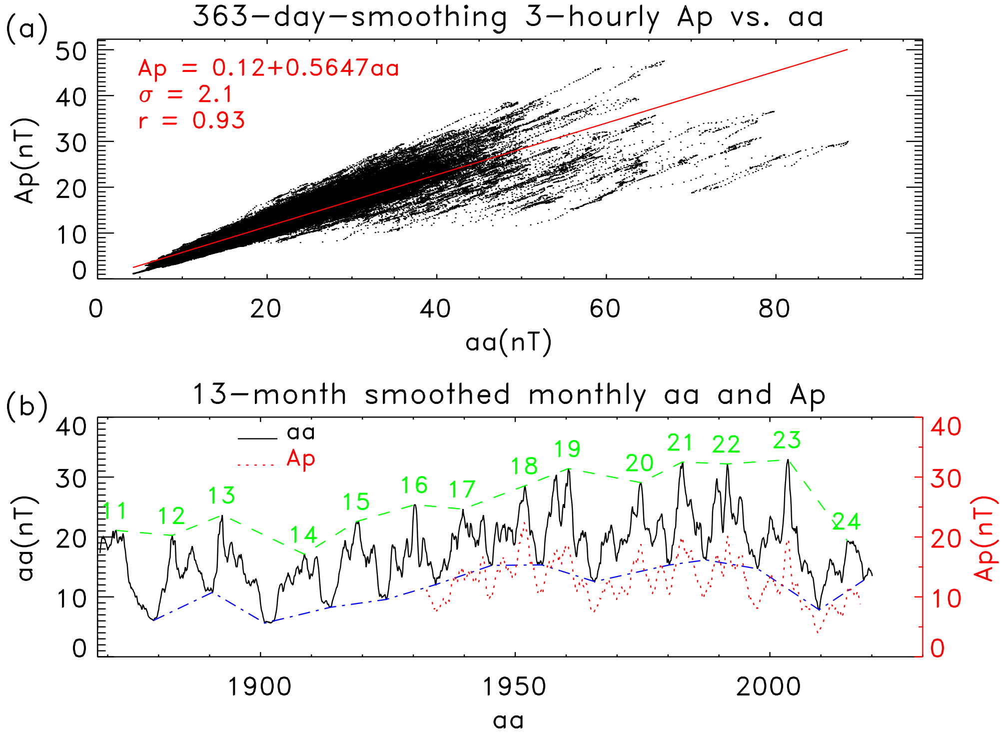

Fig. 7. (a) Scatter plot of the 363-day-smoothing 3-hourly Ap against aa indices since 1932 (dots) and the linear fit (solid). (b) The 13-month smoothed monthly mean time series of aa (solid) since 1868 and Ap (dotted) since 1932. The numbers in the figure indicate the solar cycles. The upper dashed and lower dash-dotted lines represent the maxima (aa_{max}) and minima (aa_{min}) of aa, respectively, for the 11-year solar cycle.

Table 5. Parameters of 13-month smoothed monthly mean aa and A_{p} for the solar cycle.

n	aa_{min}(nT)	aa_{max}(nT)	T_{r}(month)	Ap_{min}(nT)	Ap_{max}(nT)	T_{a}(month)
11		21.10				
12	6.07	20.25	44			
13	10.77	23.66	23			
14	5.64	17.12	93			
15	8.26	22.60	63			
16	9.57	25.39	68			
17	12.06	24.66	64	7.29	16.82	112
18	15.26	28.56	79	9.78	22.45	82
19	15.34	31.42	62	10.55	18.64	56
20	12.56	29.07	109	7.37	18.81	111
21	15.33	32.51	31	10.37	20.08	29
22	16.18	32.20	51	9.62	20.24	57
23	14.69	32.90	72	8.11	19.65	72
24	7.85	19.33	67	3.84	11.72	71
25	12.78					
Av.	11.60	25.77	63.5	8.41	18.55	73.8

4 Result for the 13-month smoothed monthly mean aa/Ap index

At last in this section, we simply analyze the previous result using the 13-month smoothed (with half weight at the two ends) monthly mean aa index (solid) since 1868 and Ap index (dotted) since 1932, as shown in Fig. 7(b). The upper dashed and lower dash-dotted lines represent the maximum (aa_{max}) and minimum (aa_{min}) of the aa index, respectively, for the 11-year solar cycle. The parameters are listed in Table 5, in which, T_{r} is the rise time from aa_{min} to aa_{max}, Ap_{max} and Ap_{min} are the maximum and minimum of the Ap

index for the 11-year solar cycle, respectively, and T_a is the rise time from Ap_{min} to Ap_{max}.

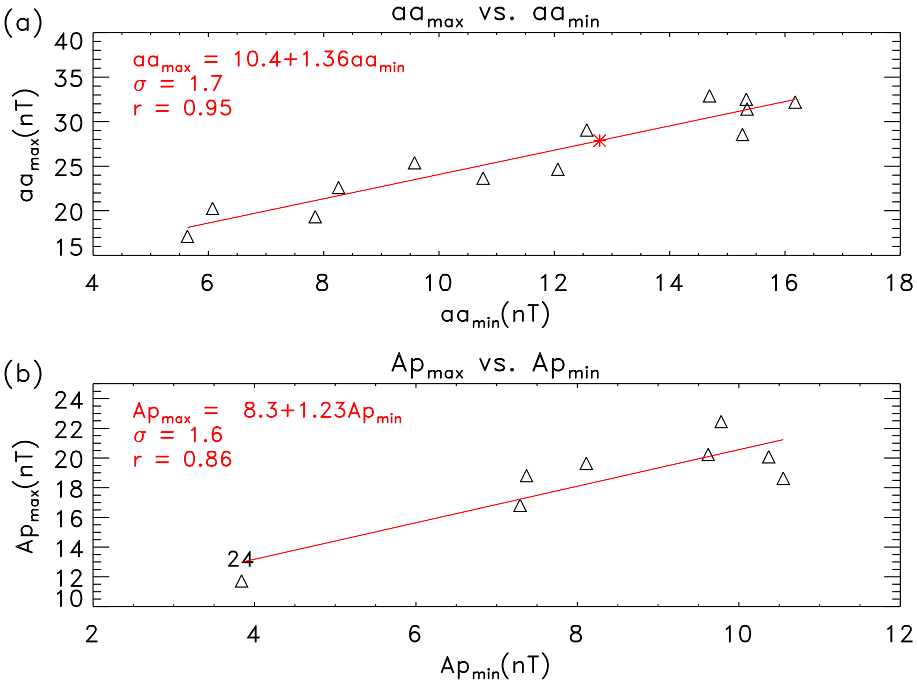

Fig. 8. (a) Scatter plot of aa_{max} against aa_{min} (triangles) and the linear fit (solid). (b) Scatter plot of Ap_{max} against Ap_{min}(triangles) and the linear fit (solid).

4.1 Relationship between aa_{max} and aa_{min}

Figure. 8(a) shows the scatter plot of aa_{max} against aa_{min} for cycles 11-24 (triangles). The solid line indicates the linear fit of aa_{max} to aa_{min} by the following equation,

$$aa_{\mathrm{max}} = 10.4 \pm 1.7 + (1.36 \pm 0.14) aa_{\mathrm{min}},\ \sigma = 1.7. \tag{10}$$

The correlation coefficient between aa_{max} and aa_{min} is $r = 0.95$ (at a confidence level greater than 99%), **slightly higher than that, $r = 0.85(0.89)$, for the correlation between aa_{Hmax} and $aa_{\mathrm{Hmin}}(aa_{\mathrm{Lmin}})$ in Fig. 2 using the 363-day-smoothing highest (lowest) 3-hourly aa index in 3-day-interval.**

Substituting the value of aa_{min} (12.78) for cycle $n = 25$ into this equation, one can estimate $aa_{\mathrm{max}}(25) = 27.9 \pm 1.7$ (asterisk), about 44% higher than that (19.33) of cycle 24. **This estimate is similar to the case in Sect. 2.2 that the estimate (91.0) of $aa_{\mathrm{Hmax}}(25)$ from aa_{Lmin} in Eq. (2) is about 40% higher than that (64.81 nT) of cycle 24 using the minimum of 363-day-smoothing lowest 3-hourly aa index in 3-day-interval.**

4.2 Relationship between Ap_{max} and Ap_{min}

Figure 8(b) illustrates the scatter plot of Ap_{max} against Ap_{min} for cycles 17-24 (triangles). It is seen in the figure that Ap_{max} is also well correlated to Ap_{min}, with a correlation coefficient of $r = 0.86$ (at a confidence level greater than 99%), **slightly lower (higher) than that, $r = 0.96(0.79)$, for the correlation between Ap_{Hmax} and $Ap_{\mathrm{Hmin}}(Ap_{\mathrm{Lmin}})$ in Fig. 5 using the 363-day-smoothing highest (lowest) 3-hourly Ap index in 3-day-interval.** The linear fitting equation of Ap_{max} to Ap_{min} (solid) is

$$Ap_{\mathrm{max}} = 8.3 \pm 2.5 + (1.23 \pm 0.29) Ap_{\mathrm{min}},\ \sigma = 1.6. \tag{11}$$

If Ap_{min} is known, Ap_{max} can be estimated from this equation.

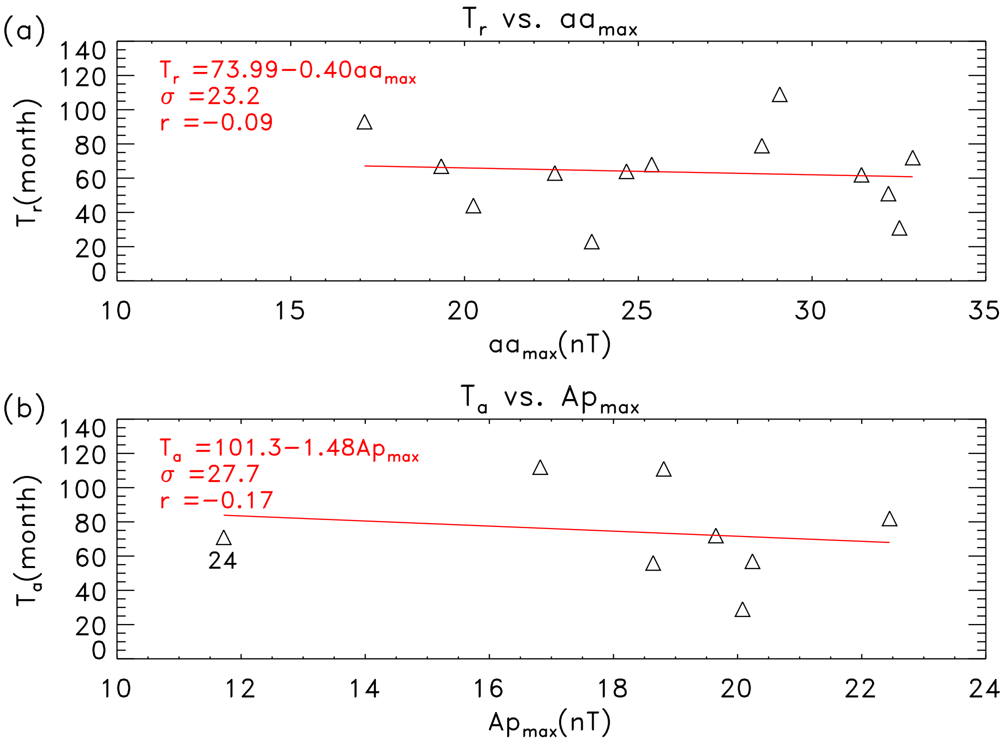

Fig. 9. (a) Scatter plot of T_a against aa_{max} (triangles) and the linear fit (solid). (b) Scatter plot of T_r against Ap_{max} (triangles) and the linear fit (solid).

4.3 Relationship between the rise time and following maximum

Figure 9(a) shows the scatter plot of the rise time (T_r) of aa_{max} from aa_{min} to aa_{max} against the maximum (aa_{max}). It is seen in this figure that the data points are much scattered, and so T_r is nearly uncorrelated to the following aa_{max}, $r = -0.09$. Similarly, the rise time (T_a) of Ap_{max} from Ap_{min} to Ap_{max} is also nearly uncorrelated to the following maximum (Ap_{max}), $r = -0.17$, as shown in Fig. 9(b) for the scatter plot of T_a against Ap_{max}. Therefore, these correlations are unable to be used to estimate the rise times of aa_{max} and Ap_{max}.

5 Discussions and Conclusions

It is well known that the aa index is positively correlated to the solar activity (as represented by R_I), since the latter is the main source of the former (Legrand and Simon, 1981; Feynman, 1982; Echer et al., 2004). In general, the stronger the solar activity, the higher the (aa) geomagnetic activity. However the relationship between aa and R_I is not a simple linear one (Borello-Filisetti et al., 1992; Mussino et al., 1994; Kishcha et al., 1999; Lockwood et al., 1999; Echer et al., 2004; Tsurutani et al., 2006; Du, 2011a,c, 2020). The aa index tends to lag behind R_I about 2–3 years around a solar cycle maximum (Wang et al., 2000; Echer et al., 2004), and (only) about 1 year around a cycle minimum (Legrand and Simon, 1981; Wang and Sheeley, 2009; Du, 2011b), as indicated in Fig. 10. The strength of geomagnetic activity can only be roughly evaluated from the strength of solar (sunspot) activity, as the linear correlation coefficient between the smoothed monthly mean aa and R_I is only 0.61 (Du, 2011c) or even lower (0.43) if using the non-smoothed series (Du, 2011b). In addition, the future solar activity is also unknown at the current time and so it can not be directly used to estimate the future geomagnetic activity.

There are many methods that can be used to predict the maximum amplitude of sunspot cycle (R_m), such as 1) statistical methods, employing the relationship between the inter-cycle parameters (Thompson, 1988; Hathaway et al., 1994) or the early rising rate (Thompson, 1988; Cameron and Schüssler, 2008; Du and Wang, 2012); 2) the functional methods, using mathematical functions of a few parameters (Hathaway et al., 1994; Du, 2011d) for extrapolating the following monthly values; 3) the geomagnetic precursor methods (Ohl, 1979; Brown and Williams, 1969; Du et al., 2009),

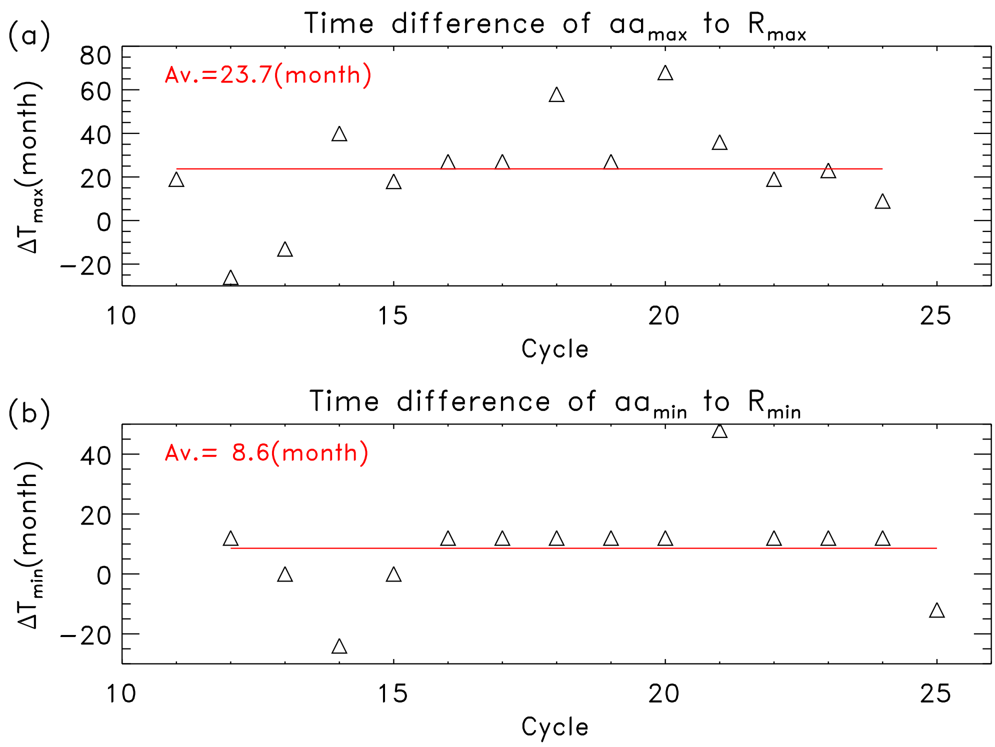

Fig. 10. The time difference between $aa_{\rm max}$ and $R_{\rm max}$ (a) and that between $aa_{\rm min}$ and $R_{\rm min}$ (b).

using the geomagnetic activity near the solar minimum; and 4) the solar precursor ones (Schatten et al., 1978; Pesnell and Schatten, 2018), using the previous cycle's polar field.

In contrast, there are less methods found to predict the maximum amplitude of geomagnetic index for the 11-year solar cycle. Geomagnetic activity forecast has been over the order of hours or days (McPherron, 1999; Abunina et al., 2013). The annual or monthly prediction on the geomagnetic activity is within a limited accuracy (over 20%) due to the irregular variation in the time series (McPherron, 1999; Gordon, 2015). In the earlier years, Kane (1988) even pointed out that it is impossible to forecast the long-term geomagnetic activity through analyzing the time series of the Ap and aa indices (refer also to Gordon, 2015). The geomagnetic activity near the solar minimum or at the decreasing phase of the solar cycle has been widely used to predict the maximum amplitude of sunspot cycle, but was seldom used to predict the maximum amplitude of the geomagnetic activity itself.

In the current work, we analyzed the **highest (aa_H) and the lowest (aa_L) 3-hourly aa index in each 3 days' interval, smoothed by 363 days (121 points) to mimic the 13 months smoothing. It is found that the maximum of aa_H (aa_Hmax) is well correlated to the preceding minimum of either aa_H (aa_Hmin,$r = 0.85$) or aa_L (aa_Lmin,$r = 0.89$) for the 11-year solar cycle.** So, these relationships can be used to estimate the strength of geomagnetic activity for the ensuing cycle by employing the time series itself, $aa_\mathsf{Hmax}(25) = 84.5 \pm 6.9$ (nT). It implies that the strength of **geomagnetic activity for** the ensuing cycle 25 would be similar to the average over the past cycles, but higher than that of cycle 24 **by about 30%. Certainly, this estimate may be an upper limit, as cycle 24 has not completely passed and we should check if there is an even smaller value of $aa_\mathsf{Hmin}(25)$ or $aa_\mathsf{Lmin}(25)$ than that used in the current work (31.05 or 2.94) in the future few months.**

Similar result can also be obtained if using the 363-day-smoothing highest/lowest 3-hourly Ap index in 3-days-interval ($Ap_\mathsf{H}/Ap_\mathsf{L}$). The maximum of Ap_H (Ap_Hmax) is found to be well correlated to the preceding minimum of $Ap_\mathsf{H}(Ap_\mathsf{Hmin}, r = 0.96)$ or $Ap_\mathsf{L}(Ap_\mathsf{Lmin}, r = 0.79)$ for the 11-year solar cycle. The rise time (T_Ha) from Ap_Hmin to Ap_Hmax is reversely correlated to the preceding minimum of $Ap_\mathsf{L}(Ap_\mathsf{Lmin}, r = -0.72)$. For the 13-month smoothed monthly mean $aa(Ap)$ index, the maximum $aa(Ap)$ index, $aa_\mathsf{max}(Ap_\mathsf{max})$, of the solar cycle is also well correlated to the preceding minimum, $aa_\mathsf{min}(Ap_\mathsf{min})$, with a correlation coefficient of $r = 0.95(0.86)$.

The well known 'Waldmeier effect' (Waldmeier, 1939) that the rise time of a solar cycle is well anti-correlated to the following maximum amplitude has been widely used to estimate the rise and peak times of a solar cycle if the amplitude has been predicted. However, such a correlation **is very weak** for the geomagnetic activity index. The rise time (T_{Hr}) from aa_{Hmin} to aa_{Hmax} for the 11-year solar cycle is found to be only weakly anti-correlated to the following maximum (aa_{Hmax}), $r = -0.42$**. The rise time** (T_{Ha}) **from** Ap_{Hmin} **to** Ap_{Hmax} **is also weakly anti-correlated to the following maximum (**Ap_{Hmax}**),** $r = -0.33$**. For the 13-month smoothed monthly mean** $aa(Ap)$ **index, the rise time of** $aa_{\mathsf{max}}(Ap_{\mathsf{max}})$ **is nearly uncorrelated to the following maximum,** $r = -0.09(-0.17)$**. These weak correlations may be related to the fact that the geomagnetic activity minimum (maximum) is not aligned to the solar (sunspot) activity minimum (maximum) in time, as shown in Fig. 10 for the time difference of** aa_{max} **to** R_{max}**,** ΔT_{max} **(a), and that of** aa_{min} **to** R_{min}**,** ΔT_{min} **(b). In most cases,** $aa_{\mathsf{max}}(aa_{\mathsf{min}})$ **lags behind** $R_{\mathsf{max}}(R_{\mathsf{min}})$**. But in some other cases,** $aa_{\mathsf{max}}(aa_{\mathsf{min}})$ **precedes** $R_{\mathsf{max}}(R_{\mathsf{min}})$**. The weak correlation between the rise time and the following maximum of geomagnetic activity for the 11-year solar cycle can hardly be used to estimate the former.**

According to the analysis above, the following conclusions may be summarized.

1. **The 363-day-smoothing highest (**aa_{H}**) and lowest (**aa_{L}**) 3-hourly** aa **indices in 3-day-interval are analyzed, finding that the maximum of** aa_{H} **(**aa_{Hmax}**) is well correlated to the preceding minimum of either** aa_{H} **(**$aa_{\mathsf{Hmin}}, r = 0.85$**) or** aa_{L} **(**$aa_{\mathsf{Lmin}}, r = 0.89$**) for the 11-year solar cycle.** As a result, the maximum aa index for the ensuing cycle 25 is estimated to be $aa_{\mathsf{Hmax}}(25) = 84.5 \pm 6.9$ (nT), about 30% higher than that of cycle 24. **This value is equivalent to the Ap index of** $Ap_{\mathsf{Hmax}}(25) = 47.8 \pm 3.9 \pm 2.1$ **(nT) if using the relationship between** Ap **and** aa **(Eq.(9)).**

2. **The maximum (**aa_{Lmax}**) of** aa_{L} **is also found to be well correlated to the preceding** aa_{Hmin}**,** $r = 0.80$**. Based on this correlation,** $aa_{\mathsf{Lmax}}(25)$ **is estimated to be** 6.1 ± 1.2 **(nT), about 16% higher than that of cycle 24.**

3. **The maximum of sunspot cycle (**R_{m}**) is much better correlated to the high geomag-**

netic activity (aa_{Hmax}, $r = 0.79$) than to the low one (aa_{Lmax},$r = 0.37$).

4. **The rise time (T_{Hr}) from aa_{Hmin} to aa_{Hmax} is found to be weakly anti-correlated to the following maximum (aa_{Hmax}) for the 11-year solar cycle, $r = -0.42$ at the 84% confidence level.**

5. **Similar correlations are found for the 363-day-smoothing highest/lowest 3-hourly Ap index in 3-day-interval ($Ap_{\mathrm{H}}/Ap_{\mathrm{L}}$). (1) The maximum of Ap_{H} (Ap_{Hmax}) is well correlated to the preceding minimum of either Ap_{H} (Ap_{Hmin},$r = 0.96$) or Ap_{L} (Ap_{Lmin},$r = 0.79$) for the 11-year solar cycle. (2) The maximum of Ap_{L} (Ap_{Lmax}) is well correlated to the preceding $Ap_{\mathrm{Hmin}}(r = 0.70)$. (3) The rise time ($T_{\mathrm{Ha}}$) from Ap_{Hmin} to Ap_{Hmax} is well anti-correlated to the preceding Ap_{Lmin} ($r = -0.72$).**

6. **For the 13-month smoothed monthly mean $aa(Ap)$ index, the maximum $aa(Ap)$ index, $aa_{\mathrm{max}}(Ap_{\mathrm{max}})$, of the solar cycle is well correlated to the preceding minimum, $aa_{\mathrm{min}}(Ap_{\mathrm{min}})$, with a correlation coefficient of $r = 0.95(0.86)$. The rise time of $aa_{\mathrm{max}}(Ap_{\mathrm{max}})$ is nearly uncorrelated to the following maximum, $r = -0.09(-0.17)$.**

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References

Abunina, M., Papaioannou, A., Gerontidou, M., Paschalis, P., Abunin, A., and more: Forecasting Geomagnetic Conditions in near-Earth space, Journal of Physics: Conference Series, 409, id. 012197, 2013.

Bartels, J.: Discussion of time variations of geomagnetic activity indices Kp and Ap 1932-1961, Ann. Geophys., 19, 1–20, 1963.

Borello-Filisetti, O., Mussino, V., Parisi, M., and Storini, M.: Long-term variations in the geomagnetic activity level. I - A connection with solar activity, Ann. Geophys., 10, 668–675, 1992.

Brown, G.M., Williams, W.R.: Some properties of the day-to-day variability of Sq(H), Planet. Space Sci., 17, 455, 1969.

Cameron, R., Schüssler, M.: A robust correlation between growth rate and amplitude of solar cycles: consequences for prediction methods, Astrophys. J., 685, 1291–1296, 2008.

Chen, S., Chai, L., Xu, K., Wei, Y., Rong, Z., and Wan, W.: Estimation of the Occurrence Probability of Extreme Geomagnetic Storms by Applying Extreme Value Theory to Aa Index, J. Geophys. Res.: Space Physics, 124, 9943–9952, 2019.

Clette, F., Cliver, E., Lefèvre, L., Svalgaard, L., Vaquero, J., Leibacher, J.: Preface to topical issue: recalibration of the sunspot number, Solar. Phys., 219, 2479–2486, 2016.

Cliver, E. W., Boriakoff, V., and Feynman, J.: Solar variability and climate change: Geomagnetic aa index and global surface temperature, Geophys. Res. Lett., 25, 1035–1038, 1998.

Dobrica, V., Demetrescu, C., Boroneant, C., and Maris, G.: Solar and geomagnetic activity effects on climate at regional and global scales: Case study-Romania, J. Atmos. Sol. Terr. Phys., 71, 1727–1735, 2009.

Du, Z. L.: The correlation between solar and geomagnetic activity – Part 1: Two-term decomposition of geomagnetic activity, Ann. Geophys., 29, 1331–1340, 2011a.

Du, Z. L.: The correlation between solar and geomagnetic activity – Part 2: Long-term trends, Ann. Geophys., Ann. Geophys., 29, 1341–1348, 2011b.

Du, Z. L.: The correlation between solar and geomagnetic activity – Part 3: An integral response model, Ann. Geophys., 29, 1005–1018, 2011c.

Du, Z. L.: The Shape of Solar Cycle Described by a Modified Gaussian Function, Solar. Phys., 273, 231–253, 2011d.

Du, Z. L.: The solar cycle: predicting the peak of solar cycle 25, Astrophys. Space Sci., 365, 104, 2020.

Du, Z. L., and Wang, H. N.: The relationships of solar flares with both sunspot and geomagnetic activity, Research in Astron. Astrophys., 12, 400–410, 2012.

Du, Z. L., Li, R., and Wang, H. N.: The Predictive Power of Ohl's Precursor Method, Astron. J., 138, 1998–2001, 2009.

Du, Z. L., and Wang, H. N.: Predicting the solar maximum with the rising rate, Science China (Physics, Mechanics & Astronomy), 55, 365–370, 2012.

Echer, E., Gonzalez, W. D., Gonzalez, A.L.C. et al.: Long-term correlation between solar and geomagnetic activity, J. Atmos. Sol. Terr. Phys., 66, 1019–1025, 2004.

El-Borie, M. A., El-Taher, A. M., Thabet, A. A., and Bishara, A. A.: The impact of asymmetrical distribution of solar activity on geomagnetic indices throughout five solar activity cycles, Adv. Spa. Res., 64, 278–286, 2019.

Feynman, J.: Geomagnetic and solar wind cycles, 1900-1975, J. Geophys. Res., 87, 6153–6162, 1982.

Gavrilyeva, G. A., Ammosov, P. P., Ammosova, A. M., Koltovskoi, I. I., and Sivtseva, V. I.: Geomagnetic activity signature in seasonal variations of mesopause temperature over Yakutia, Proceedings of the SPIE, 10466, id. 1046670, 2017.

Gonzalez, W. D., Gonzalez, A. L. C., Tsurutani, B. T. et al.: Solar wind-magnetosphere coupling during intense magnetic storms (1978-1979), J. Geophys. Res., 94, 8835–8851, 1989.

Gonzalez, W. D., Joselyn, J. A., Kamide, Y. et al: What is a geomagnetic storm?, J. Geophys. Res., 99, 5771–5792, 1994.

Gordon, R.: Forecasting geomagnetic activity at monthly and annual horizons: Time series models, J. Atmos. Sol. Terr. Phys., 133, 111–120, 2015.

Hathaway, D. H., Wilson, R. M., and Reichmann, E. J.: The shape of the sunspot cycle, Solar. Phys., 151, 177–190, 1994.

Kane, R. P.: Forecasting geomagnetic activity, PAGEOPH., 126, 85–101, 1988.

Kishcha, P. V., Dmitrieva, I. V., and Obridko, V. N.: Long-term variations of the solar - geomagnetic correlation, total solar irradiance, and northern hemispheric temperature (1868-1997), J. Atmos. Sol. Terr. Phys., 61, 799–808, 1999.

Legrand, J. P., and Simon, P. A.: Ten cycles of solar and geomagnetic activity, Sol. Phys., 70, 173–195, 1981.

Li, Y.: Predictions of the features for sunspot cycle 23, Sol. Phys., 170, 437–445, 1997.

Lockwood, M., Stamper, R., and Wild, M. N.: A doubling of the Sun's coronal magnetic field dring the past 100 years, Nature, 399, 437–439, 1999.

Lukianova, R., Alekseev, G., and Mursula, K.: Effects of station relocation in the aa index, J. Geophys. Res., 114, A02105, 2009.

Marat, D., Galina, D., Viktor, D., and Anna, D.: Dependence of the F2-layer critical frequency median at midlatitudes on geomagnetic activity, Sol. Terr. Phys., 64, 278–286, 2017.

Mayaud, P. N.: The aa indices: A 100-year series characterizing the magnetic activity, J. Geophys. Res., 77, 6870–6874, 1972.

McPherron, R.L.: Predicting the Ap index from past behavior and solar wind velocity, Physics and Chemistry of the Earth, Part C: Solar, Terrestrial & Planetary Science, 24, 45–56, 1999.

Mussino, V., Borello-Filisetti, O., Storini, M., and Nevanlinna, H.: Long-term variations in the

geomagnetic activity level Part II: Ascending phases of sunspot cycles, Ann. Geophys., 12, 1065–1070, 1994.
Ohl, A. I.: A new method of very long-term prediction of solar activity, In NASA. Marshall Space Flight Center Solar-Terrest. Predictions Proc., 2, 258–263, 1979.
Petrovay, K.: Solar cycle prediction, Living Rev. Sol. Phys., 17, 2, 2020.
Pesnell, W. D., Schatten, K. H.: An early prediction of the amplitude of Solar Cycle 25, Solar. Phys., 293, 112, 1994.
Prestes, A., Rigozo, N. R., Echer, E., and Vieira, L. E. A.: Spectral analysis of sunspot number and geomagnetic indices (1868-2001), J. Atmos. Sol. Terr. Phys., 68, 182–190, 2006.
Russell, C. T., and Mulligan, T.: The 22-year variation of geomagnetic activity: Implications for the polar magnetic field of the Sun, Geophys. Res. Lett., 22, 3287–3288, 1995.
Schatten, K. H., Scherrer, P. H., Svalgaard, L., and Wilcox, J. M.: Using dynamo theory to predict the sunspot number during solar cycle 21, Geophys. Res. Lett., 5, 411–414, 1978.
Singh, P. R., Tiwari, C. M., Saxena, A. K., Agrawal, S. L.: Quasi-biennial periodicities and heliospheric modulation of geomagnetic activity during solar cycles 22C24, Phys. Scr., 94, 105005, 2019.
Thompson, R. J.: The rise of solar cycle number 22, Solar. Phys., 117, 279–289, 1988.
Tsurutani, B. T., Gonzalez, W. D., Gonzalez, A. L. C. et al.: Corotating solar wind streams and recurrent geomagnetic activity: A review, J. Geophys. Res., 111, A07S01, 2006.
Waldmeier, M.: Über die Struktur der Sonnenflecken, Astron. Mitt. Zrich, 14, 439–450, 1939.
Wang, Y. M., Lean, J., and Sheeley, N. R.: The long-term variation of the Sun's open magnetic flux, Geophys. Res. Lett., 27, 505–508, 2000.
Wang, Y. M., and Sheeley, N. R.: Understanding the Geomagnetic Precursor of the Solar Cycle, Astrophys. J., 694, L11–L15, 2009.
Yoshida, A., Yamagishi, H.: Predicting amplitude of solar cycle 24 based on a new precursor method, Ann. Geophys., 28, 417–425, 2010.