Dear Dr. Wen Li, Handling Topical Editor:

We thank you for considering our manuscript. We re-submit a revised version of our manuscript entitled "Excitation of chorus with small wave normal angles due to BPA mechanism in density ducts" [No.: angeo-2019-83] for publication in Journal ANGEO (ANGEO Communicates). We also submit an itemized responses to Referee #1 and Referee #2 comments. The issues raised by the Editor and Referees have been fully addressed in the revised version (highlighted and clean versions of the manuscript are presented) and in the responses, and we thus hope that the manuscript will now be considered acceptable for publication.

Yours sincerely

Peter Bespalov

on behalf of the co-authors

Respond to all referee #1 comments

(1) Comment from Referee #1

I have read the above paper with interests, and I have come to the conclusion that this paper can be acceptable for publication in Angeo, but only after the authors will make the revisions listed below.

General remark: The idea of wave-particle interactions in a confined area such as whistler ducts is very acceptable, and this is theoretically investigated in this paper. Fundamentally it can be acceptable for publication, but only after the authors will make the appropriate revisions.

Specific remarks:

(1) English The English of this paper is not good enough to be accepted in an international journal like Angeo. I strongly request the authors to polish their English with the help of an native English speaker. This will definitely enhance the quality of the paper.

(2) Title: “into density ducts” is not good, and it is better to use “in density ducts”.

(3) Abstract, line 4 beam pulse amplifier (BPA) mechanism

(4) Introduction, line 14 Something is wrong, and I can suggest the following change. -somewhat lower than and just above half the minimum — - in question (see a review by Sazhin and Hayakawa (1992)).

Sazhin, S., and M. Hayakawa, Magnetospheric chorus emissions: A review, Planet. Space Sci., vol.40, 681-697, 1992.

- Line 20; I can suggest one more paper, which is published in a not so popular journal.

(Karpman and Kaufman, 1984; Ishikawa et al., 1990; Laird, 1992; ——) if you are

interested in.

Ishikawa, K., K. Hattori, and M. Hayakawa, A study of ray focusing of whistler-mode waves in the outer magnetosphere, Trans. of the IEICE (Institute of Electronics, Information and Communication Engineers of Japan), Vol. E73, 149-154, 1990. You will see it as an attachment.

(5) Line 16: in Bell et al. (2009).

(6) Line 20пijŽMust be misspelling. Laird, 1992

(7) I am very unhappy with the first paragraph of p.2. Because you have cited only the recent papers on direction finding of chorus emissions, and it seems that you are not aware of earlier work before 1990. Previous papers should be properly described in the paper. Page 2, line 4: (Muto et al., 1987; Hayakawa et al., 1990; Santolik et al., 2009)

Muto, H., M. Hayakawa, M. Parrot, and F. Lefeuvre, Direction finding of half- gyrofrequency VLF emissions in the off-equatorial region of the magnetosphere and their generation and propagation, J. Geophys. Res., 92, 7538-7550, 1987. Hayakawa, M.,

S. Shimakura, M. Parrot, F. Lefeuvre, and K. Hattori, Direction finding of chorus emissions in the outer magnetosphere and their generation and propagation, Planet. Space Sci., 38, 135-143, 1990.

(8) The authors mention that the direction finding by Taubenschuss et al. (2014) is based on the assumption of a single wave. However, the earlier DF works by Muto et al. and Hayakawa et al. are much more general, because they used the wave distribution function. So, how about including the following sentences on line 9 (after the sentence of in a cold homogeneous plasma). We here compare the THEMIS results with earlier analyses based on a more general concept of wave distribution function. For the lower band chorus, the earlier work by Hayakawa et al. (1990) is very consistent with the THEMIS result. While, there is some discrepancy between Muto et al. (1987) ‘s result and THEMIS result for the upper band chorus.

(9) Line 17: - beam pulse amplifier (BPA) mechanism of — - BPA concept appears firstly here in this paper, and you had better mention something about this BPA here.

(10) Page 3, line 6: The depleted duct (e.g. Helliwell, 1965)

(11) Line 7: enhanced duct (Helliwell, 1965; Karpman —

(12) Line 18: well-known form (Laird, 1992)

(13) Line 21: Gendrin velocity

(14) Page 4, line 4: electron cyclotron

(15) Page 6, line 4: recall the formation process of chorus frequency-time spectrogram in the implementation of the BPA mechanism

(16) Line 9: classify the duct solutions

(17) Page 7, line 7: Actually the number of —-

(18) Line 14: realization of the BPA mechanism

(19) Line 26: the BPA mechanism

(20) References: -Addional papers should cited here in References. -p8, line 27; should be Gurnett -Heliwell (1995) seems to be not cited in the text.

(2) Author's response

We would like to thank the Reviewer #1 for the time he/she spent reading, positive response, and commenting our manuscript. We have prepared a point-by-point answer to his/her comments below. The responses are marked in bold and the modified parts are marked in yellow in the new marked version of the manuscript.

Reviewer's Comments:

(1) English The English of this paper is not good enough to be accepted in an international journal like Angeo. I strongly request the authors to polish their English with the help of an native English speaker. This will definitely enhance the quality of the paper.

Response:
Some minor corrections were made to edit the manuscript.

Reviewer's Comments:

(2) Title: “into density ducts” is not good, and it is better to use “in density ducts”.

Response:

The title of the manuscript was modified in accordance to the important suggestion of the Reviewer. The new title is "Excitation of chorus with small wave normal angles due to BPA mechanism in density ducts".

Reviewer's Comments:

(3) Abstract, line 4 beam pulse amplifier (BPA) mechanism

Response:
The text is corrected.

Reviewer's Comments:

(4) Introduction, line 14 Something is wrong, and I can suggest the following change. -somewhat lower than and just above half the minimum — - in question (see a review by Sazhin and Hayakawa (1992)).

Sazhin, S., and M. Hayakawa, Magnetospheric chorus emissions: A review, Planet. Space Sci., vol.40, 681-697, 1992.

Response:
We modified the text in accordance with reviewer's suggestion and referred to the review by Sazhin and Hayakawa (1992).

Reviewer's Comments:

- Line 20; I can suggest one more paper, which is published in a not so popular journal.

(Karpman and Kaufman, 1984; Ishikawa et al., 1990; Laird, 1992; ——) if you are

interested in.

Ishikawa, K., K. Hattori, and M. Hayakawa, A study of ray focusing of whistler-mode waves in the outer magnetosphere, Trans. of the IEICE (Institute of Electronics, Information and Communication Engineers of Japan), Vol. E73, 149-154, 1990. You will see it as an attachment.

Response:

The list of references was extended.

Reviewer's Comments:

(5) Line 16: in Bell et al. (2009).

Response:

The text is corrected.

Reviewer's Comments:

(6) Line 20пijŽMust be misspelling. Laird, 1992

Response:

The text is corrected.

Reviewer's Comments:

(7) I am very unhappy with the first paragraph of p.2. Because you have cited only the recent papers on direction finding of chorus emissions, and it seems that you are not aware of earlier work before 1990. Previous papers should be properly described in the paper. Page 2, line 4: (Muto et al., 1987; Hayakawa et al., 1990; Santolik et al., 2009)

Muto, H., M. Hayakawa, M. Parrot, and F. Lefeuvre, Direction finding of half- gyrofrequency VLF emissions in the off-equatorial region of the magnetosphere and their generation and propagation, J. Geophys. Res., 92, 7538-7550, 1987. Hayakawa, M.,

S. Shimakura, M. Parrot, F. Lefeuvre, and K. Hattori, Direction finding of chorus emissions in the outer magnetosphere and their generation and propagation, Planet. Space Sci., 38, 135-143, 1990.

Response:

The list of references was extended, and the corresponding citations were added to the text of the manuscript.

Reviewer's Comments:

(8) The authors mention that the direction finding by Taubenschuss et al. (2014) is based on the assumption of a single wave. However, the earlier DF works by Muto et al. and Hayakawa et al. are much more general, because they used the wave distribution function. So, how about including the following sentences on line 9 (after the sentence of in a cold homogeneous plasma). We here compare the THEMIS results with earlier analyses based on a more general concept of wave distribution function. For the lower band chorus, the earlier work by Hayakawa et al. (1990) is very consistent with the THEMIS result. While, there is some discrepancy between Muto et al. (1987) ‘s result and THEMIS result for the upper band chorus.

Response:

We agree with the distinguished reviewer. The concept of wave distribution function is a more general in comparison with the plane wave approximation. We mentioned this in the text.

Reviewer's Comments:

(9) Line 17: - beam pulse amplifier (BPA) mechanism of — - BPA concept appears firstly here in this paper, and you had better mention something about this BPA here.

Response:

We added a couple of sentences about BPA concept.

Reviewer's Comments:

(10) Page 3, line 6: The depleted duct (e.g. Helliwell, 1965)

Response:

The citation is added.

Reviewer's Comments:

(11) Line 7: enhanced duct (Helliwell, 1965; Karpman —

Response:

The citation is added.

Reviewer's Comments:

(12) Line 18: well-known form (Laird, 1992)

Response:

The text is corrected.

Reviewer's Comments:

(13) Line 21: Gendrin velocity

Response:

The text is corrected.

Reviewer's Comments:

(14) Page 4, line 4: electron cyclotron

Response:

The text is corrected.

Reviewer's Comments:

(15) Page 6, line 4: recall the formation process of chorus frequency-time spectrogram in the implementation of the BPA mechanism

Response:

The text is corrected.

Reviewer's Comments:

(16) Line 9: classify the duct solutions

Response:

The text is corrected.

Reviewer's Comments:

(17) Page 7, line 7: Actually the number of —-

Response:

The text is corrected.

Reviewer's Comments:

(18) Line 14: realization of the BPA mechanism

Response:
The text is corrected.

Reviewer's Comments:

(19) Line 26: the BPA mechanism

Response:
The text is corrected.

Reviewer's Comments:

(20) References: -Addional papers should cited here in References. -p8, line 27; should be Gurnett -Heliwell (1995) seems to be not cited in the text.

Response:
The list of references was extended, and the corresponding citation (Lauben, D.S., Inan, U.S., Bell, T.F., and Gurnett, D.A.: Source characteristics of ELF/VLF chorus, J. Geophys. Res., 107, CiteD1429, doi:10.1029/2000JA003019, 2002.) was added to the text of the manuscript.

(3) Author's changes in manuscript (marked-up in yellow)

Title
The text
"into density"
is replaced by
"in density"

Abstract line 1

The text

"mechanism (BPA)"

is replaced by

"(BPA) mechanism"

Page 1 line 1

The text

"in (Bell et al., 2009)"

is replaced by

"in Bell et al. (2009)"

Page 1 line 14

The text

"centered somewhat lower than half the minimum electron cyclotron frequency for the magnetic tube in question"

is replaced by

"somewhat lower than and just above half the minimum electron cyclotron frequency for the magnetic tube in question (see, e.g., a review by Sazhin and Hayakawa (1992))"

Page 1 line 20

The text

"Laids"

is replaced by

"Laird"

Page 2 line 3

The text

"the experimental"

is replaced by

"the theoretical and experimental"

Page 2 line 4

The text

"Santolik et al., 2009"

is replaced by

"Muto et al., 1987; Hayakawa et al., 1990; Santolik et al., 2009"

Page 2 line 8

The text

"is based"

is replaced by

"(Taubenschuss et al., 2014) is based"

Page 2 line 11

The text

"formalism"

is replaced by

framework of a concept of wave distribution function (Muto et al., 1987; Hayakawa et al., 1990)

Page 2 line 16

The text

"not requiring significant anisotropy of the distribution function of energetic electrons"

is removed

Page 2 line 17

The text

"It has been shown that under suitable conditions a very effective amplification of short noise pulses can occur even in a stable plasma. Pulse amplification leads to the excitation of bursts of electromagnetic radiation having the same properties as would occur due to an instability resulting from a very high anisotropy in the distribution function of energetic electrons."

is added.

Page 3 line 6

The text

"(Helliwell, 1965)"

is added.

Page 3 line 7

The text

"(Helliwell, 1965)"

is added.

Page 4 line 4

The text

"electronic"

is replaced by

"electron"

Page 6 line 4

The text

"beam pulse amplifier"

is replaced by

"BPA"

Page 6 line 24

The text

"The mentioned frequencies are typical for low-band and upper-band chorus (Lauben et al., 2002)."

is added.

Page 7 line 7

The text

"Actually, that"

is replaced by

"Actually"

Page 7 line 14

The text

"beam pulse amplifier"

is replaced by

"BPA"

Page 7 line 26

The text

"beam pulse amplifier"

is replaced by

"BPA"

Page 8 line 22

The text

"Lauben, D.S., Inan, U.S., Bell, T.F., and Gurnett, D.A.: Source characteristics of ELF/VLF chorus, J. Geophys. Res., 107, CiteD1429, doi:10.1029/2000JA003019, 2002."

is added.

Page 8 line 30
The text
"Sazhin, S., and Hayakawa, M.: Magnetospheric chorus emissions, A review, Planet. Space Sci., 40, 681-697, doi:10.1016/0032-0633(92)90009-D, 1992.'

is added.

Page 9 line 15

The text

"Hayakawa, M., Shimakura, S., Parrot, M., Lefeuvre, F., and Hattori, K.: Direction finding of chorus emissions in the outer magnetosphere and their generation and propagation, Planet. Space Sci., 38, 135-143, doi:10.1016/0032-0633(90)90012-F, 1990."

is added

Page 9 line 26

The text

"Muto, H., Hayakawa, M., Parrot, M., and Lefeuvre, F.: Direction finding of half-gyrofrequency VLF emissions in the off-equatorial region of the magnetosphere and their generation and propagation, J. Geophys. Res., 92, 7538-7550, doi:10.1029/JA092iA07p07538, 1987."

is added

Respond to all referee #2 comments

(1) Comment from Referee #2

General comments

The authors study the chorus excitation in the density ducts in the frame of the beam pulse amplifier mechanism for the enhanced and depleted ducts. It is noted that the considered model allows one to explain small angles of the wave normal in the as- sumption of a single planar whistler-mode wave in a cold homogeneous plasma. The subject of the paper is significant for geoscience. The paper can be published after minor revision.

Specific comments

1) Page 2, line 9. It is written: 'In the duct, a standing wave structure occurs at the transverse coordinate.' It is not clear what the authors mean.

2) The main terms and concepts used in the paper could be explained more explicitly in the Introduction.

3) Page 3, lines 2-4. It is written that 'the WKB approximation is fulfilled'. It is better to remind the conditions for the WKB approximation validity in more detail.

4) Figure 1 is not very illustrative. Two branches are hard to see, they should be marked.There is a lot of empty place in this figure. Modification is needed.

5) Page 3, line 13. It is written: 'Let us note that there is a range of values kz, omega in which kx has not one, but two values.' It is desirable to determine this region (or regions), if this point is essential.

6) Page 3, line 18. '(laird, 1992)' should be replaced by ('Laird, 1992').

7) Page 4, line 5. It is written: 'While performing calculations, we will con- sider that the magnetic field is uniform and the electronic cyclotron frequency omega_B = 6 Au 10**4 s**-1. The plasma density outside the duct corresponds to the condition (omega p,out/omega_B)**2 = 25. Inside the enhanced duct we have (omega p, int/omega_B)**2 = 29, while inside the depleted duct, (omega p,int/omega_B)**2 = 21.' It is desirable to explain, why these values are cho- sen.

8) Figure 2. Labels 'a' and 'b' are not shown in figure. In the figure caption nothing is told about the red curves, the corresponding explanation is given only in the text. First and seventh ducted modes, probably,correspond to p=1 and p=7, respectively, which is desirable to note in the figure caption.

9) At the end of page 4 the normal angle is estimated. Why normal angle is only considered to be small? What will be in the opposite case? Some explanation is desirable. If it is determined by observations, for example, it should be mentioned.

10) There are no explanations of all designations in Eqs. (6) - (9). For example, what is Vz in Eq. (8)? There is reference to paper Bespalov and Savina (2018), however, for self-consistency it is desirable to describe all mentioned parameters.

11) Page 7, line 21. Instead of '(7)' it should be 'Eq. (7)'.

12) The new contribution related to the previous works should be emphasized more clearly.

(2) Author's response

We would like to thank the Reviewer #2 for the time he/she spent reading, positive response, and commenting our manuscript. We have prepared a point-by-point answer to his/her comments below. The responses are marked in bold and the modified parts are marked in blue in the new marked version of the manuscript.

Reviewer's Comments:

1) Page 2, line 9. It is written: 'In the duct, a standing wave structure occurs at the transverse coordinate.' It is not clear what the authors mean.

Response:
We modified the text in accordance with reviewer's comment. The text
"In the duct, a standing wave structure occurs at the transverse coordinate."
is replaced by
"In the duct, a standing wave structure occurs across the magnetic field."

2) The main terms and concepts used in the paper could be explained more explicitly in the Introduction.

Response:

We modified the introduction in accordance with reviewer's suggestion.

3) Page 3, lines 2-4. It is written that 'the WKB approximation is fulfilled'. It is better to remind the conditions for the WKB approximation validity in more detail.

Response:

We modified the text in accordance with reviewer's suggestion. The list of references was extended and the text

"For typical magnetospheric conditions (Haque et al., 2011) in the region of chorus excitation, the WKB approximation is fulfilled since the length of the whistler wave $\lambda \simeq 15\;km$ is less than the scale of the background plasma density distribution and the transverse size of the duct $d = 100 - 300\;km$."

is replaced by

"For typical magnetospheric conditions (Haque et al., 2011) in the region of chorus excitation the length of the whistler wave $\lambda \simeq 15\;km$ is less than the scale of the background plasma density distribution and the transverse size of the duct $d = 100 - 300\;km$. Therefore the inequality $\lambda/d \ll \pi$ is fulfilled and the well known WKB approximation (Budden, 1985) is valid."

4) Figure 1 is not very illustrative. Two branches are hard to see, they should be marked. There is a lot of empty place in this figure. Modification is needed.

Response:

Figure 1 and its caption are modified.

5) Page 3, line 13. It is written: 'Let us note that there is a range of values kz, omega in which kx has not one, but two values.' It is desirable to determine this region (or regions), if this point is essential.

Response:

The text is modified.

6) Page 3, line 18. '(laird, 1992)' should be replaced by ('Laird, 1992').

Response:

The text is corrected.

7) Page 4, line 5. It is written: 'While performing calculations, we will consider that the magnetic field is uniform and the electronic cyclotron frequency omega_B = 6 Au 10**4 s**-1. The plasma density outside the duct corresponds to the condition (omega p,out/omega_B)**2 = 25. Inside the enhanced duct we have (omega p, int/omega_B)**2 = 29, while inside the depleted duct, (omega p,int/omega_B)**2 = 21.' It is desirable to explain, why these values are cho- sen.

Response:

We mentioned that these values are close to the known experimental data.

8) Figure 2. Labels 'a' and 'b' are not shown in figure. In the figure caption nothing is told about the red curves, the corresponding explanation is given only in the text. First and seventh ducted modes, probably,correspond to p=1 and p=7, respectively, which is desirable to note in the figure caption.

Response:

Labels 'a' and 'b' are shown near figure 2. The figure caption is corrected.

9) At the end of page 4 the normal angle is estimated. Why normal angle is only considered to be small? What will be in the opposite case? Some explanation is desirable. If it is determined by observations, for example, it should be mentioned.

Response:
Some explanation is added. Finally normal angle is estimated theoretically.

10) There are no explanations of all designations in Eqs. (6) - (9). For example, what is Vz in Eq. (8)? There is reference to paper Bespalov and Savina (2018), however, for self-consistency it is desirable to describe all mentioned parameters.

Response:
The text is corrected.

11) Page 7, line 21. Instead of '(7)' it should be 'Eq. (7)'.

Response:
The text is corrected.

12) The new contribution related to the previous works should be emphasized more clearly.

Response:
We added paragraph to the Conclusions.

(3) Author's changes in manuscript (marked-up in blue)

Page 2 line 9
The text
"In the duct, a standing wave structure occurs at the transverse coordinate."
is replaced by
"In the duct, a standing wave structure occurs across the magnetic field."

Page 2 line 17

The text

"It has been shown that under suitable conditions a very effective amplification of short noise pulses can occur even in a stable plasma. Pulse amplification leads to the excitation of bursts of electromagnetic radiation having the same properties as would occur due to an instability resulting from a very high anisotropy in the distribution function of energetic electrons."

is added.

Page 3 line 2
The text

"For typical magnetospheric conditions (Haque et al., 2011) in the region of chorus excitation, the WKB approximation is fulfilled since the length of the whistler wave $\lambda \simeq 15\;km$ is less than the scale of the background plasma density distribution and the transverse size of the duct $d = 100 - 300\;km$."

is replaced by

"For typical magnetospheric conditions (Haque et al., 2011) in the region of chorus excitation the length of the whistler wave $\lambda \simeq 15\;km$ is less than the scale of the background plasma density distribution and the transverse size of the duct $d = 100 - 300\;km$. Therefore the inequality $\lambda/d \ll \pi$ is fulfilled and the well known WKB approximation (Budden, 1985) is valid."

Page 3 line 13
The text
"but two values"

is replaced by
"but two values (see Fig. 1)"

Page 4
Figure 1 was modified.

Page 4 line 1
The text
"Dependence of the transverse component of the wave vector on the frequency and the longitudinal component of the wave vector for ${({\omega _p}/{\omega _B})^2} = 25$."
is replaced by
"Dependence of the transverse component of the wave vector on the frequency and the longitudinal component of the wave vector for ${({\omega _p}/{\omega _B})^2} = 25$ is shown by contours with constant k_x in the surface $k_x=k_x(\omega, k_z)$. Two branches K_{x-} and K_{x+} (see Eq. (2)) are separated by a blue line."

Page 4 line 5
The text
"according to the known experimental data (see, e.g. (Hague et al., 2011; Taubenschuss et al., 2014; Agapitov et al., 2017))"
is added.

Page 5 line 1
The text
"Relationship between the frequency and the longitudinal component of the wave vector for the first and seventh ducted modes at ${({\omega _{p,out}}/{\omega _B})^2} = 25$:"
is replaced by
"Relationship between the frequency and the longitudinal component of the wave vector for the first ($p=1$) and seventh ($p=7$) ducted modes at ${({\omega _{p,out}}/{\omega _B})^2} = 25$ are shown in red:"

Page 5 line 8
The text
"If"
is replaced by

"As it will be shown in the next section $\theta < \pi /4$ for actual modes"

Page 6 line 18
The text
"where ${n_b}$ is the density of the beam coordinated with the pulse"
is replaced by
"where $\delta(\varsigma)$ is a delta function, ${n_b}$ is the density of the beam coordinated with the pulse, V_z is the beam velocity"

Page 7 line 14
The text
"The beam pulse amplifier (BPA) mechanism of chorus excitation was first studied for homogeneous plasma (Bespalov and Savina, 2018). This mechanism explains many properties of the oblique electromagnetic chorus. The proposed model with waveguide propagation explains the possibility of excitation of chorus with small angles of the wave normal when the BPA mechanism is implemented."

is added.

Page 7 line 21

The text

"(7)"

is replaced by

"Eq. (7)"

Page 8 line 10
The text
"Budden, K.G.: The propagation of radio waves: The theory of radio waves of low power in the ionosphere and magnetosphere, Cambridge University Press, Cambridge, 669 p., doi: 10.1017/CBO9780511564321, 1985."
is added.

Respond to all second referee #2 comments

(1) Comment from Referee #2

I think that the paper can be published now.

(2) Author's response

We would like to thank the Reviewer #2 for the time he/she spent reading, positive response, and conclusion: "the paper can be published now".

Excitation of chorus with small wave normal angles due to BPA mechanism ~~into~~ in density ducts

Peter A. Bespalov[1,2] and Olga N. Savina[2]

[1]Institute of Applied Physics RAS, Nizhny Novgorod, Russia
[2]National Research University Higher School of Economics, Russia

Correspondence: P. Bespalov
(PBespalov@mail.ru)

Abstract. We examine specific features of the realization of the beam pulse amplifier ~~mechanism~~ (BPA) **mechanism** of chorus excitation in the density ducts having a width of the order of $100-300\ km$ with refractive reflection. The dispersion characteristics of whistler emissions in a planar duct under conditions for the fulfillment of the WKB approximation and refractive reflection from the "walls" of the duct are analyzed. It is shown that in the enhanced duct, discrete spectral elements of chorus with a narrow angular spectrum along the external magnetic field can be excited at frequencies somewhat lower than half the electron cyclotron frequency. In the depleted duct at frequencies somewhat higher than half the electron cyclotron frequency, chorus with a narrow angular spectrum along the magnetic field can be excited. The proposed model explains the possibility of excitation of chorus with small angles of the wave normal when the BPA mechanism is implemented. It is noted that the properties of chorus, such as the intensity and a typical angle of the wave normal, can be different for the lower and upper band chorus.

1 Introduction

In accordance with the experimental data of the CLUSTER satellites and Van Allen Probes, chorus emissions are excited in the "cigar-shaped" region with a length of the order of $l = (1-2)10^8\ cm$ and average diameter $\bar{d} = 3 \cdot 10^7\ cm$ (Bell et al., 2009; Agapitov et al., 2017) near the local magnetic field minimum. Usually, the chorus spectrogram is observed in two basic spectral bands ~~centered~~ somewhat lower **and just above** than half the minimum electron cyclotron frequency for the magnetic tube in question **(see, e.g., a review by Sazhin and Hayakawa (1992))**. Relationship of the chorus excitation regions in two spectral bands with background plasma density inhomogeneities across the magnetic field was discussed in ~~(Bell et al., 2009)~~ **Bell et al. (2009)**. The authors of that paper proposed that the source region for banded chorus consists of whistler mode ducts of depleted electron density (n_p) for the upper band chorus and ducts of enhanced electron density n_p for the lower band chorus.

Theoretical aspects of the whistler wave propagation in the magnetospheric ducts have been studied for more than half a century using different approaches (see, e.g., (Karpman and Kaufman, 1984; ~~Laids~~ **Laird**, 1992; Pasmanik and Trakhtengerts, 2005; Sonwalkar, 2006; Woodroffe et al., 2013)). Known experimental data make it possible to define concretely the important characteristics of the geophysical situation. For example, according to the data given in (Haque et al., 2011), the difference in the plasma density inside and outside the duct is of the order of $10-15\%$. The wave field of chorus rapidly decays with distance from the duct in the transverse direction. This indicates that the frequency of the working emission mode is not close to the critical frequency of the duct.

In the **theoretical and** experimental studies of chorus, much attention has been given to determination of the wave normal angles (θ) in the region of excitation of emissions (**Muto et al., 1987; Hayakawa et al., 1990;** Santolik et al., 2009). In accordance with the THEMIS satellites data (Taubenschuss et al., 2014) for the rising and falling tones in the lower band of chorus, electromagnetic emissions were detected more frequently at angles of the order of 20°, and the range of angles is smaller for the lower frequencies. For the upper band chorus, the angles of the wave normal are also of the order of 20° and decrease at the higher frequencies. Let us note that the algorithm of the wave normal angle calculation **(Taubenschuss et al., 2014)** is based on the assumption that the experimental data offer a single, planar whistler-mode electromagnetic wave in a cold homogeneous plasma. In the duct, a standing wave structure occurs ~~*at the transverse coordinate*~~ **across the magnetic field**. It corresponds to two counter-propagating waves. Probably, this should be taken into account within the ~~*formalism*~~ **framework of the concept of a wave distribution function (Muto et al., 1987; Hayakawa et al., 1990)** .

The growth rate (γ) of chorus emission modification is very high (Santolik and Gurnett, 2003) and can correspond to $\gamma \approx 100\, s^{-1}$. According to experimental data and the results of numerical calculations (Li et al., 2011), for the achievement of a cyclotron instability growth rate of this order, high anisotropy of the distribution function of energetic electrons in the excitation region is necessary. According to (Fu et al., 2014), the mechanism required for producing high anisotropies is not clear. Moreover, it is noted in (Zhou et al., 2015), on the basis of the analysis of a large volume of experimental data, that chorus are frequently excited in the regions of the daytime magnetosphere with a marginal stable plasma without significant anisotropy of the distribution function. For explaining these results, one can use the beam pulse amplifier mechanism (BPA) of chorus excitation ~~*not requiring significant anisotropy of the distribution function of energetic electrons*~~ in the magnetosphere (Bespalov and Savina, 2018). **It has been shown that under suitable conditions a very effective amplification of short noise pulses can occur even in a stable plasma. Pulse amplification leads to the excitation of bursts of electromagnetic radiation having the same properties as would occur due to an instability resulting from a very high anisotropy in the distribution function of energetic electrons.** In a homogeneous plasma, the BPA mechanism is most effective at wave normal angles close to $\theta \simeq 39°$.

In this paper, we examine some specific features of the BPA mechanism realization of electromagnetic chorus excitation in the duct. This will be done in the WKB approximation for a planar duct with refractive reflection. The study will make it possible to better understand the real conditions of the chorus excitation. In particular, we will explain the experimental data about typical wave normal angles in the chorus excitation region.

2 Characteristic equation for the modes of a planar duct with refractive reflection

The motion of ions is not important for describing the whistler wave radiation in the chorus excitation region with frequency ω in the interval $\omega_{LHF} < \omega < \omega_B < \omega_p$, where ω_{LHF} is the lower-hybrid frequency, $\omega_B = eB/mc$ and $\omega_p = (4\pi n_p e^2/m)^{1/2}$ are the absolute value of the electron cyclotron and plasma frequencies, e is the absolute value of electron charge, m is the

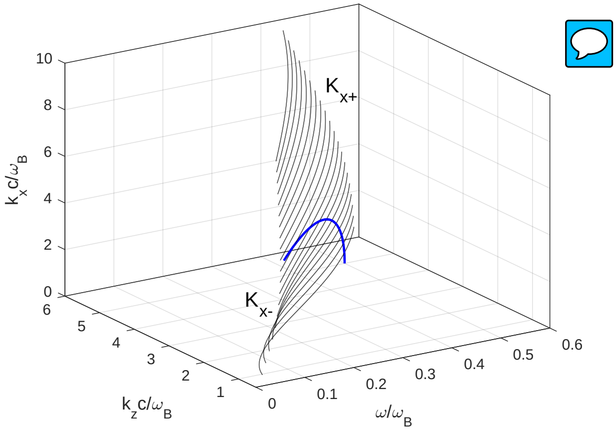

Figure 1. Dependence of the transverse component of the wave vector on the frequency and the longitudinal component of the wave vector for $(\omega_p/\omega_B)^2 = 25$ **is shown by contours with constant** k_x **in the surface** $k_x = k_x(\omega, k_z)$**. Two branches** K_{x-} **and** K_{x+} **(see Eq. (2)) are separated by a blue line**.

electron mass, n_p is the plasma density, and c is the speed of light in free space. A theoretical analysis is simplified if the conditions for the applicability of the so-called quasi-longitudinal approximation of electromagnetic wave propagation in a cold, relatively dense plasma are fulfilled (Helliwell, 1965).

$$\omega = \frac{\omega_B \, |k_z| \, (k_z^2 + k_x^2)^{1/2}}{k_z^2 + k_x^2 + \omega_p^2/c^2} \, , \tag{1}$$

where k_z and k_x are the wave vector components along and across the magnetic field, respectively.

For typical magnetospheric conditions (Haque et al., 2011) in the region of chorus excitation , ~~the WKB approximation is fulfilled since~~ the length of the whistler wave $\lambda \simeq 15\ km$ is less than the scale of the background plasma density distribution and the transverse size of the duct $d = 100 - 300\ km$. **Therefore the inequality** $\lambda/d \ll \pi$ **is fulfilled and the well known WKB approximation (Budden, 1985) is valid.** When the plasma density is inhomogeneous in the direction perpendicular to the duct axis, refraction is an important factor for the wave propagation. It is known that under conditions for refractive reflection, waves with frequencies higher than half the electron cyclotron frequency are directed by the depleted duct (**Helliwell, 1965**). Waves with frequencies lower than half the electron cyclotron frequency are directed by the enhanced duct (**Helliwell, 1965;** Karpman and Kaufman, 1984).

We now consider in more detail the characteristic equation of a refractive planar duct. For this purpose, we analyze the dependence of the transverse component of the whistler wave vector on the frequency and the longitudinal component of the wave vector with different background plasma densities (Woodroffe et al., 2013). This dependence, in accordance with Fig. 1, has two branches, which, according to Eq. (1), can be written as follows:

$$k_x = \begin{cases} K_{x+},\ if \\ \frac{\omega}{\omega_B} \le H(\frac{1}{2}-\frac{\omega}{\omega_B})\frac{|k_z|u_G}{\omega_B} + H(\frac{\omega}{\omega_B}-\frac{1}{2})\frac{1}{1+(\omega_p/k_z c)^2}; \\ K_{x-},\ if \\ \frac{1}{1+(\omega_p/k_z c)^2} \le \frac{\omega}{\omega_B} \le \frac{|k_z|u_G}{\omega_B} \le \frac{1}{2}. \end{cases} \quad . \qquad (2)$$

where $u_G = (c\omega_B/2\omega_p)$ is the Gendrin velocity (Helliwell, 1995),

$$K_{x\pm} = \left\{ \left(\frac{\omega_B k_z}{2\omega}\right)^2 \left[(1-\frac{\omega^2}{k_z^2 u_G^2})^{1/2} \pm 1 \right]^2 - k_z^2 \right\}^{1/2} .$$

$H(\varsigma)$ is the Heaviside step function. Let us note that there is a range of values k_z, ω in which k_x has not one, but two values **(see Fig. 1)**.

The property of the eigenmode wave solutions is determined by the characteristic equation, which expresses the dependence of the frequency on the longitudinal component of the wave vector. For determining the characteristic equation in the WKB approximation, it is necessary to calculate a complete transverse change in the phase during the ray propagation period. A change in the phase during one period should include phase displacements on the caustics and be multiple of 2π. Therefore, the characteristic equation takes the well-known form (Laird, 1992)

$$\Phi(\omega, k_z) = \int_{-x_{\max}}^{x_{max}} k_x dx = (p - \frac{1}{2})\pi\ , \qquad (3)$$

where p is a positive integer, and in a symmetric duct $\mp x_{\max}$ are the reflection levels, where $k_x = 0$.

Within the framework of the WKB approximation, the dependence $k_x(x)$ should be continuous, and in the regions of refraction reflection $k_x = 0$ if the density $n_p(x)$ and the Gendrin velocity $u_G(x)$ change continuously. The refraction enhanced duct can occur for frequencies $\omega < \omega_B/2$, and only branch K_{x-} (2) satisfies the continuity condition of the dependence $k_x(x)$. The refraction depleted duct can occur for frequencies $\omega > \omega_B/2$, and only the branch K_{x+} (2) satisfies the continuity condition of the dependence $k_x(x)$.

Let us assume that there is a duct with enhanced or depleted cold plasma density and which is uniform along the magnetic field. While performing calculations, we will consider **according to the known experimental data (see, e.g. (Hague et al., 2011; Taubenschuss et al., 2014; Agapitov et al., 2017))** that the magnetic field is uniform and the ~~electronic~~ **electron** cyclotron frequency $\omega_B = 6 \cdot 10^4\ s^{-1}$. The plasma density outside the duct corresponds to the condition $(\omega_{p,out}/\omega_B)^2 = 25$. Inside the enhanced duct, we have $(\omega_{p,int}/\omega_B)^2 = 29$, while inside the depleted duct, $(\omega_{p,int}/\omega_B)^2 = 21$. For determining the solutions of the characteristic equation, we used a model transverse density distribution for the enhanced and depleted ducts in the form

$$n_p(x) = n_{p,int}[1 + (\frac{n_{p,out}}{n_{p,int}} - 1)\tanh^2(2\frac{x}{d})]\ , \qquad (4)$$

where $d = (1-3)10^7\ cm$ is the transverse size of the duct.

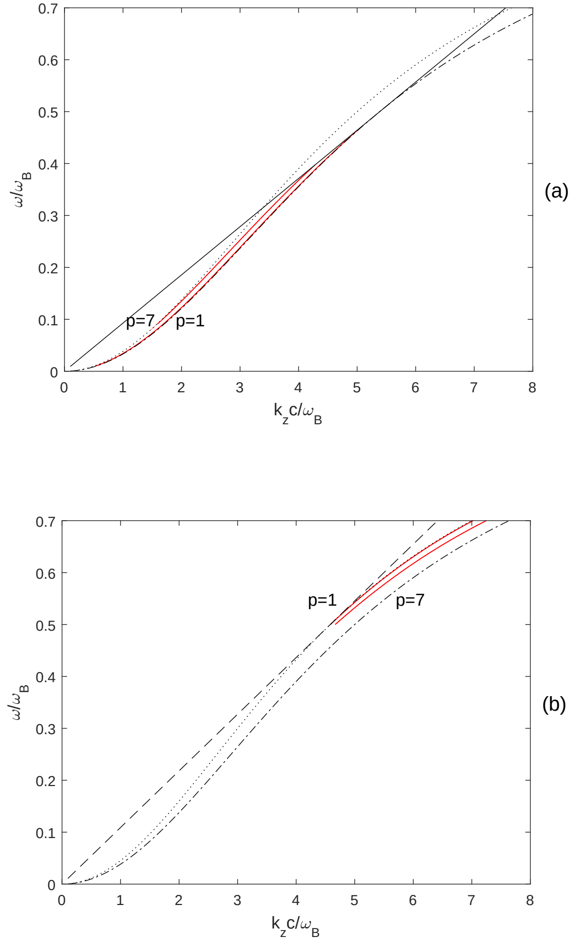

Figure 2. Relationship between the frequency and the longitudinal component of the wave vector for the first ($p = 1$) and seventh ($p = 7$) ducted modes at $(\omega_{p,out}/\omega_B)^2 = 25$ **are shown in red**: (a) in the enhanced duct with $(\omega_{p,int}/\omega_B)^2 = 29$; (b) in the depleted duct with $(\omega_{p,int}/\omega_B)^2 = 21$.

For the accepted density distributed across the duct (4), it is possible to numerically calculate, based on Eq. (2), the phase change given by Eq. (3) and obtain dispersion formulas for the enhanced and depleted ducts. Some results of calculations are given in Fig. 2. In these figures, the red lines show solutions of the characteristic equations for the first and seventh modes in the enhanced (a) and depleted (b) ducts, the dash-dot and the dotted curves corresponds to the solution of dispersion equation (1) for $k_x(x) = 0$ at the maximum and minimum density, respectively, the solid line corresponds to the Gendrin velocity at the maximum density, and the dashed straight line corresponds to the Gendrin velocity at the minimum density.

For relatively gently sloping ray trajectories, it is not difficult to estimate the average of the normal angle. Then Eq. (3) can be written in the form $(2/d)\int\limits_0^{x_{max}} \tan(\theta)dx = (2\pi/k_z d)(p-\frac{1}{2})$. ~~If~~**As it will be shown in the next section** $\theta < \pi/4$ **for actual modes,** with accuracy up to 20% we have $\tan(\theta) \simeq \theta$, and therefore

$$\bar{\theta} \simeq \frac{2\pi}{k_z d}(p-\frac{1}{2}) \,. \tag{5}$$

Note that the distance between the modes in Fig. 2 decreases with increasing transverse size of the duct.

3 Special features of implementation of the BPA mechanism of the chorus excitation in a duct with refractive reflection

Now we briefly recall the process of chorus frequency-time spectrogram formation in the implementation of the ~~*beam pulse amplifier*~~ **BPA** mechanism (Bespalov and Savina, 2018). At the input of the wave-particle interaction region ($z = 0$), there is weak noisy emission with an electric field containing a random sequence of weak shot electromagnetic pulses. Noise that does not satisfy the conditions for interaction with particles is dumped in a nearly stable plasma. An appropriate shot pulse is a trigger for the chorus discrete element. In accordance with this, we assume that a short noise pulse flies in the duct on the z axis along the magnetic field $\mathbf{B}$. Our interest is to classify the duct solutions which are close in structure to the expression

$$E_z = G_p(x)\tilde{E}_p(z,t) \,, \tag{6}$$

in which the longitudinal electric field E_z corresponds to the pth wave mode in the duct. Wave mode (6) is well localized on the transverse coordinate x because of the fast decrease in the function $G_p(x)$ with large $|x|$, but in the general case, it spreads along the longitudinal coordinate z because of the dispersion.

There is an important exception from this general regularity. A noisy electromagnetic pulse will not rapidly spread if it has frequencies for which

$$v_{phz} = v_{gz} = u \,, \tag{7}$$

where $v_{phz} = \omega/k_z$ and $v_{gz} = \partial\omega/\partial k_z$ are the phase and group velocities of the wave mode in the duct. In this case, the population of epithermal electrons, which fly in together with the pulse into the region of wave-particle interaction, starts to play an important role. These electrons for the pulse are a single-velocity effective distribution function f proportional to

$$f = n_b\delta(V_z - u) \,, \tag{8}$$

where $\delta(\varsigma)$ **is a delta function,** n_b is the density of the beam coordinated with the pulse**,** V_z **is the beam velocity**.

In a homogeneous plasma with beam (8), the electromagnetic pulse increases with the growth rate (Bespalov and Savina, 2018)

$$\gamma_{BPA} = \frac{\sqrt{3}}{4}\omega_B\left(\frac{n_b}{4n_p}\sin^2\theta\right)^{1/3}|\cos\theta| \,. \tag{9}$$

First of all, we verify the possibility of the fulfillment of condition (7) for the duct modes. For the plasma parameters introduced in the foregoing section, this condition can be fulfilled (see Fig. 2a) in the enhanced duct if the frequency spectrum is concentrated near $\omega/\omega_B \simeq 0.42$ and the longitudinal component of the wave vector is close to $k_z c/\omega_B \simeq 4.7$. Condition (7) also can be fulfilled (see Fig.2b) in the depleted duct if the frequency spectrum is concentrated near $\omega/\omega_B \simeq 0.51$ and the longitudinal component of the wave vector is close to $k_z c/\omega_B \simeq 5.0$. **The mentioned frequencies are typical for lower- and upper-band chorus (Lauben et al., 2002).**

We can use a formula for growth rate (9) since the wave modes in the duct conform to the WKB approximation, and therefore their polarization corresponds to the whistler waves in which during the inclined propagation there is a longitudinal component of the wave electric field which is taken into account in the growth rate calculation (Bespalov and Savina, 2018). We will refine now the value of the average angle $\bar{\theta}$, which should be substituted into Eq. (9). We will do this using Eq. (5), according to which

$$\bar{\theta} \simeq 2\pi \frac{\omega_B}{k_z c} \frac{c}{d\omega_B}(p - \frac{1}{2}) \simeq 0.03(p - \frac{1}{2}) \,, \tag{10}$$

where we expected that $k_z c/\omega_B \simeq 5$ and $d = 300\ km$. The angle θ is less than 20 deg for the first ten modes.

~~*Actually, that*~~ **Actually,** the number of operating modes (p) in a relatively small duct is limited by several factors. In particular, the quality of the mode should be sufficient high, the mode should be electromagnetic, and key condition (7) should be fulfilled. Formally, growth rate (9) increases with increasing mode number p. However, it should be taken into account that there is another limitation on the angle of excitation, which is caused by the Landau damping on the $\check{C}$erenkov resonance of electromagnetic waves in a plasma. We note that for the angle $\theta = 20°$ growth rate (9) decreases by 20% only in comparison with the maximum value close to $\theta = 39°$, and it is sufficient for explaining the experimental data.

4 Conclusions

The beam pulse amplifier (BPA) mechanism of chorus excitation was first studied for homogeneous plasma (Bespalov and Savina, 2018). This mechanism explains many properties of the oblique electromagnetic chorus. The proposed model with waveguide propagation explains the possibility of excitation of chorus with small angles of the wave normal when the BPA mechanism is implemented.

We have examined specific features of the realization of the ~~*beam pulse amplifier*~~ **BPA** mechanism of chorus excitation in density ducts with a width of the order of $100 - 300\ km$ with refractive reflection. It is shown that the BPA mechanism can be realized in magnetospheric density ducts. Waveguide propagation of the whistler waves excited by this mechanism in a refractive duct permits one to explain the experimentally observed small angles of the wave normal. For the modes with relatively low numbers (in the examined example Eq. (10), $p = 1 - 10$), the angle of the wave normal $\theta \leq 20°$. The rate of change in spectral forms in this case is characterized by growth rate (9) with a value smaller than the maximum possible one (up to 20%). This rate of change is sufficient for explaining the experimental data within the framework of the approach presented in the paper by Bespalov and Savina (2019). Key condition **Eq.** (7) of the BPA mechanism is satisfied in a frequency range lower than half the electron cyclotron frequency in the enhanced duct and larger than half the electron cyclotron frequency in the depleted duct (see Fig. 2). If there is no appropriate wave refraction, pulses can also be excited, but no longer as the ducted mode. For these emission, it is possible to expect the lower values of the intensity and larger angles θ.

The proposed model explains the possibility of the chorus emissions excitation with small angles of the wave normal due to the ~~*beam pulse amplifier*~~ **BPA** mechanism. The properties of chorus, such as the intensity and a typical angle of the wave normal, can be different for lower- and upper-band chorus.

Data availability. The paper is theoretical and no new experimental data are used. All figures data are obtained from numerical calculation in MATLAB codes. Corresponding parameters are listed in the text.

Author contributions. PB proposed and analyzed the BPA mechanism in the density ducts, and wrote the paper. OS analyzed the BPA mechanism in the density ducts, and wrote the paper.

Competing interests. The authors declare that they have no conflict of interest.

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