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Excitation of chorus with small wave normal angles due to BPA mechanism into density ducts

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Abstract. We examine specific features of the realization of the beam pulse amplifier mechanism (BPA) of chorus excitation in the density ducts having a width of the order of $100-300\ km$ with refractive reflection. The dispersion characteristics of whistler emissions in a planar duct under conditions for the fulfillment of the WKB approximation and refractive reflection from the "walls" of the duct are analyzed. It is shown that in the enhanced duct, discrete spectral elements of chorus with a narrow angular spectrum along the external magnetic field can be excited at frequencies somewhat lower than half the electron cyclotron frequency. In the depleted duct at frequencies somewhat higher than half the electron cyclotron frequency chorus with a narrow angular spectrum along the magnetic field can be excited. The proposed model explains the possibility of excitation of chorus with small angles of the wave normal when the BPA mechanism is implemented. It is noted that the properties of chorus, such as the intensity and a typical angle of the wave normal, can be different for the lower and upper band chorus.

1 Introduction

In accordance with the experimental data of the CLUSTER satellites and Van Allen Probes, chorus emissions are excited in the "cigar-shaped" region with a length on the order $l=(1-2)10^8\ cm$ and average diameter $\bar{d}=3\cdot 10^7\ cm$ (Bell et al., 2009; Agapitov et al., 2017) near the local magnetic field minimum. Usually, the chorus spectrogram is observed in two basic spectral bands centered somewhat lower than half the minimum electron cyclotron frequency for the magnetic tube in question. Relationship of the chorus excitation in two spectral bands with region of the background plasma density inhomogeneities was discussed in (Bell et al., 2009). The authors of that paper proposed that the source region for banded chorus consists of whistler mode ducts of depleted electron density (n_p) for the upper band chorus and ducts of enhanced electron density n_p for the lower band chorus.

Theoretical aspects of the whistler wave propagation in the magnetospheric ducts have been studied for more than half a century using different approaches (see, e.g., (Karpman and Kaufman, 1984; Laids, 1992; Pasmanik and Trakhtengerts, 2005; Sonwalkar, 2006; Woodroffe et al., 2013)). Known experimental data make it possible to define concretely the important characteristics of the geophysical situation. For example, according to the data given in (Haque et al., 2011), the difference in the plasma density inside and outside the duct is of the order of $10-15\%$. The wave field of chorus rapidly falls during the

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removal from the duct in the transverse direction. This indicates that the frequency of the working emission mode is not close to the critical frequency of the duct.

In the experimental studies of chorus, much attention has been given to determination of the wave normal angles (θ) in the region of excitation of emissions (Santolik et al., 2009). In accordance with the THEMIS satellites data (Taubenschuss et al., 2014) for the rising and falling tones in the lower band of chorus, electromagnetic emissions were detected more frequently at angles of the order of $20°$ and the range of the angles is smaller for the lower frequencies. For the upper band chorus, the angles of the wave normal are also of the order of $20°$, and decrease at the higher frequencies. Let us note that the algorithm of the wave normal angle calculation is based on the assumption that the experimental data offer a single, planar whistler-mode electromagnetic wave in a cold homogeneous plasma. In the duct, a standing wave structure occurs at the transverse coordinate. It corresponds to two counter propagating waves. Probably, this should be taking into account in the formalism.

The growth rate (γ) of chorus emission modification is very high (Santolik and Gurnett, 2003) and can correspond to $\gamma \approx 100\, s^{-1}$. According to experimental data and the results of numerical calculations (Li et al., 2011), for the achievement of cyclotron instability growth rate of this order, high anisotropy of the distribution function of energetic electrons in the excitation region is necessary. According to (Fu et al., 2014), the mechanism required for producing high anisotropies is unclear. Moreover, it is noted in (Zhou et al., 2015), on the basis of the analysis of a large volume of experimental data, that chorus are frequently excited in the regions of the daytime magnetosphere with a marginal stable plasma without significant anisotropy of the distribution function. For explaining these results, one can use the beam pulse amplifier mechanism (BPA) of chorus excitation not requiring significant anisotropy of the distribution function of energetic electrons in the magnetosphere (Bespalov and Savina, 2018). In a homogeneous plasma, the BPA mechanism is most effective at wave normal angles close to $\theta \simeq 39°$.

In this paper, we examine some specific features of the BPA mechanism realization of electromagnetic chorus excitation in the duct. This will be done in the WKB approximation for a planar duct with refractive reflection. The study will make it possible to better understand the real conditions of the chorus excitation. In particular, we will explain the experimental data about typical wave normal angles in the chorus excitation region.

2 Characteristic equation for the modes of a planar duct with refractive reflection

The motion of ions is not important for describing the whistler wave radiation in the chorus excitation region with frequency ω in the interval $\omega_{LHF} < \omega < \omega_B < \omega_p$, where ω_{LHF} is the lower-hybrid frequency, $\omega_B = eB/mc$ and $\omega_p = (4\pi n_p e^2/m)^{1/2}$ are the absolute value of the electron cyclotron and plasma frequencies, e is the absolute value of electron charge, m is the electron mass, n_p is the plasma density, and c is the speed of light in free space. A theoretical analysis is simplified if the conditions for the applicability of the so-called quasi-longitudinal approximation of electromagnetic wave propagation in cold, relatively dense plasma are fulfilled (Helliwell, 1965).

$$\omega = \frac{\omega_B\,|k_z|\,(k_z^2 + k_x^2)^{1/2}}{k_z^2 + k_x^2 + \omega_p^2/c^2}\ , \tag{1}$$

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where k_z and k_x are the wave vector components along and across the magnetic field, respectively.

For typical magnetospheric conditions (Haque et al., 2011) in the region of chorus excitation the WKB approximation is fulfilled since the length of the whistler wave $\lambda \simeq 15\,km$ is less than the scale of the background plasma density distribution and duct transverse size is $d = 100 - 300\,km$. When the plasma density is inhomogeneous in the direction perpendicular to the duct axis, refraction is an important reason for the wave propagation. It is known that under conditions for refractive reflection, waves with frequencies higher than half the electron cyclotron frequency are directed by the depleted duct. Waves with frequencies lower than half the electron cyclotron frequency are directed by the enhanced duct (Karpman and Kaufman, 1984).

We now consider in more detail the characteristic equation of a refractive planar duct. For this purpose, we analyze the dependence of the transverse component of whistler wave vector on the frequency and the longitudinal component of the wave vector with different background plasma densities (Woodroffe et al., 2013). This dependence, in accordance with Fig. 1 has two branches, which according to Eq. (1), can be written as follows:

$$k_x = \begin{cases} K_{x+},\ if \\ \frac{\omega}{\omega_B} \le H(\frac{1}{2} - \frac{\omega}{\omega_B})\frac{|k_z|u_G}{\omega_B} + H(\frac{\omega}{\omega_B} - \frac{1}{2})\frac{1}{1+(\omega_p/k_z c)^2}; \\ K_{x-},\ if \\ \frac{1}{1+(\omega_p/k_z c)^2} \le \frac{\omega}{\omega_B} \le \frac{|k_z|u_G}{\omega_B} \le \frac{1}{2}. \end{cases} \quad . \tag{2}$$

where $u_G = (c\omega_B/2\omega_p)$ is the Gendrin velocity (Helliwell, 1995),

$$K_{x\pm} = \left\{ \left(\frac{\omega_B k_z}{2\omega}\right)^2 \left[(1 - \frac{\omega^2}{k_z^2 u_G^2})^{1/2} \pm 1 \right]^2 - k_z^2 \right\}^{1/2} .$$

$H(\varsigma)$ is the Heaviside step function. Let us note that there is a range of values k_z, ω in which k_x has not one, but two values.

The performance of the eigenmode wave solutions is determined by the characteristic equation, which expresses the dependence of frequency on the longitudinal component of the wave vector. For determining the characteristic equation in the WKB approximation, it is necessary to calculate a complete transverse change in the phase during the ray propagation period. A change in the phase during one period must consider phase displacements on the caustics and be multiple of 2π. Therefore, the characteristic equation takes the well known form (laird, 1992):

$$\Phi(\omega, k_z) = \int_{-x_{\max}}^{x_{max}} k_x dx = (p - \frac{1}{2})\pi\ , \tag{3}$$

where p is a positive integer, and in a symmetric duct $\mp x_{\max}$ are the reflection levels, where $k_x = 0$.

Within the framework of the WKB approximation, the dependence $k_x(x)$ should be continuous, and in the regions of refraction reflection $k_x = 0$ if the density $n_p(x)$ and the Cendrin velocity $u_G(x)$ change continuously. The refraction enhanced duct can occur for frequencies $\omega < \omega_B/2$, and only branch K_{x-} (2) satisfies the continuity condition of the dependence $k_x(x)$.

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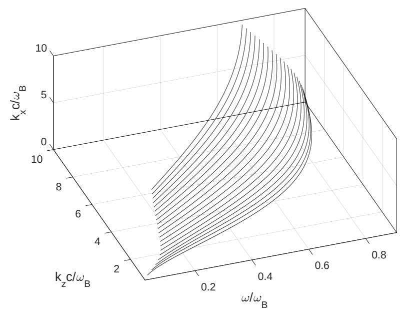

Figure 1. Dependence of the transverse component of the wave vector on the frequency and the longitudinal component of the wave vector for $(\omega_p/\omega_B)^2 = 25$.

The refraction depleted duct can occur for frequencies $\omega > \omega_B/2$, and only branch K_{x+} (2) satisfies the continuity condition of the dependence $k_x(x)$.

Let us assume that there is a duct with enhanced or depleted cold plasma density and which is uniform along the magnetic field. While performing calculations, we will consider that the magnetic field is uniform and the electronic cyclotron frequency $\omega_B = 6 \cdot 10^4\ s^{-1}$. The plasma density outside the duct corresponds to the condition $(\omega_{p,out}/\omega_B)^2 = 25$. Inside the enhanced duct we have $(\omega_{p,int}/\omega_B)^2 = 29$, while inside the depleted duct, $(\omega_{p,int}/\omega_B)^2 = 21$. For determining the solutions of the characteristic equation, we used model transverse density distribution for the enhanced and depleted ducts in the form

$$n_p(x) = n_{p,int}[1 + (\frac{n_{p,out}}{n_{p,int}} - 1)\tanh^2(2\frac{x}{d})]\,, \qquad (4)$$

where $d = (1-3)10^7\ cm$ is the transverse size of the duct.

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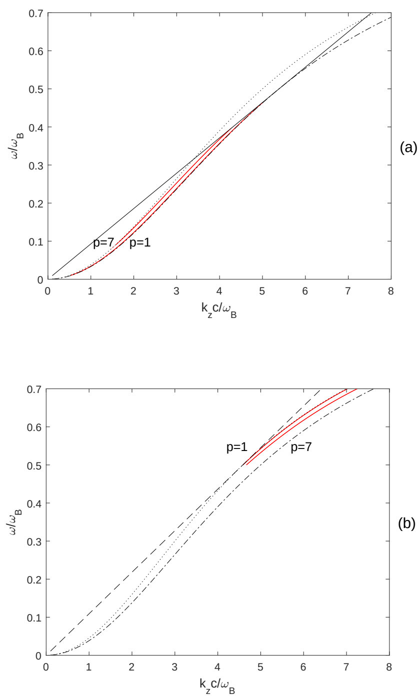

Figure 2. Relationship between the frequency and the longitudinal component of the wave vector for the first and seventh ducted modes, if $(\omega_{p,out}/\omega_B)^2 = 25$: (a) in the enhanced duct with $(\omega_{p,int}/\omega_B)^2 = 29$; (b) in the depleted duct with $(\omega_{p,int}/\omega_B)^2 = 21$.

For the accepted density distributed across the duct (4), it is possible to numerically calculate, based on Eq. (2), the phase change given by Eq. (3) and obtain dispersion formulas for the enhanced and depleted ducts. Some results of calculations are given in Fig. 2. In these figures the red lines show solutions of the characteristic equations for the first and seventh modes in the enhanced (a) and depleted (b) ducts, the dot-dash and the dotted curves corresponds to the solution of dispersion equation (1) for $k_x(x) = 0$ at the maximum and minimum density respectively, the solid line corresponds to the Gendrin velocity at the maximum density, the dashed straight line corresponds to the Gendrin velocity at the minimum density.

For relatively gently sloping ray trajectories, it is not difficult to estimate the average of the normal angle. Then Eq. (3) can be written in the form $(2/d) \int_0^{x_{max}} \tan(\theta) dx = (2\pi/k_z d)(p - \frac{1}{2})$. If $\theta < \pi/4$, with accuracy up to 20% we have $\tan(\theta) \simeq \theta$, and therefore

$$\bar{\theta} \simeq \frac{2\pi}{k_z d}(p - \frac{1}{2}) \ . \qquad (5)$$

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Note that the distance between the modes in Fig2 decreases with increasing transverse size of the duct.

3 Special features of implementation of the BPA mechanism of the chorus excitation in a duct with refractive reflection

Now we briefly recall the process of chorus frequency-time spectrogram formation in the implementation of the beam pulse amplifier mechanism (Bespalov and Savina, 2018). At the input of the wave-particle interaction region ($z = 0$), there is weak noisy emission with an electric field containing a random sequence of weak shot electromagnetic pulses. Noise that does not satisfy the conditions for interaction with particles is dumped in a nearly stable plasma. An appropriate shot pulse is a trigger for the chorus discrete element. In accordance with this, we assume that a short noise pulse flies in the duct along the axis z along the magnetic field $\mathbf{B}$. Our interest is to classify of the duct solutions which are close in structure to the expression

$$E_z = G_p(x)\tilde{E}_p(z,t) \; , \tag{6}$$

in which the longitudinal electric field corresponds to the p-th wave mode in the duct. Wave mode (6) is well localized on the transverse coordinate because of the fast decrease of function $G_p(x)$ with the large $|x|$, but in the general case, it spreads along the longitudinal coordinate z because of the dispersion.

There is an important exception from this general regularity. Pulse will not rapidly spread if noisy electromagnetic pulse has frequencies for which

$$v_{phz} = v_{gz} = u \; , \tag{7}$$

where $v_{phz} = \omega/k_z$ and $v_{gz} = \partial\omega/\partial k_z$ are the phase and group velocities of the wave mode in the duct. In this case, the population of epithermal electrons, which fly in together with the pulse into the region of wave-particle interaction, starts play an important role. These electrons for the pulse are a single-velocity beam with a distribution function proportional to

$$f = n_b \delta(V_z - u) \; , \tag{8}$$

where n_b is the density of the beam coordinated with the pulse.

In a homogeneous plasma with beam (8), the electromagnetic pulse increases with growth rate (Bespalov and Savina, 2018)

$$\gamma_{BPA} = \frac{\sqrt{3}}{4}\omega_B \left(\frac{n_b}{4n_p} \sin^2\theta \right)^{1/3} |\cos\theta| \; . \tag{9}$$

First of all, we verify the possibility of the fulfillment of condition (7) for the duct modes. For the plasma parameters introduced in the foregoing section, this condition can be fulfilled (see Fig. 2a) in the enhanced duct if the frequency spectrum is concentrated near $\omega/\omega_B \simeq 0.42$ and the longitudinal component of the wave vector is near to $k_z c/\omega_B \simeq 4.7$. Condition (7) also can be fulfilled (see Fig.2b) in the depleted duct if the frequency spectrum is concentrated near $\omega/\omega_B \simeq 0.51$ and the longitudinal component of the wave vector is near to $k_z c/\omega_B \simeq 5.0$.

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We can use a formula for growth rate (9) since the wave modes in the duct conform to the WKB approximation, and therefore their polarization corresponds to the whistler waves in which during the inclined propagation there is a longitudinal component of the wave electric field which is taken into account in the growth rate calculation (Bespalov and Savina, 2018). We will refine now the value of the average angle $\bar{\theta}$, which should be substituted into formula (9). We will make this by means of formula (5), according to which

$$\bar{\theta} \simeq 2\pi \frac{\omega_B}{k_z c} \frac{c}{d\omega_B} (p - \frac{1}{2}) \simeq 0.03(p - \frac{1}{2}) \, , \tag{10}$$

where we expected that $k_z c/\omega_B \simeq 5$ and $d = 300\ km$. The angle θ is less than 20 degrees for the first ten modes.

Actually, that the number of operating modes (p) in a comparatively small duct is limited by several factors. In particular, the quality of the mode should be sufficient high, the mode should be electromagnetic, and key condition (7) should be fulfilled. Formally, growth rate (9) increases with increasing mode number p. However, it should be taken into account that there is another limitation on the angle of excitation, which is caused by the Landau damping on the $\check{C}$erenkov resonance of electromagnetic waves in a plasma. We note that for the angle $\theta = 20°$ growth rate (9) decreases by 20% only to comparison with the maximum value close to $\theta = 39°$, and it is sufficient for explaining the experimental data.

4 Conclusions

We have examined specific features of the realization of the beam pulse amplifier mechanism of chorus excitation in density ducts with a width of the order of $100 - 300\ km$ with the refractive reflection. It is shown that the BPA mechanism can be realized in magnetospheric density ducts. Waveguide propagation of the whistler waves excited by this mechanism in a refractive duct permits one to explain the experimentally observed small angles of the wave normal. For the modes with relatively low numbers (in the examined example, $p = 1 - 10$), the angle of the wave normal $\theta \leq 20°$. The rate of change in spectral forms in this case is characterized by growth rate (9) with a value smaller than the maximum possible one (up to 20%). This rate of change is sufficient for explaining the experimental data within the framework of the approach presented in the paper (Bespalov and Savina, 2019). The BPA mechanism key condition (7) is satisfied in a frequency range lower than half the electron cyclotron frequency in the enhanced duct and larger than half the electron cyclotron frequency in the depleted duct (see Fig. 2). If there is no appropriate wave refraction, pulses can also be excited, but no longer as the ducted mode. For these emission, it is possible to expect the lower values of the intensity and larger angles θ.

The proposed model explains the possibility of the chorus emissions excitation with small angles of the wave normal due to the beam pulse amplifier mechanism. The properties of chorus, such as the intensity and a typical angle of the wave normal, can be different for the lower and upper band chorus.

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