PAPER

A Study of Ray Focussing of Whistler-Mode Waves in the Outer Magnetosphere

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SUMMARY The purpose of this paper is to investigate the ray focussing of whistler-mode waves in the outer magnetosphere which results in an enhanced wave-particle interactions. The critical frequency in a homogeneous plasma is first studied, at which the refractive index surface of whistler-mode waves indicates a zero curvature at a longitudinal wave normal angle. This critical frequency is also found to be consistent with the zero diffraction coefficient in the full-wave theory for a slightly inhomogeneous plasma. The two-dimensional ray-tracing computations for varying the frequency and initial wave normal direction in an inhomogeneous realistic model of the outer magnetosphere, have yielded that although the critical frequency for the homogeneous case has its importance even in the inhomogeneous plasma, the strongest ray focussing seems to occur at a frequency slightly below the above critical frequency, and hence that an enhanced gyroresonance wave-particle interaction is anticipated at this frequency.

1. Introduction

Gyroresonance wave-particle interaction is important in studying not only the generation of VLF/ELF emissions in the magnetosphere, but also the structure and stability of the magnetospheric energetic particles (e. g., Helliwell and Crystal[1]). In order to enhance efficiently the phase-bunching of incoming gyroresonant electrons by counter-propagating waves, the focussing of radiation along the magnetic field lines is highly required[1],[2]. Hence, it is of great significance to investigate the propagation of whistler-mode waves near the equator in a wide frequency range with normalized frequency from 0.1 to 0.7 in the outer magnetospere, with respect to paying a particular attention to the efficient wave-particle interactions. There is ample experimental and theoretical evidence of wave refraction in the plasmasphere, but little study is done on the topic of wave refraction in the outer magnetosphere (see Muto and Hayakawa[3], and the references therein). In Sect. 2 we discuss the propagation of whistler-mode waves in a homogeneous magnetospheric plasma based on the study of refractive index surface and then a full wave study. Then, Sect. 3 deals with the whistler-mode wave propagation in an inhomogeneous, realistic magnetosphere by means of the two-dimensional ray-tracing computations.

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2. Propagation in a Homogeneous Magnetosphere

The propagation of whistler-mode waves in a homogeneous plasma can be studied with the aid of refractive index surfaces. For dense plasmas such that the electron plasma frequency f_p is much larger than the electron gyrofrequency $f_H (f_p \gg f_H)$, we have plotted the change in morphology of the whistler-mode refractive index surface above and below the critical frequency of $f_H/2$. Figure 1 illustrates this situation, and Fig. 1(a) refers to the frequency below the critical frequency and Fig. 1(b) above the critical frequency. As seen from the figure, the surface below the critical frequency is convex at small wave normal angles, while above the critical frequency we find the surface simply concave. Hence, we

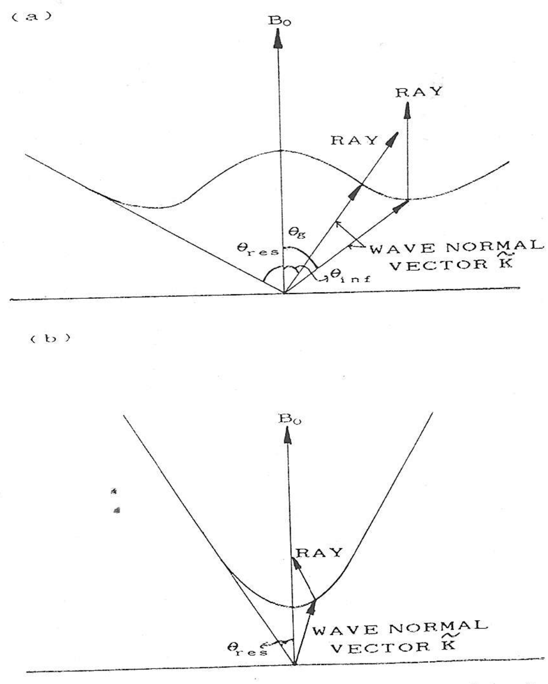

Fig. 1 Change in morphology of whistler-mode refractive index surface (a) below and (b) above the critical frequency. The relationship of ray and wave normal directions is indicated. Three characteristic angles, θ_{inf}, θ_g and θ_{res} are illustrated, which are defined in the text.

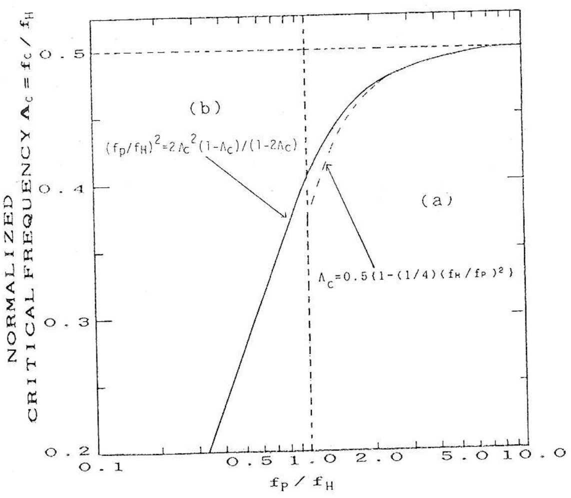

Fig. 2 Critical normalized frequency (in a full line) as a function of the ratio of f_p/f_H. An approximate curve for the critical frequency for $f_p/f_H>1.0$ is illustrated in a broken line. The change in morphology of the whistler-mode refractive index is recognized above and below the critical frequency.

can anticipate that the curvature of the surface vanishes at zero wave normal angle just at this critical frequency. Because the ray direction is perpendicular to the refractive index surface[4], this suggests a possibility of enhanced phase-bunching of incoming electrons caused by focussing of the radiation in the direction of the Earth's magnetic field $\tilde{B}_0$.

In the outer magnetosphere the f_p becomes, on some occasions, smaller than the f_H, and so we will derive the general equation for this critical frequency for different plasma conditions. By using the full, but collisionless, Appleton-Hartree's equation for the refractive index[5] and by putting $d^2(n\cos\theta)/d\theta^2|_{\theta=0}=0$, the critical frequency is found to satisfy the following equation[2].

$$2\Lambda_c^2(1-\Lambda_c)/(1-2\Lambda_c)=(f_p/f_H)^2 \qquad (1)$$

where $\Lambda_c=f_c/f_H$ is the critical frequency (f_c) normalized by f_H. The relationship of Λ_c with f_p/f_H is presented in Fig. 2. This figure indicates that the critical frequency Λ_c decreases considerably below 0.5 for the more teneous plasma ($f_p<f_H$). The variation of the curvatures of the refractive index surface at $\theta=0°$ is plotted in Fig. 3 for varying the normalized frequency ($\Lambda=f/f_H$) along the broken line in Fig. 2 (i. e. $f_p/f_H=1.0$). When the frequency Λ is increased, the curvature is negative (i. e. concave), then crosses zero at Λ_c and we have a positive curvature (convex surface).

The importance of the above-obtained critical frequency derived from the concept of refractive index surfaces (or in other words, the ray theory) is reexamined by means of a full-wave concept. We consider that the plasma is weakly inhomogeneous across the magnetic field line (or in x direction) and the corre-

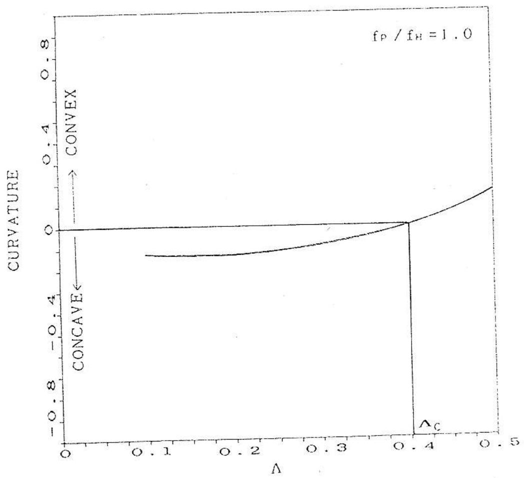

Fig. 3 Variation of the curvature of whistler-mode refractive index for a longitudinal wave normal direction with the normalized frequency for the case of $f_p/f_H=1.0$.

sponding wave equation of whistler-mode waves propagating along the magnetic field $\tilde{B}_0$ (z direction) in a slab-shaped inhomogeneous magnetoplasma is derived as follows by means of the reductive perturbation method[6].

$$i\{(1/v_g)\partial E/\partial t+\partial E/\partial z\}+p\partial^2 E/\partial x^2-U(x)E=0 \qquad (2)$$

where E is the wave electric field and expressed by $\tilde{\varepsilon}=1/2\{E\exp i(kz-\omega t)+\text{c. c(complex conjugate)}\}$ in which k is the wave number and ω, wave frequency. v_g is the group velocity ($=\partial\omega/\partial k$). $U(x)$ is related with the plasma inhomogeneity and expressed by

$$U(x)=-k/2(\Delta\varepsilon(x)/\varepsilon_0) \qquad (3)$$

where ε_0 and $\Delta\varepsilon$ are the homogeneous and inhomogeneous parts of the dielectric constant ε, respectively ($\varepsilon=\varepsilon_0+\Delta\varepsilon$). Then, the important coefficient p is called the "diffraction term", which represents the degree of wave focussing and defocussing and expressed by[6]

$$p=\nu/2k \qquad (4)$$

where

$$\nu=1+(1/2)(\varepsilon/P-1)$$
$$=1+(1/2)[-(f_p^2/f^2)\{f_H/(f-f_H)\}] \qquad (5)$$

when P in Eq. (5) is the notation used in Stix[4]. Figure 4 illustrates the diffraction p value as a function of the normalized frequency Λ for the same fixed plasma parameter, $f_p/f_H=1$ as in Fig. 3. This figure indicates that the p value decreases with increasing wave frequency and changes its sign from positive to negative at a specific frequency. This specific frequency can be obtained by putting p (in Eq. (4))$=0$, which is found to give exactly the same relationship with Eq. (1). That is, no diffraction ($p=0$) in Eq. (4) corresponds to the

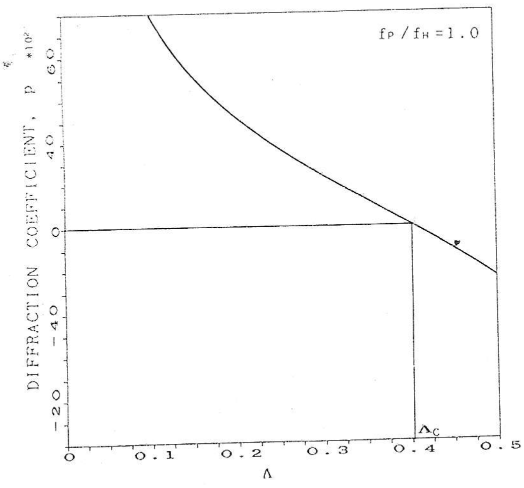

Fig. 4 Variation with the normalized frequency of the diffraction p value in the same plasma condition of $f_P/f_H = 1.0$.

strongest ray focussing. And, this critical frequency Λ_c is well approximated for higher f_P/f_H by the following equation.

$$\Lambda_c \simeq 0.5\{1-(1/4)(f_H/f_P)^2\} \quad (f_P > f_H) \qquad (6)$$

This approximate relation is also plotted in Fig. 2 in a broken line, which indicates the validity of Eq. (6) for $f_P/f_H > 2.0$.

In conclusion, both from the refractive index surface studies and full-wave considerations, the critical normalized frequency is found to be an important frequency for having a strong ray-focussing in a homogeneous plasma and correspondingly, we can expect a strong phase bunching of incoming electrons, leading to the enhancement of wave-particle interactions.

3. Propagation in an Inhomogeneous Magnetosphere

The propagation properties of whistler-mode waves including the ray focussing and so forth, are investigated with the aid of two-dimensional ray-tracing computations. The outer magnetosphere where chorus and many other emissions are generated, is characterized by a realistic magnetospheric model. A dipole model is adopted for the Earth's magnetic field and the plasma density is assumed to follow the diffusive equilibrium model. The L dependence of the equatorial density for our model is indicated in Fig. 5, and the details on the specification of the model are already described in Muto and Hayakawa[3] and Muto et al.[7] The initial L value was fixed at $L_0 = 6.6$ and the normalized wave frequency at the equator which is the starting (or initial) position is widely varied from $\Lambda_0 = 0.1$ through 0.7 with a step of 0.1 in order to know the general behaviours of ray paths. The initial wave normal angle θ_0 is also widely varied from 0° to large values with a spacing of 10° up to the oblique resonance angle θ_{res}. The definitions of wave

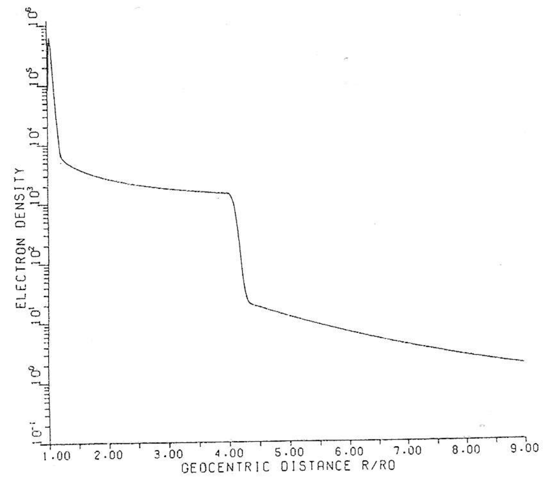

Fig. 5 Variation of equatorial electron density with geocentric distance, which is used for the ray-tracing computations.

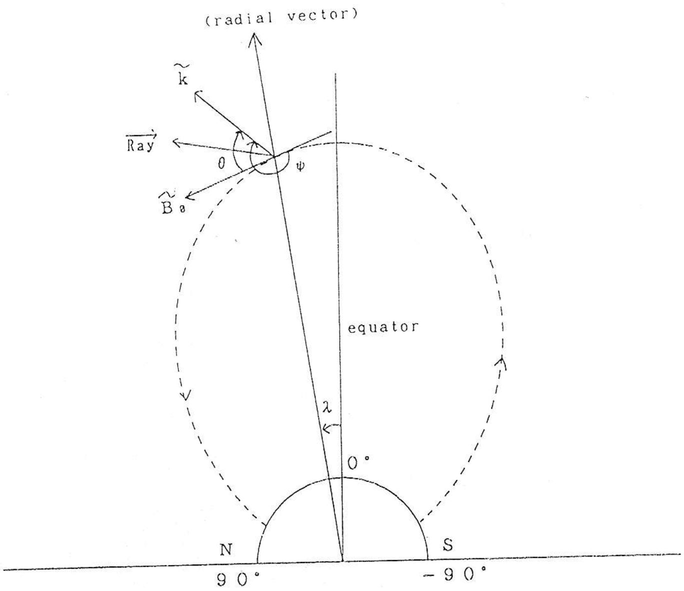

Fig. 6 Definitions of wave normal direction of a ray. θ is the angle with respect to the Earth's magnetic field $\tilde{B}_0$ and ψ is measured from the anti-parallel of B_0. The variations of this ψ are indicated in Fig. 7.

normal direction are indicated in Fig. 6 and we show the results of ψ, where positive means the angle measured clockwise from the opposite direction of $\tilde{B}_0$. The ray-tracing results for varying Λ_0 are presented in Fig. 7. At the initial equatorial point, the ratio of $f_P(=20.04$ kHz) to $f_H(=2.953$ kHz) is $f_P/f_H = 6.786$, and so looking at Fig. 2 gives $\Lambda_c = 0.497$. Three important characteristic angles are defined and used; inflection (θ_{inf}), Gendrin (θ_g) and oblique resonance angles. The inflection angle θ_{inf} is defined as the wave normal direction at which the curvature changes from convex to concave, and is given by

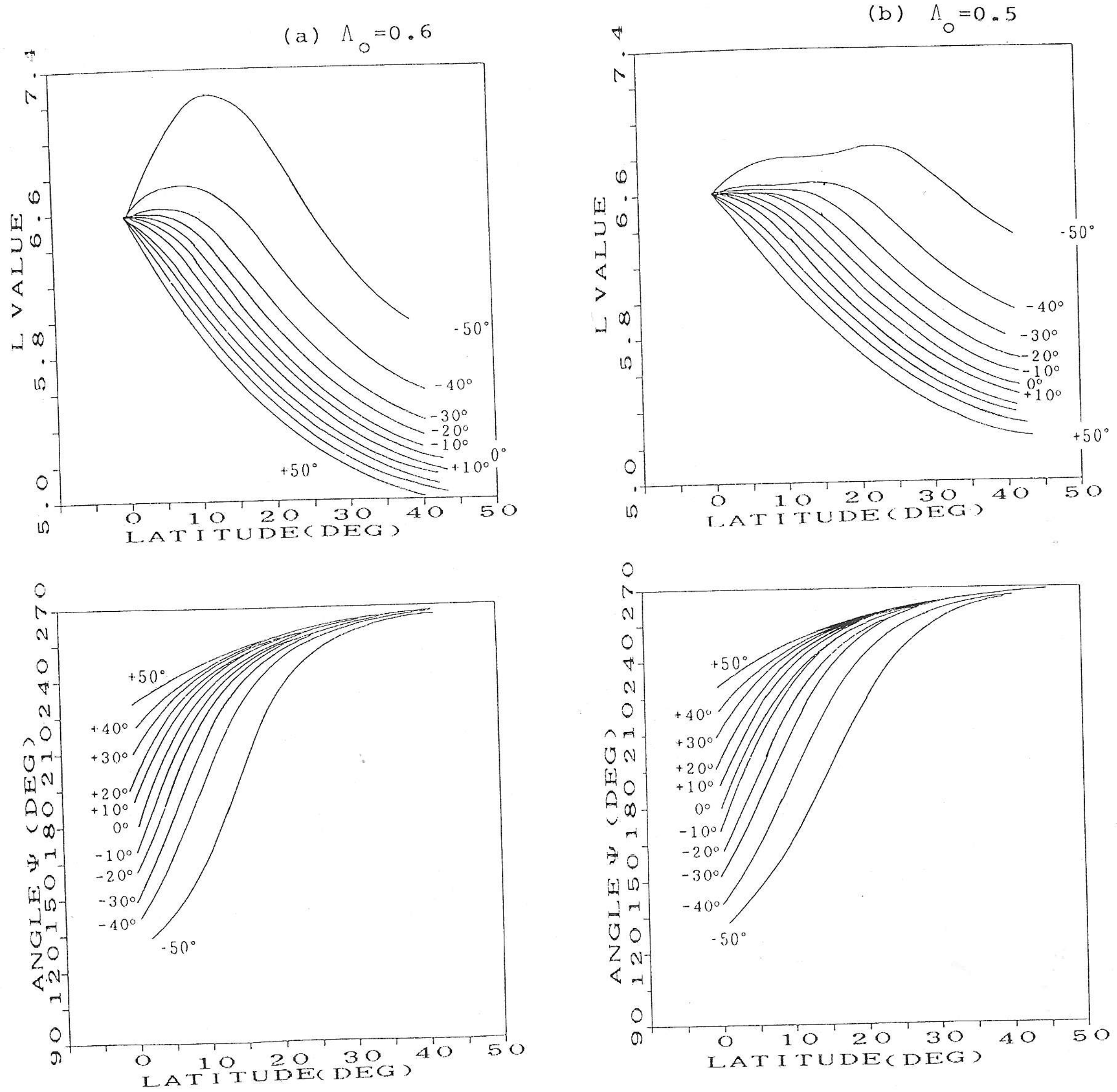

Fig. 7 Results of ray-tracing computations. In each figure the upper panel refers to the latitudinal variation of L value and the lower, the corresponding latitudinal variation of wave normal direction ψ. The initial wave-normal angle θ_0 which is indicated by each curve is widely varied. (a) $\Lambda_0=0.6$, (b) $\Lambda_0=0.5$, (c) $\Lambda_0=0.4$ and (d) $\Lambda_0=0.3$.

$$\theta_{\mathrm{inf}}=\cos^{-1}\{f/f_H+\sqrt{(1+(f/f_H)^2)/3}\} \tag{7}$$

Other two angles are well known as follows[(5),(8)].

$$\theta_{\mathrm{res}}=\cos^{-1}\{(f/f_H)\sqrt{1+(f_H^2-f^2)/f_P^2}\} \tag{8}$$

$$\theta_g=\cos^{-1}(2f/f_H) \tag{9}$$

In each figure of Fig. 7 there are two plots for each initial wave normal frequency, Λ_0; the upper panel illustrates the variation of L value of ray path as a function of geomagnetic latitude, and the lower, the latitudinal variation of the wave normal angle ψ defined in Fig. 6. The negative initial wave normal angle ($\theta_0<0$) indicates that the initial wave normal is directed inward from $\vec{B}_0$, while $\theta_0>0$, the outward-directed initial wave normal direction. For $\Lambda_0=0.6$ in Fig. 7(a) well above the critical frequency, the rays except those with larger initial positive θ_0's (such as $+40°$, $+50°$) exhibit a downward curvature in the smaller latitude range and followed by an upward curvature for higher latitudes. The rays with $\theta_0=+40°$ and $+50°$ are found to indicate an upward curvature at all latitudes. At this normalized frequency, the simple concave refractive index surface

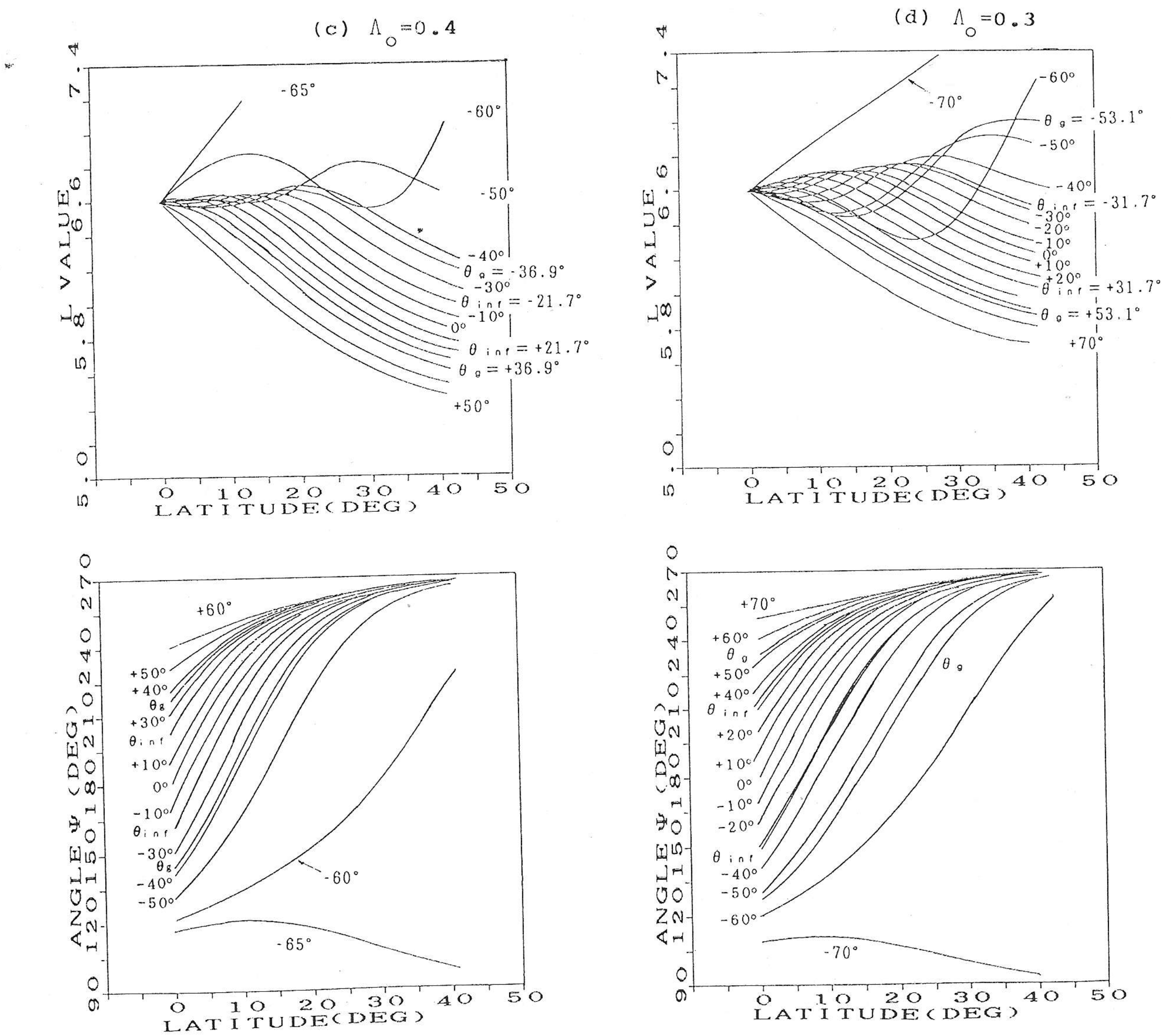

Fig. 7 (Continued)

provides no spatial concentration of rays, and the inhomogeneity of the medium results in no one ray remaining parallel to $\tilde{B}_0$ over considerable distances.

In Fig. 7(b) with $\Lambda_0=0.5$, slightly above the critical frequency, we notice strong focussing of some of the rays, with θ_0's in a range from $-40°$ to $-10°$ in the latitude range up to about 10°. Wave normals that are initially inward with respect to $\tilde{B}_0$, rotate very quickly, because of field line curvature, outward across the flat portion of the refractive index surface as seen from the lower panel in Fig. 7(b), so that the ray direction (normal to the surface) remains nearly parallel to $\tilde{B}_0$. This results in a concentration of rays as mentioned above.

Slightly below the critical frequency as in Fig. 7(c) for $\Lambda_0=0.4$, the strongest ray focussing is found. There is a critical cone of radiation about $\tilde{B}_0$ within which rays with three different wave normal angles may all propagate in the same direction. In particuler, rays parallel to $\tilde{B}_0$ result from wave normal angles of zero and $\pm\theta_g$. The edge of the critical cone is a type of caustic surface, and the intensity of radiation is greatest in this direction. Because of the inhomogeneous magnetic field, the ray with $\theta_0=0°$ (and $\theta_0=-10°$) exhibit a slight outward shift in L in the low latitudes up to about 10° and the subsequent decrease in L value. The rays with $\theta_0=-20°$, $-30°$, and $-40°$ (just above θ_g) are found to show a slight initial decrease in L, a subsequent slight outward slight in L and then a sharp decrease in L. With the increase of θ_0, the latitude where we expect the onset of sharp decrease in L, is found to shift towards higher latitudes. The ray with $\theta_0=-50°$ well above θ_g is found to exhibit a path considerably different from the previous ray paths such that it swings around the initial L value. The caustic cone is generated by wave normal angle θ_{inf}, at which the ray direction reaches a maximum with respect to $\tilde{B}_0$. A comparison of this figure with the previous figures may imply that the most enhanced ray focussing is expected at a frequency between those frequencies, that is, it is expected at a frequency slightly below the critical frequency.

In Fig. 7(d) for $\Lambda_0=0.3$ well below the critical frequency, the caustic cone similar to that in Fig. 7(c) is again found, but it is considerably widened, due to the increased θ_{inf} value for this smaller Λ_0 value. No rays remain parallel to the field line.

4. Concluding Remarks

The critical frequency at which the whistler-mode refractive index surface exhibits a zero curvature at a longitudinal wave normal, is estimated for the general plasma conditions. This critical frequency determined by the ratio of electron plasma- to gyro-frequency in a homogeneous plasma, is also found from the full-wave theory in a slightly inhomogeneous case. Then, the two-dimensional ray-tracing analyses for an inhomogeneous outer magnetospheric model, have indicated that the strongest ray focussing in the vicinity of the geomagnetic equator is expected slightly below the above critical frequency. This kind of study of ray focussing is important in the study of enhanced gyro-resonant wave-particle interactions in the outer magnetosphere.

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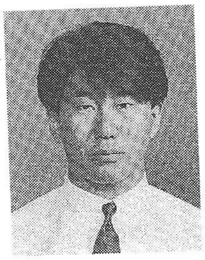

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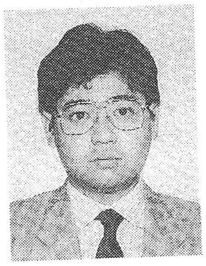

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