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Mirror mode physics: The amplitude limit

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Abstract. The mirror mode evolving in collisionless magnetised high-temperature thermally anisotropic plasmas is shown to resemble a macro-quantum state. Starting as a classical zero frequency ion fluid instability it saturates quasi-linearly at very low magnetic level, while forming extended magnetic bubbles.It traps the electron component into an adiabatic bounce motion along the magnetic field which causes a bulk electron anisotropy. This can drive an electron mirror mode (see Treumann & Baumjohann, 2018b, who identified it in old spacecraft data). More important, however, we show that trapped electrons play the dominant role of further evolution towards a stationary state. Interaction of the trapped bouncing electrons with the thermal level of ion sound waves causes attractive potentials between electrons and forms electron pairs in the lowest-energy singlet state of two combined electrons. This happens preferentially near the electron mirror points resulting in a diamagnetic current effect which ultimately drives evolution of the magnetic field into large amplitude mirror bubbles causing diamagnetism and expelling a larger fraction of magnetic flux from the interior of the initial quasi-linearly stable mirror mode bottle. Estimates given in view of mirror modes in the magnetosheath are in reasonable numerical agreement with observation. We derive the self-consistent final state of the mirror bubbles. This analysis demonstrates that the observed mirror mode in high temperature space plasmas (solar wind, magnetosheath, magnetotail) is not a simple magnetohydrodynamic instability. It resembles a classical super-conducting, super-fluid state in high temperature plasma under conditions when electron pairs form. This is a most interesting observation which suggests that pair formation can become relevant in space and astrophysics.

Keywords. Mirror modes, magnetosheath, solar wind, turbulence, macro-quantum states

1 Introduction

There seems to be nothing particularly interesting left about a very low frequency effect in high temperature magnetised plasma known as the mirror mode (see, e.g., Tsurutani et al., 2011, for a more recent observational review). It was formally discovered some sixty years ago (Chandrasekhar, 1961; Hasegawa, 1975; Gary, 1993) as a theoretical complement of the zero-frequency hose instability, two purely growing linear instabilities in the presence of pressure anisotropies. The hose instability excites propagating Alfvén waves when the magnetically parallel temperature $T_\| > T_\perp$ exceeds the perpendicular temperature, the mirror mode grows under the opposite condition $T_\| < T_\perp$ that the perpendicular temperature is higher than the parallel by a certain amount, passing a threshold. The mirror mode generates magnetically elongated magnetic bottles thereby providing the plasma a local texture. Later it was found that in the presence of weak plasma gradients the mirror mode assumed a small

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but finite real frequency (Hasegawa, 1969). Various formal properties of the instability were added under different plasma conditions and in different wavenumber ranges like finite gyroradius effects, dependencies on electron temperature and electron anisotropies, as well as on streaming plasma conditions. It was moreover shown that the instability saturates quasilinearly at a rather low level by exhausting the bulk thermal anisotropy (cf., e.g., Treumann & Baumjohann, 1997; Noreen et al., 2017, for the quasilinear numerics). Finally it was shown that the trapped particle components give rise to the excitation of ion cyclotron and electron whistler waves if only a thermal anisotropy of *resonant particles* evolves. Identification in real plasmas became possible when measuring the pressure balance between the magnetic field and plasma. It was shown that in observations the two pressures in ion-mirror modes are indeed anti-correlated, a condition which generally is considered the key identifier of mirror modes: mere pressure balance. Thus it seemed that the physics of mirror modes was completely understood though it remained unclear how in an ideally conducting plasma at high temperature the magnetic field could become expelled to a high degree from the interior of the magnetic mirror bottle, an effect resembling the Meissner effect in low frequency low-temperature superconductivity which, however, would be forbidden in classical physics as it requires the presence of quantum correlations known to be restricted to very dense very low-temperature conditions only. In superconductivity the Meissner effect arises from the close interaction between electrons and phonons in the crystal lattice of a superconducting metal when electron pairs form in a way that the antiparallel spins of the paired electrons which occupy the same quantum state compensate and the pairs, though remaining fermions, together with the interacting phonon assume quasi-particle-bosonic properties, and can condensate in the lowest energy level in the conduction band just above the Fermi energy, evolving into a Landau-Fermi fluid. The theory of this effect was given by Ginzburg & Landau (1950) based on Landau's Fermi-liquid theory (Landau, 1941), ultimately culminating in the famous microscopic superconducting BCS theory (Bardeen et al., 1957) which relies on Cooper pair formation (Cooper, 1956). Only when this happens, the current of the condensate becomes capable of generating a sufficient amount of correlated bulk diamagnetism, rather different from simple pressure balance, which then compensates and thus expels the magnetic field from the superconducting region. Interestingly this effect is based on microscopic annihilation of oppositely directed magnetic fields thus resembling kind of microscopic reconnection that has not yet been investigated! In superconductivity it becomes strong indeed until the magnetic field is completely expelled and the lattice resistance is not only circumvented but violently excluded. In the mirror mode, on the other hand, the comparable quasi-diamagnetic effect – generally believed to be caused by pressure balance only – amounts to roughly 50%. A substantial strong interior magnetic field remains whose amount, however, is much less than any plasma theory like quasilinear theory predicts. It has also been claimed that it could be due to weakly kinetic plasma turbulence based on wave-wave and wave-particle interaction. However no such theory is in sight as there are no substantially large amplitude plasma waves of any kind available in the required frequency and wave number range which could do the job. This discrepancy is disturbing because it suggests that some fundamental effect has not yet been understood in the evolution of the mirror mode.

Recently we exhumed kind of a parallelism between superconductivity and the growth of the ion mirror mode (Treumann & Baumjohann, 2018a). Here we demonstrate that the mirror mode can be understood as a combination of a classical plasma ion effect which generates magnetic bottles at very low quasilinear saturation level (Noreen et al., 2017), while the main large mirror effect is caused by the trapped electron component in a similar though classical way as in the BCS theory (Bardeen et

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al., 1957) of superconductivity. It are the trapped mirroring electrons, not the ions, which interact with the available general thermal ion-sound wave population in the plasma at either thermal or non-thermal level to produce correlated trapped electron pair singlets which are dynamically distributed over the volume of a mirror bottle. Together with the ion sound fluctuations they form kind of quasiparticles which condensate at an energy level near the Debye temperature, lead to strong electron boundary currents and cause sufficient diamagnetism to produce the large amplitude mirror bubbles.

Here we show that the mirror mode is a combination of the classical ion effect causing the evolution of low amplitude mirror bottles which in a plasma provide a magnetic texture. This combines with a macro-quantum effect on the mirror trapped low energy electron component which is responsible for the destruction of the quasilinear level transforming it into a diamagnetic macro-Meissner effect. The latter causes the blow up of the bubbles in the collisionless high temperature plasma. The physical mechanism behind this effect is the interaction of the trapped electrons with the thermal background noise of the ion-acoustic wave spectrum excited inside the small-amplitude mirror mode

2 Electron trapping

Once the ion-mirror mode starts growing at the well known ion-mirror growth rate

$$\frac{\gamma_m(\boldsymbol{k})}{\omega_{ci}} \approx \frac{k_\|\lambda_i}{1+A}\sqrt{\frac{\beta_\|}{\pi}}\Big[A-\frac{k^2}{k_\perp^2\beta_\perp}\Big], \qquad A\equiv\frac{T_{i\perp}}{T_{i\|}}-1\gtrsim 0 \tag{1}$$

with approximately vanishing real frequency $\omega_m \approx 0$, neglecting the effect of density gradients which would cause a finite real part on the frequency, $\lambda_i = c/\omega_i$ ion inertial length, a magnetic bottle evolves in slightly oblique direction $k_\| \ll k_\perp$ with magnetic disturbances $|\delta B_\|| \gg |\delta B_\perp|$. This bottle is elongated along the ambient magnetic field $\boldsymbol{B}$ and has a narrow opening angle θ given by $\tan\theta = k_\|/k_\perp \ll 1$. Instability corresponds to a second order phase transition in plasma which happens whence the magnetic field locally drops below a critical threshold value

$$B < B_c \approx \sqrt{2\mu_0 N T_{i\perp} A}\,|\sin\theta| \tag{2}$$

Though substantial, this growth rate is just a fraction of the ion cyclotron frequency $\omega_{ci} = eB/m_i$ for $k_\|\lambda_i \ll 1$ and B respectively A near threshold, the usual case (Treumann & Baumjohann, 2018a). As noted above, the instability readily stabilises quasi-linearly (Noreen et al., 2017; Treumann & Baumjohann, 1997) at very low level $|\delta B|^2 \ll B^2$ via depleting the anisotropy A.

2.1 Electron dynamics, energy limit, trapped density fraction

The conventional ion mirror mode provides a quasi-stationary magnetic bottle (see, e.g., Constantinescu, 2002, for a geometric analytical model) structure, which necessarily traps electrons of sufficiently small magnetic moment μ_e. Because the mirror mode frequency practically vanishes and the mirror mode grows slowly compared to the electron dynamics, electrons react adiabatically to the presence of the mirror instability. They conserve their magnetic moment $\mu_e = \mathcal{E}_{e\perp}/B = \text{const}$ when

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moving along the magnetic field $B(s)$. Trapping occurs between the two mirror magnetic fields $B_{\pm m} = B(\pm s_m)$, with s coordinate along the magnetic field. The trapped-electron perpendicular kinetic energy $\mathcal{E}_{e\perp}(s) = \mu_e B(s) \equiv V(s)$ plays the role of a retarding potential

$$\mathcal{E}_{e\|}(s) = \mathcal{E}_e - V(s) \tag{3}$$

At the mirror points the parallel energy of trapped electrons vanishes, and $\mathcal{E}_{e\perp}(\pm s_m) = \mathcal{E}_e$. Thus trapping occurs for all $\mu_e \leq \mu_m \equiv \mathcal{E}_e / B_{\pm m}$, a well-known fact. Though it does not bunch them, mirroring keeps these electrons together by confining them to the volume of the bottle, inside which they perform the oscillatory bounce motion between mirror points $\pm s_m$. The parallel electron equation of motion is

$$\frac{dv_\|(s,t)}{dt} = -\frac{\mu_e}{m_e}\nabla_\| B(s), \qquad \nabla_\| = \frac{\partial}{\partial s} \tag{4}$$

For symmetric bottles and motion around and not too far away from the minimum $B(s_0) = \min\{B(s)\} \equiv B_0$ of the magnetic field we have

$$B(s) \approx B_0 + \tfrac{1}{2} B_0''(s - s_0)^2, \qquad B_0'' = \frac{\partial^2 B}{\partial s^2}\Big|_{s_0} \tag{5}$$

which immediately gives the bounce frequency

$$\omega_b = \sqrt{\frac{\mu_e}{m_e} B_0''} \ll \omega_{ce} \tag{6}$$

a frequency much less than the electron cyclotron frequency $\omega_{ce} = eB/m_e$. We shall show below that this kind of trapping, in the case of the mirror mode, becomes advantageous for electron pairing, an effect otherwise observed only under solid state conditions in superconducting metals.

In order to get an idea on the trapping energy condition we consider the mirror point $s = \pm s_m$. Here all the energy is in the perpendicular direction, i.e. the local gyro motion of electrons. Hence de-trapping of electrons occurs once their gyroradius exceeds the opening radius of the bottle neck $r_{ce,s} \gtrsim R_s = \mathcal{L}_\| \tan\theta$, where $\mathcal{L}_\| \sim 2\pi/k_\|$ is the half length of the ion mirror bottle. This yields immediately that electrons remain trapped as long as their energy satisfies the condition

$$\mathcal{E}_e \lesssim \frac{1}{4\pi^2}\frac{T_e}{k_\perp^2 r_{ce}^2} \equiv \mathcal{E}_{trap}, \qquad r_{ce} = v_e/\omega_{ce,s} \tag{7}$$

with r_{ce} the electron gyro-radius and $k_\perp$ the perpendicular wave number of the ion-mirror mode. All electrons with this energy remain trapped in the magnetic mirror bottle. Larger energy electrons escape from the bottle along the magnetic field. (We do not discuss the subtle problem that quasi-neutrality requires them to be replaced by electron inflow.)

Since trapped electrons fill the volume of the ion mirror mode, the question is whether they contribute to diamagnetism and blow up the mirror bottle to become a real large-amplitude magnetic mirror bubble. It is clear that such an effect cannot be a simple pressure balance because this is already taken care of in the quasi-linearly stable ion mirror mode via charge neutrality where all particles contribute. The difference can thus only be the order of at most a factor of 2. Otherwise, if it can be shown

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that the trapped electrons are correlated behaving like quasi-particles, then they would form kind of a quasi-particle condensate behaving collectively and adding their diamagnetism coherently in a way similar to superconductivity in the Fermi fluid. Below we will investigate a condition which, if fulfilled, causes macroscopic mirror bubbles to evolve not as an ion but an electron effect.

2.2 Fractional trapping conditions

The fractional number density of maxwellian mirror trapped electrons is

$$\begin{aligned}\frac{N_{trap}}{N_0} &= C\int_0^{\mathcal{E}_{trap}} d\epsilon_e\,\epsilon_e^{\frac{1}{2}}\exp\Big(-\frac{\epsilon_e}{T_e}\Big) = \Gamma^{-1}\big(\tfrac{3}{2}\big)\gamma\Big(\tfrac{3}{2},\frac{\mathcal{E}_{trap}}{T_e}\Big)\\ &= \frac{1}{4}\mathrm{erf}\Big(\sqrt{\frac{\mathcal{E}_{trap}}{T_e}}\Big) - \sqrt{\frac{\mathcal{E}_{trap}}{T_e}}\exp\Big(-\frac{\mathcal{E}_{trap}}{T_e}\Big)\end{aligned} \tag{8}$$

where C is a normalisation constant, and $\gamma(a,b)$ is the incomplete Gamma function.

The trapped electron dynamics is basically along the magnetic field $\boldsymbol{B}(s)$. It consists of the bounce motion at systematically varying parallel velocity $v_\|(s)$, vanishing at the mirror point $v_\|(s_{\pm m}) = 0$ and maximising in the symmetric plane cross section, and the fast gyration with perpendicular velocity $v_\perp(s)$. Under isotropic conditions, the maximum parallel velocity is $v_\|(0) = v_\perp(0)$. Hence, the trapped electron energy distribution along the bounce path is anisotropic with

$$T_\perp(s) > T_\|(s) \qquad \text{for} \qquad s \neq 0 \tag{9}$$

This is important to remember. At the one hand, if the bulk electron anisotropy $T_\perp(s)/T_\|(s) > 1$ becomes sufficiently large somewhere along the magnetic field, electrons may themselves evolve into an electron-scale electron-mirror branch (Noreen et al., 2017) which appears as a small-scale low-amplitude structure on the ion-mirror mode. Such structures have recently been identified in both old (Treumann & Baumjohann, 2018b) and recent high resolution (Breuillard et al., 2018; Ahmadi et al., 2018) spacecraft data. Trapped electrons may also, as will be demonstrated below, go into resonance with ion-sound waves which at finite temperature are always present in the plasma at least as thermal background.

The constancy of the magnetic moment, as is well known for long (cf., e.g., Baumjohann & Treumann, 1996, for a textbook presentation), can also be exploited to represent the parallel particle velocity respectively energy through the magnetic mirror ratio

$$\mathcal{E}_\|(s) = \mathcal{E}_e\Big(1 - \frac{B(s)}{B(s_m)}\Big) \tag{10}$$

which defines the angle between velocity and magnetic field for the trapped electrons

$$\theta(s) = \cos^{-1}\sqrt{1 - \frac{B(s)}{B(s_m)}} \tag{11}$$

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3 Single electron wake potential

In preparing for the investigation of ion-mirror-mode trapped electrons, we consider the interaction of an electron with the bath of ion sound waves. This is most easily done in the naked test particle picture, assuming that we grab one of the electrons and ask for its reaction to the presence of the dielectric in which it moves.

The electron is a point charge $-e$ with velocity $\boldsymbol{v}$ that is located at its instantaneous position $\boldsymbol{x}' = \boldsymbol{x} - \boldsymbol{v}t$ in the observers frame $(\boldsymbol{x}, t)$. This is represented by the point charge density function $N(\boldsymbol{x}, t) = -e\delta(\boldsymbol{x} - \boldsymbol{v}t)$. We assume that the electron is non-relativistic which for trapped electrons under the conditions in the magnetosheath (Lucek et al., 2005) or the solar wind is good enough. The relative dielectric constant of the plasma it experiences is $\epsilon(\omega, \boldsymbol{k})$ where $\omega, \boldsymbol{k}$ are frequency and wavenumber of the plasma wave which changes the dielectric properties. In general, we have a whole spectrum of waves which is taken care of below by integrating over the entire spectrum. The naked charged electron polarises the plasma. The total electric potential the moving non-relativistic charge at location $\boldsymbol{x}'$ causes at location $\boldsymbol{x}$ is obtained from Poisson's equation with above charge density and has the form

$$\Phi(\boldsymbol{x}, t) = -\frac{e}{(2\pi)^3\epsilon_0}\int d\omega d\boldsymbol{k}\frac{\delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v})}{k^2\epsilon(\boldsymbol{k}, \omega)}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{v}t)} \tag{12}$$

This can easily be shown (cf., e.g., Neufeld & Ritchie, 1955; Krall & Trivelpiece, 1973, for a textbook description) by Fourier-transformation. In this representation the action of the δ-function on the exponential has already been taken care of. Integration is over wave numbers and frequencies, the wave spectrum responsible for the dielectric properties the electron experiences. Integration with respect to frequency ω implies the substitution $\omega \to \boldsymbol{k}\cdot\boldsymbol{v}$ also in the dielectric response function $\epsilon(\omega, \boldsymbol{k})$, which we shift until having discussed the latter.

In solid state physics it is assumed that the oscillations of the ion lattice generate a thermal spectrum of phonons. In plasmas these waves are not restricted to the Brillouin zones but are freely propagating waves or thermal noise obeying a response function which either accounts for the excitation of the waves or simply refers to the thermal jitter motion of the plasma particles which leads to spontaneous emission and modifies the dielectric properties of the plasma. Any general linear, even nonlinear electrostatic response function in a plasma reads

$$\epsilon(\omega, \boldsymbol{k}) = 1 + \frac{1}{k^2\lambda_e^2} + \chi_e(\omega, \boldsymbol{k}) + \chi_i(\omega, \boldsymbol{k}) \tag{13}$$

with $\chi_{e,i}(\omega, \boldsymbol{k})$ the electron and ion susceptibilities. One may wonder why for wave lengths usually much longer than the Debye length $\lambda_e \ll \lambda$ the second term in this expression is not neglected. The reason that it must be retained here, is that the uncompensated charge of the test particle when immersed into the plasma excites short wavelengths waves on the Debye scale in order to screen the charge. Therefore, independent on the wavelength of plasma waves, the test particle dielectric response must include the Debye term.

The dielectric response function of the thermal spectrum of ion-sound waves at frequencies far below the electron plasma frequency $\omega \ll \omega_e$ is

$$\epsilon(\omega, \boldsymbol{k}) = 1 + \frac{1}{k^2\lambda_e^2} - \left(\frac{\omega_i}{\omega}\right)^2 \tag{14}$$

where ω_i is the ion plasma frequency, $\lambda_e \approx v_e/\omega_e$ the Debye screening distance, and the frequency of ion sound waves $\omega_{\boldsymbol{k}}$ is obtained putting the real part of this expression to zero, which as usually yields

$$\left(\frac{\omega_{\boldsymbol{k}}}{\omega_i}\right)^2 = \frac{k^2\lambda_e^2}{1+k^2\lambda_e^2} \qquad \text{or} \qquad \omega_{\boldsymbol{k}}^2 = \frac{c_s^2k^2}{1+k^2\lambda_e^2} < \omega_i^2 \tag{15}$$

Here $c_s^2 = \omega_i^2\lambda_e^2 \approx (m_e/m_i)v_e^2 \approx 2T_e/m_i$ is the ion-sound speed square. It is simple matter to show that the inverse response function becomes

$$\frac{1}{\epsilon(\omega,\boldsymbol{k})} = \frac{k^2\lambda_e^2}{1+k^2\lambda_e^2}\left(1+\frac{\omega_{\boldsymbol{k}}^2}{\omega^2-\omega_{\boldsymbol{k}}^2}\right) \tag{16}$$

Actually, this is also the general inverse form of any response, if only $\omega_{\boldsymbol{k}}$ is understood as the solution of the general response function $\epsilon(\omega,\boldsymbol{k})=0$ for electrostatic waves, and $\epsilon(\omega,\boldsymbol{k})-k^2c^2/\omega^2=0$ for very-low frequency electromagnetic waves like magneto-sonic or Alfvén waves. In the latter case one has, including kinetic effects,

$$\epsilon_A(\omega,\boldsymbol{k}) = 1+\frac{1}{k^2\lambda_e^2}+\frac{c^2}{V_A^2}\left[1+(\boldsymbol{k}\cdot\boldsymbol{r}_{ci})^2\left(\frac{3}{4}+\frac{T_e}{T_i}\right)\right]^{-1} \tag{17}$$

with $\boldsymbol{r}_{ci} = \boldsymbol{v}_{i\perp}/\omega_{ci}$ the vectorial ion gyro-radius. The relevant wave frequency is $\omega_{\boldsymbol{k}A}^2 \approx k^2V_A^2$ for the ordinary Alfvén wave, with $V_A \ll c$ the Alfvén speed (if wanted including the bracketed modification factor). The electric potential resulting from its kinetic nature is along the magnetic field. Hence any attractive effect will be in this direction, a very interesting fact in itself which we do not investigate here leaving it for separate investigation.

Inserting Eq. (16) into the above electrostatic potential of the test electron

$$\Phi(\boldsymbol{x},t) = -\frac{e\lambda_e^2}{(2\pi)^2\epsilon_0}\int\frac{d\omega\, k_\perp dk_\perp dk_\parallel}{1+k^2\lambda_e^2}\left(1+\frac{\omega_{\boldsymbol{k}}^2}{\omega^2-\omega_{\boldsymbol{k}}^2}\right)\delta\Big(\omega-\boldsymbol{k}\cdot\boldsymbol{v}\Big)e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{v}t)} \tag{18}$$

shows that Φ consists quite generally of two contributions, the screened Coulomb potential of the test electron, and another wave induced term which multiplies the screened potential by the frequency dependent term in the last expression. This form demonstrates the well known self-screening Debye effect of the naked point charge, which leads to the first term in the above expression and causes the Debye-Yukawa potential to exponentially compensate for the electron charge field in a spherical region of radius λ_e. We are not interested here in the deformation of the Debye sphere introduced by the electron motion as this is a higher order effect.

The zero order effect of the test electron contained in the wave-independent term, the proper self-screening is, in the wave-dependent term, multiplied by the wave-induced factor. For frequencies $\omega^2 = \boldsymbol{k}^2\cdot\boldsymbol{v}^2 > \omega_{\boldsymbol{k}}^2$ higher than ion sound, this factor is positive adding to the screening but changes sign for frequencies $\omega^2 = \boldsymbol{k}^2\cdot\boldsymbol{v}^2 < \omega_{\boldsymbol{k}}^2$, thereby indicating the possibility of over-screening at wavelengths larger than the Debye radius λ_e. Under certain conditions it may come into play outside the Debye radius where the charge-electric field is practically already compensated, and the long range wave electric field adds up over some distance, may dominate and cause a spatially restricted deficiency of repulsion. In this case the potential may even turn negative, eliminates the repulsive nature of the electron locally and becomes attractive for electrons. This was first shown (Neufeld & Ritchie, 1955) for high frequency Langmuir waves even before the discovery of Cooper pairs in superconductivity

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and solid state physics. In a bath of Langmuir waves this attraction turned out to be unimportant however, while in an isotropic non-magnetic plasma it survives for low-frequency ion sound, first suggested by Nambu & Akama (1985). With θ_k the angle between electron speed and wavenumber, it happens at resonant electron speeds

$$v^2\cos^2\theta_k \lesssim \omega^2_{\boldsymbol{k}}/k^2 \tag{19}$$

requiring the parallel electron speed to be less than the wave phase velocity. The above expression depends on angle θ_k between velocity $\boldsymbol{v}$ and wavenumber $\boldsymbol{k}$, which in our case will turn out to be crucially important.

For completeness we note that in magnetised plasma the ion acoustic wave is azimuthally symmetric with respect to the magnetic field $\boldsymbol{B}$. However, its frequency depends itself on the angle of propagation between $\boldsymbol{k}=(k_\perp,k_\|)$ and $\boldsymbol{B}$ according to (Baumjohann & Treumann, 1996)

$$\omega^2_{\boldsymbol{k}} = \frac{c_s^2\Lambda_0(\eta_i)k_\|^2}{\Lambda_0(\eta_e)+k^2\lambda_e^2} \tag{20}$$

with $\Lambda_0(\eta_j)=I_0(\eta_j)\exp(-\eta_j)$, $\eta_j=\frac{1}{2}k_\perp^2r_{cj}^2$, and the index $j=e,i$ on the gyroradius is for electrons and ions. $I_0(\eta_j)$ is the Bessel function of imaginary argument. $r_{cj}=v_{\perp,j}/\omega_{cj}$ is the gyroradius, and ω_{cj} is the cyclotron frequency. One has that, moreover, $k_\perp\lambda_e\ll k_\perp r_{ci}\ll 1$ and $k_\|/k_\perp<1$. Long-wavelength ion sound in magnetised plasma thus propagates essentially along the magnetic field, a well known fact which in observations, for instance in the magnetosheath (Rodriguez & Gurnett, 1975), manifests itself as a complete drop out of the electrostatic low frequency thermal ion noise spectrum when the antenna points strictly perpendicular to the ambient magnetic field (cf., e.g., Treumann & Baumjohann, 2018b, for an example and discussion).

The interaction between electrons and ion sound waves thus opens up the option that electrons in a Debye-screened potential may, under certain conditions, experience an attractive potential which compensates and overcomes the Coulomb repulsion between two negatively charged electrons, resembling a the famous effect of Cooper pairing in solid state physics though here in the realm of classical physics. The paired electrons and the propagating ion sound wave form a quasiparticle in both these cases.

It is important to insist that this attraction is not due to trapping of the electron by a large amplitude wave in the wave potential trough; at the contrary, it is an electron-induced change in the dielectric properties of the wave-carrying plasma causing the electron to evolve an attractive electrostatic wake potential it carries along when moving across the plasma. We have previously shown (Treumann & Baumjohann, 2014) that this can happen also with other waves than ion-sound. Below we demonstrate that it becomes crucial in the evolution of mirror modes to which plasma wave trapping does not contribute in no sense.

Since the waves are longitudinal propagating along the magnetic field and the bounce motion of the electrons is as well along the magnetic field the coordinate s of interest is parallel to the magnetic field, and the gyration of the electrons decouples from the interaction. In this case we have for the wave number $\boldsymbol{k}=(k_\|,k_\perp)$ and velocity

$$k_\| = \boldsymbol{k}\cdot\hat{\boldsymbol{s}}, \qquad v_\|(s) = v\cos\theta(s)$$

parallel to the local magnetic field. The problem then consist in solving Eq. (18) under the conditions of a bouncing test electron. This task resembles the solution under non-magnetised conditions which had been given in our previous paper (Treumann &

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Baumjohann, 2014). In the known form it cannot be applied here but has to be substantially modified in order to become adapted to the conditions of electron trapping in mirror modes.

3.1 Conditions for an attractive potential

In the light of the previous discussion we rewrite Eq. (18) in the magnetic field as

$$\Phi(\boldsymbol{x},t) = -\frac{e\lambda_e^2}{2(2\pi)^2\epsilon_0}\int \frac{\omega_{\boldsymbol{k}}\,d\omega\,d\boldsymbol{k}\,e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{v}t)}}{(1+k_\perp^2\lambda_e^2+k_\parallel^2\lambda_e^2)}\left[\frac{\delta(\omega-k_\parallel v_\parallel)}{(\omega-\omega_{\boldsymbol{k}})}-\frac{\delta(\omega-k_\parallel v_\parallel)}{(\omega+\omega_{\boldsymbol{k}})}\right] \tag{21}$$

Here we left the Debye-potential term out as it is of no interest, and resolved the denominator. We also refer to the parallel particle velocity $v_\parallel = v\cos\theta$ which in our case of mirror trapped test particles is along the magnetic field. It selects the parallel wavenumber of the wave in the Dirac δ-function to replace the frequency ω. In the same spirit the argument of the exponential becomes $i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{v}t) = ik_\perp\rho\sin\phi + ik_\parallel(s-tv\cos\theta)$ with ρ the independent perpendicular spatial coordinate. It is assumed that the magnitude v of the velocity remains constant in this kind of interaction, which holds for the adiabatic motions along the magnetic field where no further external force acts on the electron except for the stationary restoring magnetic force. (Note also that the wave frequency $\omega_{\boldsymbol{k}}$ depends on $k_\parallel, k_\perp$ but not anymore on angle ϕ because it has been determined independently from kinetic wave theory not using the test particle picture.)

These assumptions reduce the integral to integrations over the perpendicular wavenumber $k_\perp, \phi$, and frequency ω. Moreover, since the problem has become cylindrically symmetric with respect to $\boldsymbol{B}$, integration over ϕ can easily be performed by using the representation of the exponential as a series of Bessel functions (**?**) which reduces to the zero-order Bessel function $J_0(k_\perp\rho)$. The formal result before final integration is

$$\Phi(s,\rho,t) = -\frac{e\lambda_e^2}{2(2\pi)^2\epsilon_0}\int \frac{\omega_{\boldsymbol{k}}\,d\omega\,k_\perp dk_\perp dk_\parallel\;J_0(k_\perp\rho)\,e^{ik_\parallel(s-v_\parallel t)}}{(1+k_\perp^2\lambda_e^2+k_\parallel^2\lambda_e^2)}\left[\frac{\delta(\omega-k_\parallel v_\parallel)}{\omega-\omega_k}-\frac{\delta(\omega-k_\parallel v_\parallel)}{\omega+\omega_k}\right] \tag{22}$$

where one understands $v_\parallel = v\cos\theta(s)$, and the ion sound wave frequency is

$$\omega_k^2 \approx \frac{\Lambda_0(\eta_i)c_s^2k_\parallel^2}{\Lambda_0(\eta_e)+k^2\lambda_e^2} \approx \frac{\Lambda_0(\eta_i)k_\parallel^2c_s^2}{1+k^2\lambda_e^2} \tag{23}$$

with the right-hand side holding since the electron term in the denominator is $\Lambda_0(\eta_e)\approx 1$. In the low frequency approximation applicable here, the frequency is proportional to the parallel wave number. In the following we simplify this dispersion relation setting $\Lambda_0(\eta_i)\approx 1$, which is its maximum value, and in the resonant denominators neglecting the inverse dependence of ω_k on $k\lambda_e$, only keeping it in the nominator of the integral. Then one may perform the integration with respect to $k_\perp$ which gives, with $\xi=\lambda_e k_\perp$, $\rho'=\rho/\lambda_e$, $\zeta = k_\parallel\lambda_e$,

$$I(\rho',\zeta) \equiv \int\limits_0^\infty \frac{\xi d\xi\,J_0(\xi\rho')}{\left(1+\zeta^2+\xi^2\right)^{3/2}} = \frac{\exp\left(-\rho'\sqrt{1+\zeta^2}\right)}{\sqrt{1+\zeta^2}} \tag{24}$$

In order to perform the integral, its singular properties have to be elucidated. The dominant contribution will come from the resonant denominators in the bracketed terms. Any possible resonances in the Coulomb factor do not play any role here. The

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Dirac δ-functions prescribe replacing the frequency everywhere with $k_\| v_\|$. It is, however, convenient to delay this action until integrating out the singularities in the complex ω plane. To see their effect, one temporarily replaces $k_\|$ in the argument of the exponential with ω as the δ-function prescribes as an inverse action. Then we have for $ik_\|(s-v_\|t) = i\omega(s/v_\| - t)$. Since the waves are damped, the imaginary part of the frequency is required to be negative. This forces demanding $s/v_\| - t < 0$, consequently taking the ω-integration over the lower complex ω-half plane, which in surrounding the poles in the positive sense adds a factor $2\pi i$ to the integral and includes the sum of residua $\omega = \pm\omega_k$ in the integral in this order. The result is

$$\Phi(s,\rho,t) = -\frac{i\,e}{4\pi\epsilon_0\lambda_e}\frac{c_s}{v_\|}\int \frac{\zeta\, d\zeta\, e^{-\rho'\sqrt{1+\zeta^2}+i\zeta(s-v_\|t)/\lambda_e}}{\sqrt{1+\zeta^2}}\left[\delta\Big(\zeta - \frac{\omega_k\lambda_e}{v_\|}\Big) - \delta\Big(\zeta + \frac{\omega_k\lambda_e}{v_\|}\Big)\right] \tag{25}$$

Performing the substitution prescribed by the delta functions in the exponential only yields the sum of two exponentials which turns into a sinus function. One then obtains for the potential of the particle in the presence of ion sound waves

$$\Phi(s,\rho,t) = \frac{e}{2\pi\epsilon_0\lambda_e}\frac{c_s}{v_\|}\int_0^1 \frac{\zeta\, d\zeta\, e^{-\rho'\sqrt{1+\zeta^2}}}{\sqrt{1+\zeta^2}}\sin\Big[\zeta(s-v_\|t)/\lambda_e\Big] \tag{26}$$

What remains is the ζ integration with $\zeta = k_\|\lambda_e < 1$ limited. To simplify, we can either neglect ζ or replace it by unity in the arguments of the roots. To be conservative and decide for the weakest case, we chose the latter, what yields the integral

$$\Phi(s,\rho,t) = -\frac{e}{2\sqrt{2}\pi\epsilon_0\lambda_e}\frac{c_s}{v_\|}e^{-\sqrt{2}\rho'}\int_0^1 \zeta\, d\zeta \sin\Big[\zeta\big|s-v_\|t\big|/\lambda_e\Big] \tag{27}$$

The argument of the sinus function is negative. So we have taken its sign out and use its absolute value. Integration gives

$$\Phi(s,\rho,t) = -\frac{e}{2\sqrt{2}\pi\epsilon_0}\frac{c_s}{v_\|}\frac{e^{-\sqrt{2}\rho/\lambda_e}}{|\sigma|^2}\big\{\sin|\sigma| - |\sigma|\cos|\sigma|\big\} \tag{28}$$

where

$$\sigma = (v_\|t - s)\lambda_e^{-1} > 0$$

The condition for an attractive potential follows immediately as

$$\tan|\sigma| > |\sigma| \qquad \text{or} \qquad 0 < \sigma < \frac{\pi}{2}\bmod(2\pi) \tag{29}$$

Depending on the parallel velocity $v_\| > 0$ there is an entire range of distances $s < v_\|t < \pi\lambda_e/2$ in which the conditions for an attractive potential are satisfied. We may note that for negative velocities $v_\| < 0$ there is no range where the potential can become attractive as the braced expression is always positive. It is the scalar product $\boldsymbol{k}\cdot\boldsymbol{v}$ between the wave number of the ion-sound and the test particle velocity which selects those speeds which are parallel to the sound velocity, not anti-parallel. One should keep in mind that this attraction has nothing in common with wave trapping, however! It is the over-screening effect of the particle, which is moving on the background of the wave noise and experiences the modified dielectric properties of the plasma.

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It should also be noted that in this condition the time explicitly appears because the test electron is seen from the stationary observers frame in which the electron moves. Instead, σ is measured in the moving electron frame. This distinction is important to make as it will be picked up again below.

The restriction on the velocity is obtained from that $\omega^2 < \omega_k^2$ when referring to the replacement $\omega = k_\| v_\| = k_\| v\cos\theta$ prescribed by the δ-functions. Rescaling $\omega_k \sim c_s k_\|$, it follows that the parallel particle speed is limited as

$$|v_\|| \lesssim c_s \qquad \text{or} \qquad |\cos\theta| \lesssim \frac{c_s}{v} \tag{30}$$

This is in fact a condition on the angle θ. For small speeds $v < c_s$ the condition is trivial. The largest effect is caused when the particle speed is parallel, below and close to the phase velocity c_s of the ion-sound wave. For large velocities $v > c_s$ the angle between the phase speed and velocity must be close to $\pi/2$, in agreement with the above requirement on the potential becoming attractive.

This is an important point in application to a plasma. In thermal plasmas we have generally $c_s \approx \sqrt{m_e/m_i}v_e$ which is far below thermal speed. Hence there are only few electrons in the distribution sufficiently far below thermal speed which would satisfy the resonance condition $v < c_s$. Higher speed electrons can be in resonance and thus contribute to attraction only at strongly oblique wave and electron speeds. Consequently under normal conditions in a plasma the generation of attractive potentials becomes obsolete, a point which had been missed in previous work (Neufeld & Ritchie, 1955; Nambu & Akama, 1985). In the particular case of mirror modes it becomes the crucial ingredient, as will be demonstrated below.

3.2 Correlation length

In all cases the attraction exceeds the repulsion *outside* the Debye sphere of the electron in its wake and, therefore and most important, can be felt by other electrons. From here it is clear that two electrons must move at distance somewhat larger than λ_e and at nearly same speed in the same direction in order to be held together by their attractions and form a pair. This is the important point when applying our model to the mirror mode below.

Having obtained the conditions under that the wake potential behind the moving test electron becomes attractive, we would like to know the distance over that the negative potential extends. This distance is measured in the instantaneous frame of the electron and is, hence, given by the above absolute normalised value of $|\sigma| < \pi/2$ which repeats itself periodically. It is, however, clear that it is only the zeroth period which counts as the effect of the dielectric polarisation on the electron diminishes with increasing distance $s' = \sigma\lambda_e$. In absolute numbers this distance becomes

$$\lambda_{corr} = |s - v_\| t| < \frac{\pi}{2}\lambda_e \approx 1.57\,\lambda_e \tag{31}$$

which can be understood as an electron "correlation length" between neighboured electrons. Any electrons within such a distance will behave about coherently, an important conclusion which, however, has to be extended below to many electrons.

This correlation length is to be compared with the particle spacing in the plasma. Plasmas are defined for particle densities $N\lambda_e^3 \gg 1$, which implies that the distance between the particles is $\ll \lambda_e$. Consequently the extension of the attractive potential in the electron wake is much larger than the spatial distance between two electrons. It thus affects many electrons, an effect which cannot be neglected when speaking about attraction.

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As for an example, in the magnetosheath which is the preferred domain where the mirror mode is permanently excited, the average density is, say, $N \approx 3\times 10^7$ m^{-3} at temperature $T_e \approx 50-100$ eV. For the Debye length we have $\lambda_e \approx 10$ m, while the inter-particle distance is a mere of ≈ 0.005 m. Roughly $\approx 10^4$ electrons should experience the presence of the attraction behind the test particle, which thus becomes a many-electron effect. Because pair formation depends on the quite severe condition on the particle velocity, not all those electrons of course will form pairs though however, in reality, the attractive potential involves a substantial fraction of electrons which necessarily will cause modifications of the plasma conditions. Normally such modifications will only cause minor effects in the wave spectrum and will be negligible. Below we show that in the evolution of the mirror mode they become important.

3.3 Ensemble averaged potential

If we understand the plasma as a compound of a large number of electrons, we can ask for the ensemble averaged potential $\langle\Phi\rangle$ of the single electron averaging over the particle energy distribution. In an isotropic plasma this is the Boltzmann distribution. Writing for the parallel velocity $v_\| = v\cos\theta$ the average potential becomes

$$\langle\Phi\rangle = \frac{e}{\epsilon_0}\frac{Cc_s}{\sqrt{2}\lambda_e}e^{-\sqrt{2}\rho/\lambda_e}\int_0^\infty v dv\, e^{-v^2/v_e^2}\int_{s/tv}^{(s+\pi\lambda_e/2)/tv}\frac{d\cos\theta}{\sigma^2\cos\theta}\big[\sin\sigma-\sigma\cos\sigma\big] \tag{32}$$

which immediately tells that the mean potential taken over the full Boltzmann distribution in repulsive. This is clear, however, because it accounts for all electrons in the Debye sphere. To calculate the cos-integral we expand the trigonometric functions to obtain

$$\frac{\pi}{6}\int \sigma d\cos\theta \approx \frac{\pi}{6\lambda_e}\left[\frac{\pi^2}{12}vt - s\log\left(1+\frac{\pi}{2}\frac{vt}{s}\right)\right] \tag{33}$$

We now exclude the Debye sphere by restricting the integration with respect to v over a shell between the thermal and trapped speeds. This gives

$$\int_{T_e}^{\mathcal{E}_{trap}} d\mathcal{E}e^{-\mathcal{E}/T_e}\left[\frac{\pi^2 t}{12}\sqrt{\frac{2\mathcal{E}}{m_e}} - s\log\left(1+\frac{\pi t}{2s}\sqrt{\frac{2\mathcal{E}}{m_e}}\right)\right] \approx -\left(1-\frac{\pi}{6}\right)\frac{\pi t}{2}\sqrt{\frac{2T_e^3}{m_e}}\int_1^y x^{\frac{1}{2}}e^{-x}dx \tag{34}$$

For a mean attractive potential the last integral should be positive. Doing it yields (Gradshteyn & Ryzhik, 1965)

$$\int_1^y x^{\frac{1}{2}}e^{-x}dx = \frac{2}{3}\left(y^2e^{-y}-e^{-1}\right) \approx -y^3+2y^2-1 \tag{35}$$

which is positive only if $y = 1+\Delta$ and $\Delta = \big(\mathcal{E}_{trap}-T_e\big)/T_e < 1$ in which case there is a narrow energy range (or energy "gap") for trapped electrons where the mean potential $\langle\Phi\rangle < 0$ becomes attractive for the electrons when averaging over their energy distribution and warranting that they behave coherently. The latter we will show can under certain condition be the case.

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4 Two-electron potential

We saw that, under a certain condition, an electron moving in the plasma in resonant interaction with an ion-sound background may give rise to an attractive potential in its wake where another electron can be captured and thus be forced to accompany the first electron. First of all, in plasma all electron are in permanent motion. Hence, if an electron satisfies the resonance condition with an ion sound fluctuation, it acts attracting on another one moving nearly at same speed. We have seen that this attractive potential in the presence of a large number of thermally distributed electrons becomes depleted. This holds when just one electron contributes to the potential. We now extend this to the combined effect of two electrons in the interaction, in which case we can immediately use the above solution when, however, accounting for the slightly different velocities $v_{\|1}, v_{\|2}$ and initial locations s_1, s_2 of the electrons along the magnetic field. In view of the later application to mirror modes, we again consider only motion along the magnetic field not yet specifying to the particularities introduced by bouncing in the mirror field. Then the two-electron potential becomes

$$\Phi(s,\rho,t) = -\sum_j \frac{e}{2\sqrt{2}\pi\epsilon_0}\frac{c_s}{v_{\|j}}\frac{e^{-\sqrt{2}\rho/\lambda_e}}{|\sigma_j|^2}\left\{\sin|\sigma_j| - |\sigma_j|\cos|\sigma_j|\right\} \tag{36}$$

with $j = 1,2$ counting the electrons. Here

$$\sigma_j = (v_{\|j}t - s_j)\lambda_e^{-1} > 0$$

As before, the requirement $\sigma_j > 0$ results from the condition that the waves in resonance with the electrons must be damped. In order to obtain the combined effect of the two electrons, we transform to their centre-of-mass frame

$$2Z \;=\; s_1 + s_2, \quad 2z = s_1 - s_2 \tag{37}$$

$$2U \;=\; v_{\|1} + v_{\|2}, \quad 2u = v_{\|1} - v_{\|2} \tag{38}$$

From the previous we saw that the large correlation length implies that many electrons are affected. Any attractive potential couples two ore more particles together. The most probable state to be formed is the two-particle (singlet) state. These will be distributed over the plasma, resembling the Cooper states in solid state superconductivity while not being a quantum effect here. Rather it is the polarisation effect moving particles produce in the high temperature collisionless plasma which causes singlet states of pairs.

In order to be realistic, we now derive the condition for singlet states do evolve. To simplify the algebra, let us define

$$\begin{aligned} \Sigma \;&=:\; \tfrac{1}{2}(\sigma_1 + \sigma_2) \equiv (Ut - Z)\lambda_e^{-1} > 0 \\ \sigma' \;&=:\; \tfrac{1}{2}(\sigma_1 - \sigma_2)\lambda_e \equiv (ut - z)\lambda_e^{-1} \end{aligned} \tag{39}$$

The restriction on $\Sigma > 0$ maps the ω-resonance onto the new variables. At the contrary, σ' can be positive or negative. With these expressions and after some rather tedious though simple calculations, Eq. (36) can be brought into the form

$$\Phi(s,\rho,t) \approx -\frac{\sqrt{2}e}{\pi\epsilon_0}\frac{c_s t}{(\lambda_e\Sigma + Z)}\frac{e^{-\sqrt{2}\rho/\lambda_e}}{|\Sigma|^2}\Big\{\sin|\Sigma| - |\Sigma|\cos|\Sigma|\Big\}\cos\sigma' \tag{40}$$

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where we made use of the above representations and replaced

$$v_{\|1,2}t = \tfrac{1}{2}\lambda_e\big(\Sigma \pm \sigma'\big) + Z \pm z \tag{41}$$

This expression holds under the reasonable assumptions $\sigma' \ll \Sigma$ and $z \ll Z$ that the difference between the two electrons in location is small enough to be found within the correlation length. Only under this condition one expects that the electrons will be correlated. Interestingly, the form of the potential remains the same as that for the one-particle case with the only difference that the potential is multiplied by $\cos\sigma'$. Hence the condition for attraction depends on the value of σ'. Closely spaced electrons of similar and, as required, resonant speeds not differing too much from the phase speed of the ion-sound give indeed rise to attraction between the two electrons if the following conditions are satisfied:

$$\begin{aligned} \tan\Sigma &> \Sigma \qquad \text{if} \qquad \cos\sigma' > 0 \\ \tan\Sigma &< \Sigma \qquad \text{if} \qquad \cos\sigma' < 0 \end{aligned}$$

which yields

$$\begin{array}{rcccl} 0 &<& \Sigma &<& \dfrac{\pi}{2} \qquad \text{if} \qquad \cos\sigma' > 0 \\ -\dfrac{\pi}{2} &<& \Sigma &<& 0 \qquad \text{if} \qquad \cos\sigma' < 0 \end{array} \tag{42}$$

These conditions are essentially the same as in the one-electron case. There modification is due to $\cos\sigma'$ being positive or negative and that they apply to the centre of mass coordinate system Z and mean particle speed U which both are contained in the variable Σ.

We remark that these conditions are very general. They substantially generalise the conditions found earlier by Nambu & Akama (1985) to the much more important interaction between two electrons, the lowest order singlet state and thus most realised state in a plasma. Higher order states like interaction of three electrons leading to triplets and so on are in principle also possible but will not play any important role because the interaction decays with distance even though they may be located within the correlation length and form “quasi-particles”. In the singlet state the electrons behave like one particle of double charge and double mass for the time of their interaction, the time they remain inside one correlation length. This length for the singlet is the same as given above that produced by one electron with the difference that it now applies to the centre of mass of the two electrons. Measured from the centre of mass it extends to its both sides over a length of roughly $\lambda_{corr} \approx 1.5\lambda_e$.

In physical units the first singlet state, for instance, is realised for

$$0 < Ut - Z < \frac{\pi}{2}\lambda_e, \qquad |ut - z| < \frac{\pi}{2}\lambda_e \tag{43}$$

In these cases resonant electrons in the presence of an ion-sound wave background will arrange into loosely bound electron pairs. In high temperature plasma a substantial number of such pairs will exist. However, they will mostly not play any role in the dynamics. In order to do so the plasma must offer additional ways for the bound singlet pair states to cause any susceptible effect in the plasma. Such conditions are provided by the quasilinearly stable mirror mode.

5 Mirror bottle and pairs

Being in the possession of the conditions under which electron pairs can form in a high temperature plasma in interaction with a thermal background of ion sound waves, we now intend to apply them to the case of mirror modes. We saw that the correlation length between electrons provided by one single test electrons is of the order of $\lambda_{corr} \sim 1.5\lambda_e$. This value is only slightly increased by the interaction of two electrons, such that we can roughly take $\lambda_{corr} \sim 2\lambda_e$ for the singlet. A mirror bottle is a preferred place for pair formation. This is in contrast to an extended plasma. Firstly, the bottle confines trapped electrons which cannot easily escape. Secondly, the parallel velocity of a bouncing electrons varies along the mirror magnetic field and at some place may get in resonance with the thermal ion-sound spectrum present in the entire plasma volume. If this happens at some location along the mirror magnetic field, electrons might form pairs and remain correlated for some time, bunch, perform like bound orbits and thus represent resonant correlated states which due to the correlation become coherent states.

5.1 Centre of mass pair bounce motion

The application of these finding to mirror modes is not an easy task. The electrons perform a complicated bounce motion along the inhomogeneous magnetic field with periodically changing bounce velocity and bounce frequency depending on the value of their constant magnetic moment. Under these conditions we need to now the variation of their bounce velocity as function of the location along the magnetic field be tween the mirror points. We moreover need to satisfy the common resonance condition of the pairs with respect to the phase velocity of the ion sound. Since the electron velocity is generally much larger than the latter this immediately suggests that the best conditions for attraction will be found near the mirror points s_m. There the parallel velocity of the electrons drops to zero, and there will be a certain range Δs at distance $s \lesssim s_m$ where the resonance condition is satisfied most easily. Here, near s_m one expects that attraction will become important.

In order to understand this process we thus need to transform to the moving frame of the pairing electrons. For this purpose we use the electron bounce motion to define the new pair-electron quantities

$$\mathcal{M} =: \tfrac{1}{2}\big(\mu_1 + \mu_2\big), \qquad \mu =: \tfrac{1}{2}\big(\mu_1 - \mu_2\big) \tag{44}$$

$$\Omega^2 =: \mathcal{M}B_0''/m_e, \qquad \varpi^2 =: \mu B_0''/m_e \tag{45}$$

$$U^2 =: \frac{2}{m}\mathcal{E} - \Omega^2 Z^2, \qquad \mathcal{E} =: \tfrac{1}{2}\big(\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{M}B_0\big) \tag{46}$$

$$u^2 =: \frac{2}{m}\tilde{\epsilon} - \varpi^2 z^2, \qquad \tilde{\epsilon} =: \tfrac{1}{2}\big(\mathcal{E}_1 - \mathcal{E}_2 - 2\mu B_0\big) \tag{47}$$

The mean bounce velocity U of the pair becomes a function of the location Z of the centre of mass along the magnetic field. This requires knowledge of its displacement as a function of the bounce phase which again requires solution of the two dynamics of the two electrons. Note the adiabatic constants $\mathcal{E}, \mathcal{M}, \mu, \Omega, \varpi$. The only variables are the mean and difference velocities $U(Z), u(z)$. In the magnetic mirror symmetry $U(t)$ is the bounce velocity of the trapped electron pair, and $Z(t)$ is its location along the magnetic field at time t. The difference speed $u(z)$ is measured in the centre of mass frame relative to Z and U. The mean speed U along the magnetic field must be expressed either as function of time t or distance Z. For this to

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accomplish one needs to solve the parallel equation of motion:

$$\frac{dU}{dt} = -\Omega^2 Z - \varpi^2 z \approx -\Omega^2 Z, \qquad U = \frac{dZ}{dt} \tag{48}$$

which is given in the reasonable approximation of small $\varpi^2 z$. Obviously the mean speed along the field obeys the mean bounce equation, an oscillation at frequency Ω. Integrating the bounce equation of motion with $U(Z) = \sqrt{2\mathcal{E}/m - \Omega^2 Z^2}$ yields

$$Z(t) = Z_m \sin\left(\frac{\pi t}{2t_m}\right), \qquad Z_m = \Omega\sqrt{\frac{2\mathcal{E}}{m_e}} \tag{49}$$

Z_m is the distance of the centre of mass mirror point along the magnetic which is reached by the pair at mirror time $\Omega t_m = \frac{1}{2}\pi$. Note that symmetric mirror bottles have been assumed, which implies time symmetry $\pm t_m$. For the lag in distance z as measured relative to the centre of mass Z we obtain formally and similarly

$$z(t) = z_\epsilon \sin\left(\frac{\pi t}{2t_\epsilon}\right), \qquad z_\epsilon = \varpi\sqrt{\frac{2\tilde{\epsilon}}{m_e}} \tag{50}$$

with $\varpi t_\epsilon = \frac{1}{2}\pi$ the lag in time in the electron pair to reach the mirror point at relative location z_m to the mirror location of the centre of mass Z_m.

These expressions give the centre of mass and jitter velocities as functions of time

$$U(t) = \frac{\pi}{2}\frac{Z_m}{t_m}\cos\left(\frac{\pi t}{2t_m}\right) \tag{51}$$

$$u(t) = \frac{\pi}{2}\frac{z_\epsilon}{t_\epsilon}\cos\left(\frac{\pi t}{2t_\epsilon}\right) \tag{52}$$

5.2 Condition for pair formation

Electron pair formation proceeds if, in addition to the conditions for attraction which have been given above, the pair electron are in resonance with the ion sound. This condition is non-trivial. We mentioned already that electrons participating in attraction move at speed comparable to the thermal speed v_e which exceeds c_s substantially. Under non-mirror conditions pair formation will thus barely take place. However, magnetic mirrors as provided by the mirror instability are a rare exception. The resonance condition is in fact not a condition on $U(Z)$ but on the angle between the pair velocity and the direction of the magnetic field, as the latter is the direction of the propagation of the ion sound. During the bounce motion the particle velocities are adiabatically conserved. It is only the angle $\theta(s)$ that changes along the magnetic field. Thus writing $U(s) = \frac{1}{2}v\big(\cos\theta_1(s) + \cos\theta_2(s)\big)$, assuming that $v_1 \approx v_2, \mu_1 \approx \mu_2$, we have

$$\cos\theta_1 = \frac{U+u}{v}, \qquad \cos\theta_2 = \frac{U-u}{v} \tag{53}$$

Introducing the mean angles $\Theta = \frac{1}{2}(\theta_1 + \theta_2), \vartheta = \frac{1}{2}(\theta_1 - \theta_2) \ll \Theta$ we obtain

$$\frac{\langle U\rangle}{v} = \cos\frac{\Theta}{2}\cos\frac{\vartheta}{2} \approx \cos\frac{\Theta}{2}, \qquad \frac{\langle u\rangle}{v} = -\sin\frac{\Theta}{2}\sin\frac{\vartheta}{2} \approx -\frac{\vartheta}{2}\sin\frac{\Theta}{2} \tag{54}$$

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Note that u can be negative as it is measured in the centre of mass frame. The condition of resonance $U(Z) \lesssim c_s$ then reduces to

$$\begin{aligned} \langle U \rangle &\lesssim c_s \longrightarrow \cos\frac{\Theta}{2} \lesssim \frac{c_s}{v} \ll 1 \\ \left|\frac{\vartheta}{2}\right| &\sim \frac{\langle u \rangle}{v} \ll 1 \end{aligned} \tag{55}$$

This condition shows that our assumption of about equal magnitudes $v_1 \approx v_2$ is not crucial because of the smallness of this ratio. It shows moreover that the resonance condition is nicely satisfied near the mirror points s_m where the average angle $\Theta/2 \approx \frac{\pi}{2}$. As for an example, $c_s/v_e \approx \sqrt{m_e/m_i} \approx 0.023$ which shows that the average cosine is very small, and the effective angle $\Theta/2 \approx 88.7^\circ$ is close to 90°. Allowing for a deviation in the ratio of ~ 0.002 the angular variation would amount to $\vartheta \approx 0.2^\circ$ as obtained from the average jitter velocity $\langle u \rangle$, as is suggested by the second above condition with $\sin\Theta/2 \approx 1$, which gives the angular spread in case of attraction.

This analysis shows that once a mirror bottle evolves there is a narrow range near the mirror points $\pm s_m$ for the trapped electrons to generate attractive potentials in their wake during their bounce motion inside the magnetic mirror trap. This attractive potential extends over approximately one or two Debye lengths along the magnetic field outside the Debye sphere of the acting electron (roughly some ten meters in the magnetosheath!) whose charge field is compensated by the bulk of the surrounding electrons populating its Debye sphere. This lengths is much larger than the mutual particle distance. It thus affects a substantial number of electrons which in case their velocities do not differ much form pairs inside the correlation length which the attractive potential attributes to them. As a consequence, there is a substantial number of paired electrons inside the mirror bottle along the magnetic field around all the many mirror points of trapped electrons of different initial angle and velocity. The distribution of those mirror points depends on the distribution of pitch angles of the electrons trapped in the field minimum $B = B_0$ at the centre of the mirror bottle. One thus expects that over a certain length along the mirror magnetic field an almost homogeneous distribution of electron pairs will evolve.

5.3 Dynamics of pair population: Magnetic susceptibility

The pitch angle distribution of trapped electrons in a mirror bottle is not known a priori. The equatorial pitch angle θ_0 is given as

$$\sin^2\theta_0 = B_0/B(s_m) \tag{56}$$

the ratio of minimum magnetic field to the mirror field of trapped electrons. Electrons with large equatorial pitch angle mirror very close to the minimum magnetic field. It is thus clear that there is practically a continuous distribution of mirror points along the mirror magnetic field in the bottle depending on the given initial distribution of equatorial pitch angles. Moreover this applies to all magnetic field lines, not only the central one. As a consequence, the entire narrow volume of the quasi-linearly stable mirror bottle will be subject to the presence of pairs each of which is located at and along the magnetic field centred around its mirror point. Under these conditions the pairs become an important population of a mirror bottle and contribute to its dynamics. This pair distribution has dropped out of the main particle distribution which does not evolve into pairs because it does not fulfil the requirements, i.e. the resonance condition for attraction. The normal population simply continues in the bounce motion and for an observer will dominate the measurement. The pairs at the contrary after becoming formed are locked. They drop out from bouncing remaining in their locked positions along their magnetic field line and jittering at velocity u around it.

Formation of pairs in the magnetic bottle has an immediate and profound effect on the dynamics of the magnetic bottle. By producing an attractive potential and at the same time forming pairs in the narrow environment of extension of a Debye length (as noted above ~ 10 m in the magnetosheath) around their common mirror points at the magnetic field the two paired electrons do not anymore participate in bouncing. To repeat, they have become locked at the personal mirror points with all their energy being now in the perpendicular gyration. They form a particle of twice its mass $m^* = 2m_e$ and twice its charge $q^* = -2e$. Dropping out of the bounce by being locked in the attraction implies energetically that their parallel motion condensates in the lowest energy bounce level which is classically. This level has not anymore thermal energy but energy of the order of $\tilde{\epsilon} \sim (m^*/2)\langle u^2\rangle \ll \mathcal{E}_e$, the energy in their average jitter motion around the mirror point, negligibly small with respect to their gyration energy.

The main consequence of this effect of pairing is thus that the paired electrons have all the same mean speed $v_\perp \approx v_e$ in their gyration, which is rather close to the thermal speed v_e, while their common mean energy which equals their temperature is less by a factor 2 than the electron temperature of the non-paired electron plasma. On the other hand, their kinetic energy and thus magnetic moment is larger by the same factor 2 for the simple reason that they are pairs.

Being locked in pairing, at lowest bounce level, and in steady gyration, they behave coherently. In this way they form a coherent gyration current $\boldsymbol{J}_{pair}$ which gives rise to an orbital diamagnetism which expels the magnetic field from the interior of the mirror bottle thereby increasing its radius and diminishing the magnetic field until the depletion of the field substantially exceeds the original quasi-linear saturation value of the depletion.

This becomes a difficult problem to solve in detail as it requires the construction of the grand partition function for the paired electrons which is hardly known in the inhomogeneous situation provided by the mirror mode. Though this is possible to do in the homogeneous quantum mechanical approach to superconductivity. This was done in the celebrated Bardeen-Cooper-Schrieffer theory of superconductivity. The classical case is unfortunately less transparent as it does not allow the simplifying use of a wave function and operator formalism.

In a more heuristic approach we refer to the definition of the diamagnetic susceptibility χ. This is given in the pair case as

$$\chi_{pair} = \mu_0 \frac{\partial \mathcal{M}_{pair}}{\partial B} \tag{57}$$

where $\mathcal{M}_{pair}$ is the average magnetic moment induced by the total pair distribution in unit volume, and B_0 is the undisturbed field reached after quasi-linear stabilisation of the quasi-linearly stable mirror mode. Of course and again, $\mathcal{M}_{pair}$ is not a priori known microscopically in the inhomogeneous medium of the interior of a mirror bubble, even though we have extensively discussed the formation of pair singlets.

However we may assume that for a single pair it is given as $\mathcal{M}\,\delta(U-U_{pair})\delta(Z-Z_{pair})$. The summation over all pair velocities and locations, assuming coherence, requires knowledge of the pair distribution. Assuming that $\mathcal{M}_{pair}$ is the wanted

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average magnetic moment, this gives that

$$\mathcal{M}_{pair} \approx \mathcal{N}_{pair}\mathcal{M}, \qquad \mathcal{M} \approx \frac{\langle\mathcal{E}_\perp\rangle}{B} \tag{58}$$

where $\mathcal{N}_{pair} = \alpha N$ is the total density of pairs in the volume with α their fractional density. Strictly speaking, this is a crude approximation because each pair contributes a slightly different magnetic moment. When the phase transition sets on near some critical mean energy corresponding to some critical temperature of the order of the average trapping energy $\langle\mathcal{E}_{trap}\rangle = T_{e,crit} \gtrsim T_e$ the mean magnetic moment will have a slightly weaker dependence on the magnetic field $\mathcal{M} \sim \mathcal{B}^{-\delta}$ with critical exponent $\delta \lesssim 1$ related to three other critical exponents α, β, γ which have to be determined experimentally, all determining the exact way in which the phase transition proceeds. Here, for our purpose of demonstrating the main effect, we leave the subtle problem of phase transition aside. Taking then the derivative we obtain that

$$\chi_{pair} = -\mu_0 \frac{\mathcal{N}_{pair}}{B}\mathcal{M} \approx -\mu_0\alpha \frac{N\langle\mathcal{E}_\perp\rangle}{B^2} \tag{59}$$

which suggests that the coherent effect of the trapped paired electrons causes the expected diamagnetism which reduces the magnetic field inside the mirror bottle.

The dynamics of the pairs in the mirror bottle is thus responsible for the further demagnetisation of its interior. This proceeds by the coherent gyration of the pairs. Since they are locked in the lowest bounce level at frequency $\omega = \frac{1}{2}\omega_b$, their parallel temperature is practically zero compared to the plasma temperature, with jitter temperature $\delta T \sim 2m_e\langle u^2\rangle/2 \ll T_e$ and all their energy in the gyromotion at average velocity $\langle v_\perp\rangle \gtrsim v_e$. The grand partition function of this problem consists of the integral over the continuum of (classical) Landau states in the inhomogeneous magnetic field, a problem which cannot easily be solved in our non-quantum case, as it requires knowledge of the coherent Hamiltonian of trapped paired bounce-locked electrons.

5.4 Self-consistent mirror amplitude limit

To obtain a physically motivated approximation we can make use of the famous quantum mechanical Landau solution in a homogeneous magnetic field (cf., e.g., Huang, 1987, pp. 258 ff.). Making the transition to large perpendicular temperatures and taking the classical limit by replacing in Landau's final solution the Bohr magneton, the quantum of an electron magnetic moment, by the average canonical magnetic moment of the typical electron pair yields that

$$\chi_{pair} \approx -\frac{\mu_0\alpha N}{T_{e\perp}}\mathcal{M}^2\Big(1-\mathrm{erf}(1)\Big) \approx -0.59\frac{\mu_0\alpha N}{T_{e\perp}}\mathcal{M}^2 \tag{60}$$

where we accounted for the condition that the Landau summation is maintained while the momentum integration applies only to the paired electrons, excluding the Debye sphere. This resembles the above expression given for the magnetic susceptibility. The magnetic susceptibility obtained is negative, as expected also in this case. The above bracketed expression shows that including the Debye electrons would deplete the susceptibility effect which is diamagnetic, decreasing the magnetic field.

Contrary to the solid state homogeneous Landau diamagnetism, where the magnetic susceptibility is independent of the magnetic field, being universally constant and proportional to Bohr's magneton, the diamagnetism in the inhomogeneous

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mirror state depends inversely on the magnetic field $\chi \propto B^{-2}$. The diamagnetism in this case is dynamical. Decreasing the magnetic field acts amplifying on its effect until the field becomes so weak that the gyration effect ceases and external pressure inhibits further expansion of the bottle. This is a most important finding. It is the clue to the description of large amplitude mirror modes as being caused by the presence of the paired electron population in a mirror unstable plasma. This will be demonstrated below.

In the final state, assuming it has been reached, we have for the minimum magnetic field in the bottle

$$B_{fin} \sim \Big(1 - |\chi_{pair}|\Big) B_0 \tag{61}$$

Let us take the following (initial) conditions: $T_\perp \sim 50$ eV, $B_0 \sim 10$ nT, $N \sim 3\times 10^7$ m^{-3}. Then we have

$$B_{fin} \sim \Big(1 - 2\alpha\Big) B_0 \tag{62}$$

In order to obtain a 50% reduction of the magnetic field (as is frequently observed in the magnetosheath) thus requires that $\alpha \sim 0.25$. Hence roughly $\sim 25\%$ of the electrons trapped in the quasi-linearly stable ion-mirror bottle are required to interact with the ion-sound background in this case to evacuate half of the magnetic flux from the mirror bottle. This is a substantial though finite and not overwhelmingly large fraction of electrons which participate in the generation of an attractive potential between electrons in their wakes and form singlets of electron pairs.

The second last expression enables estimating the magnetic self-quenching of the field due to pair formation. Define

$$B_{fin} = B_0 \left(1 + \frac{\Delta_B}{B_0}\right) \tag{63}$$

where $\Delta_B < 0$ measures the depletion of the magnetic field. Inserting for B in $\mathcal{M}$ and χ_{pair} gives a third order equation for Δ_B/B_0. When neglecting the third order term, it yields the condition on

$$\alpha < \frac{B_0^2}{4.7\mu_0 N T_\perp} \tag{64}$$

as the maximum possible absolute fraction of pairs which self-consistently support the expansion of the mirror bottle until becoming self-stabilised. Assuming pressure balance in the quasilinear mirror mode state requires that $\alpha \lesssim 1/2.4 \approx 0.42$, which is a fairly large fraction indeed, though of course just the severe upper limit. The resulting limiting maximum achievable depletion of the magnetic field then becomes

$$\Delta_B \approx -\frac{1}{2}\left(1 - \frac{0.07\mu_0\alpha N T_\perp}{B_0^2}\right) \approx -0.49\, B_0 \tag{65}$$

Here in the final state we assumed already that the pressure balance $B_0^2/2\mu_0 = NT_\perp$ is maintained in any mirror bottle. When comparing with observations in the magnetosheath of Earth it is most interesting that this is close to the maximum observed depletions (Lucek et al., 2001; Constantinescu et al., 2003; Tsurutani et al., 2011; Treumann & Baumjohann, 2018b; Yao et al., 2019) in the largest mirror mode bubbles.

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6 Conclusions

Natural mirror modes are an exception in high-temperature collisionless plasmas. They start from a simple magnetohydrodynamic instability in an anisotropic pressure configuration far from thermal equilibrium that has been produced for instance in the magnetosheath by the forced flow across the bow shock (cf., e.g., Balogh & Treumann, 2013, for the relations around the bow shock) and may be a general property of shocked plasma flows (Lucek et al., 2005). Linear theory shows that this instability produces magnetic field-elongated magnetic bottles which stabilise by quasilinear interaction between the anisotropic ions and the magnetic field in which course the thermal anisotropy is depleted to a low stable rudimentary value. The amplitude of the magnetic depletion, as numerical simulations have demonstrated, is very low. It is in fact so low that the quasilinear mirror mode would in observations not be noticed but added to the ordinary fluctuations of the magnetic field and thermal pressure. It does not explain the notorious though not persistent observation of very large amplitude chains of mirror modes of up to 50% magnetic depletion.

In the present communication we have demonstrated that the physics of those large amplitude mirror modes is rather complicated. So far no weak turbulence theory has been developed to explain their existence, mainly because it requires a spectrum of very low frequency plasma waves which is barely available in this frequency range.

The solution to this difficult problem is found in quite a different direction. It is given by accounting for the dynamics of the electron component trapped in the magnetic mirror bottle. These electrons perform their bounce motion and can in the vicinity of their mirror points, where their parallel speed drops to near zero, get into resonance with the always present thermal ion-acoustic noise spectrum. They then generate an attractive potential in the wake outside the charge-compensating Debye sphere that may affect another close electron, attract it and form an electron pair.

This process has been applied in the present communication to the formation of electron pair singlets consisting of two electrons and the wave spectrum, and the conditions for this to happen have been obtained. Such electron pairs need not to be spin compensated in our case because at the high temperatures in classical plasmas the Pauli principle plays no role and does not work. The jitter energies are still high above the spin energies.

However, the pairs fulfil an important function in the case of the mirror modes. By becoming trapped in the attraction, they drop out of the bounce motion, becoming locked near their mirror points along the quasi-linearly stable mirror bottle, spending all their kinetic energy into their gyration. Thus the pair distribution in a mirror mode becomes highly and very narrowly peaked just above and near their thermal velocity, an effect which should be very interesting to investigate in its further consequences.

The main effect of pair formation is that the gyration-locked pair distribution produces a finite diamagnetic susceptibility which acts depleting on the mirror bottle-magnetic field. The conditions for this to happen have been given above. It turns out that the magnetic bottle in this pairing-diamagnetic process, which in fact is a classical-physics Meissner effect, stabilises in a self-consistent manner by determining both the fraction of pairs which can be sustained by it and, at the same time, the saturation amplitude of the magnetic depletion. It is interesting that both these fractions are not unreasonable when compared with observations.

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Aside from the theoretically difficult problem of determining the grand partition function of the pairs respectively the free thermodynamic energy density related to the pair gas, and the implicated phase transition they produce in the trapped plasma-magnetic field configuration, a number of problems remain open. These concern the observation of the paired distribution, the theoretical problem of its classical effect concerning the possible generation of secondary instabilities and, finally, the maintenance of pressure balance. The latter is probably not crucial as the quasilinear theory is not violated by the effect of the pairs. The pairs take the energy they feed into the diamagnetism from their bounce motion, which is a process completely interior to the quasi-linear mirror mode which already is in very near pressure balance. Nevertheless would it be interesting to check experimentally whether pairing causes additional changes.

Altogether, the present communication has discovered an interesting new effect in high-temperature plasma which might have other consequences as well. It brings the theory of mirror modes to an intermediate conclusion by contributing to the understanding of the so far badly understood large magnetic amplitudes of mirror bubbles. It also eliminates the sometimes claimed necessity of considering spiked mirror modes where the saturated magnetic field overshoots. If this actually happens then it would be caused by pressure imbalance as consequence of the pairing effect. It also has brought an interesting important and unexpected application of the strange generation of attractive potentials of electrons proposed long ago for a plasma but never having found any application nor effect in a plasma. This has been shown here to become important when extending the theory of mirror modes to the inclusion of pair formation and generation of electron singlets, a close analogy to the behaviour of electrons in solid state physics in particular superconductivity.

It would be of interest investigating which effects the process elucidated here might have in turbulence theory and as well in astrophysical applications, in particular in view of our above finding that kinetic Alfvén waves are also capable of generating attractive potentials and may form electron pairs. Since they naturally have high phase speeds, satisfaction of the resonance condition seems to be easier for them than with ion sound waves. One example where this capability of kinetic Alfvén waves could become involved is in the aurora coupling region to the plasma sheet. Here, electrons are naturally trapped in the geo-magnetic field and perform bounce motions. The relaxed kinetic Alfvén pairing condition may generated a fraction of trapped pairs along the auroral magnetic field in this case to produced a substantial diamagnetic effect here with possible consequences for auroral physics.

In mirror-modes kinetic Alfvén waves are of little interest, as there is no obvious reason for them to be generated. They have, moreover, never been identified in relation to mirror observations while ion sound waves are generally present within and outside them. In ion-inertial range turbulence kinetic Alfvén waves seem to play some role as various observations indicate and theory also supports for the reason that the scale of the ion-inertial range coincides with the perpendicular wave lengths of kinetic Alfvén waves. Even though there is no obvious reason for the expectation that a coherent state would evolve for instance in turbulence, our pair-singlet mechanism should work in those cases as well and might have consequences for turbulence, entropy generation, and turbulent dissipation.

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