



1 **Invariants of the Spatial-Energy Structure and Modeling of the**
2 **Earth's Ion Radiation Belts**

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6 **Abstract** The spatial-energy distributions of proton fluxes in the Earth's radiation belts (ERB)
7 are well studied and the NASA averaged empirical models constructed for them (the latest
8 versions are AP8 and AP9). These models are widely used in space research. However, for
9 heavier ERB ions (helium, oxygen, etc.), much less measurements were made on satellites,
10 especially in the energy range from tens to hundreds of MeV, and there are no sufficiently
11 complete and reliable models for them. Meanwhile, such ions, although there are much smaller
12 than protons, play a very important role in the physics of ERB, especially in their dynamics, as
13 well as in solving problems of ensuring the safety of space flights. The data on such ions
14 represent a rather fragmentary picture, in which there are significant “white spots”. Using the
15 methods considered in this paper, these fragmentary data can be streamlined, linked to each
16 other and get a regular picture that has a simple physical meaning. Spatial-energy distributions
17 of the stationary fluxes of protons, helium ions and ions of the CNO group with energy from
18 100 keV to 200 MeV at $L \sim 1-8$ considered here on the data of the satellites for 1961–2017. It is
19 found, that results of the measurements of the ion fluxes are arrange in certain regular patterns
20 in the spaces $\{E, L\}$ and $\{L, B/B_0\}$. This effect connected with the existence of invariant
21 parameters of these distributions of ion fluxes. These invariant parameters are very useful and
22 necessary for constructing the ion models of the ERB. The physical mechanisms leading to
23 formation spatial-energy structure of the ERB ion fluxes and the values of its invariant
24 parameters discussed here. In the course of this work, solar-cyclic (11-year) variations in the
25 distributions of helium and carbon-nitrogen-oxygen ions fluxes in the ERB studied for the first
26 time. It shown that, as compared with such variations in the proton fluxes studied earlier, the
27 amplitude of the variations of heavier ions is much larger and increases with increasing their
28 mass.

29
30 **Keywords.** Magnetospheric physics (energetic particles trapped)

31



32 1 Introduction

33 The Earth's radiation belts (ERB) consist of charged particles with energy from $E \sim 100$ keV to
34 several hundreds of MeV. They trapped by the geomagnetic field at altitudes from ~ 200 km to \sim
35 50–70 thousands kilometers. The ERB consist mainly of electrons and protons. In the ERB there
36 are also ions of helium, oxygen, and other elements with the atomic number $Z \geq 2$ (Z is the charge
37 of the atomic nucleus with respect to the charge of the proton). Fluxes of ions and its distributions
38 vary during geomagnetic disturbances. These fluxes depend also on the phase of the solar cycle,
39 conditions in the interplanetary space, and other factors.

40 Particles with different energy and with different pitch-angles α (α is the angle between the
41 local vector of the magnetic field and vector of the particle velocity) which injected into some
42 point of the geomagnetic trap, drifting with conservation of the invariants μ and K gradually
43 populate a narrow layer around the Earth, so-called drift shell (Alfvén and Fälthammar, 1963;
44 Northrop, 1963). Therefore, experimental data on the ERB most simply represented in coordinates
45 $\{L, B\}$, where L is a parameter of a drift shell and B is the local induction of a magnetic field
46 (McIlwain, 1961). For a dipole magnetic field parameter L is a distance, in the equatorial plane,
47 from the given magnetic field line to the center of the dipole (in the Earth's radii R_E).

48 Outer and inner regions of the ionic ERBs formed and maintained in dynamic equilibrium with
49 the environment by the various mechanisms (see review Kovtyukh, 2018).

50 The outer belt ($L > 3$) of ions is formed mainly by the mechanisms of radial diffusion of the
51 particles of the hot plasma from the periphery regions of the magnetosphere into the geomagnetic
52 trap under the action of low-frequency fluctuations of electric and magnetic fields which resonate
53 with drift periods of the trapped particles. This transport accompanied by betatron acceleration of
54 ions, and by ionization losses and charge exchange of ions with atoms of the residual atmosphere.

55 The inner belt ($L < 2.5$) of protons with $E > 10$ MeV is forming mainly as result of decay of
56 neutrons knocked from the atomic nuclei of the atmosphere by the Galactic Cosmic Rays. For
57 protons with $E < 10$ MeV, this mechanism (CRAND) supplemented by the radial diffusion of
58 particles from the outer to the inner belt. The inner belt of ions with $Z > 4$ formed mainly from
59 ions of the Anomalous component of Cosmic Rays (ACR).

60 In the intermediate region ($2.5 < L < 3.5$) during strong magnetic storms is operated also the
61 mechanism of capture ions from the Solar Cosmic Rays (see, e.g., Selesnick et al., 2014).

62 Thus, the main mechanisms of formation of the ERB, sources and losses of these ions we know.
63 However, for the comprehensive verification of the physical models and to identification of the
64 mathematical models parameters it is necessary primarily to create sufficiently complete and
65 reliable empirical models of the ERB for each ion component.

66 These models one can create only on the base of experimental data obtained over many years
67 and decades. Such models (see, e.g., Ginet et al., 2013) created for protons (AP8/AP9). However,
68 measurements of fluxes of ions with $Z \geq 2$ represent a difficult technical problem, due to the small
69 fluxes of these ions and high background fluxes of protons and electrons. We facing here with the
70 problems of limited and incomplete information. Therefore, sufficiently complete empirical
71 models of the ERB for ions with $Z \geq 2$, similar to models for protons, we do not have, although
72 there are separate fragments of empirical and semi-empirical models for these ions. For example,
73 such models presented for the oxygen ions trapped in the ERB from the ACR (Selesnick et al.,
74 2000; Selesnick, 2001).

75 When constructing such models, it is necessary to coordinate the data of various experiments.
76 More or less significant discrepancies in the results of these experiments connected with
77 differences in the construction of instruments and in the trajectories of the satellites, differences in
78 the energy ranges and in the angular characteristics of the instruments, differences in the phase of
79 the solar cycle, and with many other physical factors. These factors influence on the fluxes of ions
80 with $Z \geq 2$ in the ERB more significantly than on the proton fluxes (see, e.g., Kovtyukh, 2018).



81 The creation of the ERB models for ion with $Z \geq 2$ is also necessary in connection with
82 estimates of the radiation hazard of space flights for humans and with the widespread introduction
83 of new radiation-sensitive technologies into the space equipment.

84 This paper consider some approaches to solving the problem of creation of the empirical models
85 of the ERB for ions with $Z \geq 2$ and presents the first model versions of the stationary ERB for
86 helium ions and for ions of the CNO group in the range $E \sim 0.1\text{--}100$ MeV/nucleon at $L \sim 2\text{--}6$.

87 In the following sections, we consider the experimental data on the spatial-energy structure of
88 the ERB for protons, helium ions and ions of the CNO group. We consider invariants of the
89 structure of the ERB (Sect. 2), distributions of ion fluxes in the spaces $\{E, L\}$ and $\{L, B/B_0\}$ and
90 solar-cyclic variations of these distributions (Sect. 3), physical mechanisms of these structure
91 formation and the values of its invariant parameters (Sect. 4). Section 5 concludes the paper.

92 2 Invariants of the ion Earth's radiation belts structure

93 Five invariant parameters of the stationary distributions of the ERB ions in the range $\mu = 0.01\text{--}50$
94 keV/nT at $L > 2\text{--}3$ were found as result of cross-analysis of satellite's data in (Kovtyukh, 1984,
95 1985a, 1985b, 1989, 1999a). For this analysis were used data of the satellites Explorer-12 (1961),
96 Explorer-14 (1962), Injun-4 (1965–1966), Explorer-33 (1966), Injun-5 (1968), ESRO-2 (1968),
97 1968-26B (1968), OVI-19 (1969), Molniya-1 (1970–1974), Explorer-45 (1971–1972), 1972-076B
98 (1972–1973), ATS-6 (1974), Molniya-2 (1975), ISEE-1 (1977–1979), SCATHA (1979),
99 AMPTE/CCE (1984-1986), Gorizont-21 (1985-1986), Akebono (1989-1991), CRRES (1991),
100 Gorizont-35 (1992) and ETS-VI (1994).

101 Invariant parameters of the ERB ion fluxes structure include the following quantities:

102 μ_m is corresponds to maximum at $E_m(L)$ in the energy spectra (exist only for solar origin ions);

103 μ_0 is corresponds to index $E_0(L)$ of the exponential part of the energy spectra at $E_m < E < E_b$: $J \propto$
104 $\exp(-E/E_0)$;

105 μ_b is corresponds to the boundary E_b between exponential spectral segment and a power-law tail;

106 γ is corresponds to exponent of the power-law tail of the energy spectra: $J \propto E^{-\gamma}$;

107 ξ_i is corresponds to the ratio of parameters μ_m , μ_0 , or μ_b for different ion components (scaling
108 parameter).

109 Specific values of μ_m and μ_b depends on geomagnetic and solar activity; sometimes the segment
110 $\mu_m < \mu < \mu_b$ degenerates, but usually it is clearly expressed and well approximated by the exponent
111 function.

112 These parameters of the ions radiation belts are invariants relatively L shells, and are displayed
113 not only in the energy spectra, but also in all other ion distributions by E , L and B/B_0 for all main
114 ion components of the ERB. Here B and B_0 are values of the magnetic field at the point of
115 measurement and in the equatorial plane on the same L shell. For each of the main ion components
116 of the ERB it was established that spatial, energy and pitch-angle distributions of fluxes are
117 connected to each other with by means of the invariant parameters μ_m , μ_0 , μ_b , γ and ξ_i (Kovtyukh,
118 1984, 1994, 1999a, 2001). These parameters are completely determines the spatial-energy structure
119 of the ERB ion fluxes in a wide region of the space $\{\mu, L\}$.

120 For protons with equatorial pith-angles $\alpha_0 = 90 \pm 50^\circ$, these parameters have the following
121 values:

122
$$\mu_m = 0.55 \pm 0.10 \text{ keV/nT},$$

123
$$\mu_b = 1.16 \pm 0.29 \text{ keV/nT},$$

124
$$\mu_0 = 0.29 \pm 0.10 \text{ keV/nT}, \text{ for } \mu_m < \mu < \mu_b,$$

125
$$\gamma = 4.25 \pm 0.75, \text{ for } \mu > \mu_b.$$



126 For helium ions and for CNO group ions with $\alpha_0 = 90 \pm 50^\circ$ these parameters have following
127 values:

128
$$\mu_m/\xi_i = 0.5 \pm 0.2 \text{ keV/nT},$$

129
$$\mu_b/\xi_i = 1.4 \pm 0.8 \text{ keV/nT},$$

130
$$\mu_0/\xi_i = 0.3 \pm 0.2 \text{ keV/nT}, \text{ for } \mu_m/\xi_i < \mu/\xi_i < \mu_b/\xi_i,$$

131
$$\gamma = 4.7 \pm 2.2, \text{ for } \mu/\xi_i > \mu_b/\xi_i.$$

132 According to Kovtyukh (1985a, 1985b, 1989, 1999a), parameters μ_0 , μ_b and γ applicable for ion
133 distributions only at $L > 3$, and parameters μ_m and ξ_i applicable only at $L > 3.5$. Such restrictions
134 connected with the ionization losses of ions.

135 The scatter of the values of these parameters exceed considerably the statistical errors of the
136 measurements and it is connected with the averaging the experimental data of many satellites for
137 thirty years. These measurements obtained in different phases of the solar activity, in different
138 ranges of E , α_0 , L and B/B_0 , by different devices which have different resolution, different periods
139 of accumulation and averaging of the data.

140 From minimum to maximum of solar activity, ξ_i changes from M_i to Q_i (M_i and Q_i are a mass
141 and charge of ions with respect to the corresponding values for protons), the exponential segment
142 of the spectra become softer (μ_0/ξ_i decreases in 1.5–2.0 times), and μ_b/ξ_i increases slightly
143 (Kovtyukh, 1999a).

144 Parameter γ depends slightly on B/B_0 , but μ_m , μ_0 and μ_b decreases with increasing B/B_0 at $L <$
145 6.6.

146 Usually, $\mu_b \approx \gamma\mu_0$ (the exponential and the power-law parts of the spectrum smoothly transfer one into
147 other) and $\mu_m \neq \mu_0$. Since for the Maxwellian distributions $\mu_m = \mu_0$ and parameter ξ_i should be close to
148 unity, which does not correspond to the experimental results, an exponential part of the ion spectra, as
149 well as a power-law tail, are non-equilibrium.

150 The simplest explanation of the exponential segment and maximum in the spectra of the ERB
151 ions is that they reflect, in some degree, the quasi-Maxwellian distributions of the ions in the solar
152 wind, the main source of the ERB ions. The power-law tail of the ions ERB spectra formed, most
153 likely, by statistical mechanisms of particle acceleration in the magnetosphere (see Sect. 4 for more
154 details).

155 3 Modelling of the spatial-energy structure of ion fluxes in the radiation belts

156 As examples of modeling of the ERB for ions with $Z \geq 2$, in this section are presented the
157 distributions of the fluxes of helium ions and ions of the CNO group in the spaces of variables $\{E,$
158 $L\}$ and $\{L, B/B_0\}$. Such presentations of fragmentary experimental data obtained in different ranges
159 of E , L and B/B_0 are the most capacious envelopes of these data and make it possible to organize
160 them most effectively. For comparison, there are presented also the distributions of the fluxes of
161 protons of the ERB, which constructed by the same method. All of these distributions based on the
162 satellite data averaged for quiet periods.

163 For these distributions, only reliable data on ion fluxes were used which obtained in those
164 regions of E , L and B/B_0 where these fluxes not distorted by the background of other particles. For
165 helium ions and for CNO group ions one have much smaller such data than for protons. In many
166 important experiments, the instruments did not allow separate fluxes of ions with $Z \geq 2$ by charge
167 of ions. For ions of the CNO group, separation by mass also were not performing usually. For
168 heavier ions, for example for Fe ions, we have even smaller such data. Therefore, this paper



169 presents only helium ions (without separating them by charge) and ions of CNO group (without
170 separating them by mass and charge).

171 Figures 1–6 shows the experimental results for differential fluxes of protons, helium ions, and
172 ions of the CNO group in the ERB near the plane of the geomagnetic equator, averaged for quiet
173 geomagnetic conditions and presented in space $\{E, L\}$. The values E and L presented in the
174 logarithmic scales. The ion fluxes J have a dimension $(\text{cm}^2 \text{ s ster M}\ddot{\text{a}}\text{B/n})^{-1}$ and corresponds to the
175 energies E (MeV/n) and the equatorial pitch-angle $\alpha_0 \sim 90^\circ$. In some cases in these figures, an
176 average ion energies in the instrument channels corrected for the steepness of corresponding
177 energy spectra. For the data on these figures, lines of equal intensity of ion fluxes are plotted (by
178 the least squares method), and a decimal logarithms of the fluxes are shown near each line.

179 There can be trapped on the drift shell only ions with energy less than some maximum values
180 determining by the Alfvén's criterion: $\rho_i(L, E, M_i, Q_i) \ll R_c(L)$, where ρ_i is the gyroradius of ions,
181 and R_c is the radius of curvature of the magnetic field near the equatorial plane. According to this
182 criterion and the theory of stochastic motion of particles, a geomagnetic trap can capture and
183 durably hold only ions with E (MeV) $< 2000 \times (Q_i^2/M_i) L^{-4}$ (Ilyin et al., 1984). This boundary, for
184 protons and atomic nuclei, presented in Figs. 1–6 by the green line.

185 Figures 1 and 2 shows exp results for proton fluxes in space $\{E, L\}$ averaged for quiet periods.

186 Figure 1 presents a results of the satellites 1968–81A (Stevens et al., 1970), Injun-5 (Krimigis,
187 1970; Venkatesan and Krimigis, 1971; Pizzella and Randall, 1971), OV1-19 (Croley et al., 1976),
188 Azur (Hovestadt et al., 1972; Westphalen and Spjeldvik, 1982), Molniya-1 (Panasyuk and
189 Sosnovets, 1973), GEOS-2 (Wilken et al., 1986), CRRES (Albert et al., 1998; Vacaresse et al.,
190 1999), GEO-3 (Selesnick et al., 2010) and Van Allen Probes (Selesnick et al., 2014, 2018),
191 obtained near maxima of solar activity in 20th, 22th, 23th and 24th solar cycles (1968–1971, 1990–
192 1991, 2000, 2012–2017).

193 Figure 2 presents a results of the satellites Relay-1 (Freden et al., 1965), OHZORA, ETS-VI and
194 Akebono (Goka et al., 1999), obtained near minima of solar activity between 19th and 20th, 21th and
195 22th, 22th and 23th solar cycles (1963, 1984–1985, 1994–1996).

196 The data of the satellites Explorer-45 (Fritz and Spjeldvik, 1979, 1981; Spjeldvik and Fritz,
197 1983) and ISEE-1 (Williams, 1981; Williams and Frank, 1984) refer to the years with intermediate
198 solar activity (1971–1972, 1977–1978) and at $L > 2.5$ are used in Figs. 1 and 2. The dependence of
199 proton fluxes on solar activity variations rapidly decreases with increasing L , and at $L > 2$ these
200 variations practically does not show in the satellite data (see, e.g., Vacaresse et al., 1999).

201 From the comparison of Figs. 1 and 2, one can see that at $L < 2.5$, and especially on $L < 1.4$,
202 the proton fluxes in the minima of solar-cyclic variations (Fig. 2) are higher than in the maxima of
203 solar activity (Fig. 1). Moreover, at the minima of solar activity, the inner edge of a proton flux
204 radial profiles with $E > 1$ MeV is less steep and achieves smaller L shells.

205 Solar-cyclic (11-year) variations of proton fluxes with $E > 1$ MeV in the inner region of the
206 ERB connected mainly with the variations in the concentrations of atoms in the atmosphere (see,
207 e.g., Pizzella et al., 1962; Hess, 1962; Blanchard and Hess, 1964; Filz, 1967; Nakano and
208 Heckman, 1968; Vernov, 1969; Dragt, 1971; Huston et al., 1996; Vacaresse et al., 1999;
209 Kuznetsov et al., 2010; Qin et al., 2014). These variations achieves one order of magnitude at $L =$
210 1.14 and reduced rapidly with increasing L (see, e.g., Vacaresse et al., 1999). The shape of the
211 proton energy spectra also undergo by solar-cyclic variations in the inner region of the ERB.

212 The atmospheric density depends on the intensity of the ultraviolet radiation of the Sun and
213 determines the loss rates of the ERB protons. Decreases in the amplitude of these variations with L
214 is connected with increases in the lifetime of protons with increasing L ; at $L > 2$, this time
215 approach to the main period of the solar cycle. In these variations expressed some inertness of the
216 changes in the atmospheric density, and this lag increases with increasing L .

217 In Figs. 1 and 2, the red line shows the proton energy values corresponding to the average value
218 of the invariant μ_b , and the blue line corresponds to the average value of the invariant μ_m ; these



219 values obtained from experimental data in (Kovtyukh, 1984, 1985a, 1985b, 1989, 1999a) and
220 presented in Sect. 2. These lines do not distinguished in Figs. 1 and 2, since the solar-cyclic
221 variations of the ion fluxes in the outer belt (at $L > 2.5$) are significantly weaker than in the inner
222 belt (see, e.g., Vacaresse et al., 1999). The maximum deviations from the mean values of μ_b and μ_m
223 plotted on the red and blue lines with vertical segments; on a logarithmic scale, the magnitudes of
224 these segments do not depend on L shell.

225 The isolines of the proton fluxes in Figs. 1 and 2, at $L > 3$ above the red line ($\mu > \mu_b$), go almost
226 parallel to it and are separated from each other approximately equal distances on a logarithmic
227 scale of energy. This conforms to adiabatic transformations of fluxes for the energies
228 corresponding to the power-law tail of the proton spectra. It results from these figures that in the
229 region of a near-dipole magnetic field, at $L = 3-6$, the parameter $\gamma = 4.8 \pm 0.5$. At $L > 6$, the
230 distance between these isolines increases with L , and the parameter γ decreases, from $\sim 4.7-5.0$ at
231 $L = 6$ to $\sim 4.1-4.5$ at $L = 8$. This is due to the deviation of the magnetic field from the dipole
232 configuration (L shells correspond here to the dipole magnetic field), as well as to the increasing
233 variability of this field with increasing L . In the interval between the red and blue lines ($\mu_m < \mu <$
234 μ_b), the spectra have a close to exponential form and corresponds to $\mu_0 \sim 0.32$ keV/nT.

235 Thus, the values of the invariant parameters μ_b , μ_0 and γ of the proton flux distributions in the
236 ERB, obtained from Figs. 1 and 2, are in good agreement with the values of these parameters given
237 in Sect. 2 and obtained in (Kovtyukh, 1985a, 1985b, 1989, 1999a) by other methods and with other
238 set of the experimental data.

239 However, a data representation, accepted here, does not allow determine the values of the
240 parameter μ_m for protons of the ERB and compare these with the values given in Sect. 2. For this,
241 it is necessary to reduce the step between the isolines of the fluxes ($\Delta \log J$) of protons with $E < 1$
242 MeV at $L > 3.5$ by ~ 10 times, which leads to significant systematic errors in such representation of
243 the data.

244 In addition, it must take into account that the ionization loss mechanisms generates in the proton
245 spectrum of the ERB also a less energetic maximum at $L < 5.5$ (this maximum considered in
246 details in Kovtyukh, 1989). Its energy increases from $\sim 0.01-0.03$ MeV at $L = 5.3$ to ~ 0.4 MeV at
247 $L = 3.2$ (the values of the adiabatic maximum on these L are ~ 0.11 MeV and ~ 0.52 MeV,
248 respectively). These two maxima in the spectra of protons at $L \sim 3-5$ separated by a small local
249 minimum and, at insufficiently high resolution of the spectrometers, have a view an extended
250 plateau in the spectra.

251 According to the experimental data considered in (Kovtyukh, 1985a, 1985b, 1999a), it was found
252 that the parameters μ_b and γ are detected only at $L > 3$. Here are considered more complete data of
253 satellites, and these parameters for protons with $E > 10$ MeV can be traced to $L \sim 2$: at $L = 2$, $\gamma = 4.7 \pm$
254 1.3 (Fig. 1) and $\gamma = 4.4 \pm 0.6$ (Fig. 2). This is due to the fact that, compare with (Kovtyukh, 1985a,
255 1985b, 1999a), the energy range here is significantly extended toward higher energies, but the
256 ionization losses rapidly decreases with increasing of the energy of the ERB protons (see, e.g., Schulz
257 and Lanzerotti, 1974; Kovtyukh, 2016a, 2016b).

258 Figure 2 show that at very high energies the proton spectra tail becomes steeper, which
259 corresponds to the limit of the magnetic confinement of protons in the ERB.

260 Figures 3 and 4 show averaged stationary fluxes of helium ions in the space $\{E, L\}$.

261 Figure 3 presented the data of the satellites OV1-19 (Blake et al., 1973; Fennell and Blake
262 1976), Explorer-45 (Fritz and Spjeldvik, 1978, 1979; Spjeldvik and Fritz, 1981) and SCATHA
263 (Blake and Fennell, 1981; Chenette et al., 1984), obtained near maxima of solar activity in 20th and
264 21th solar cycles (1968–1971, 1979).

265 Figure 4 presented the data of the satellites Molnija-2 (Panasyuk et al., 1977), Prognoz-5
266 (Lutsenko and Nikolaeva, 1978), ISEE-1 (Hovestadt et al., 1981) and Akebono (Goka et al., 1999),
267 obtained near minima of solar activity between 20th and 21th and between 22th and 23th solar cycles
268 (1975–1977, 1996).



269 From the comparison of Figs. 1–4, it can see that for helium ions with $E > 1$ MeV/n at $L \sim 2$ –3
270 an amplitude of the solar-cyclic (11-year) variations is more than for protons. This difference
271 connected with the ionization losses of the ERB ions: for helium ions these losses more than for
272 protons.

273 In Figs. 3 and 4, the red line shows an energy of the helium ions corresponding to the average
274 value of the invariant μ_b/ξ_i , and the blue line corresponds to the average value of the invariant
275 μ_m/ξ_i , which are represent in Sect. 2. Here $\xi_i = Q_i$ for Fig. 3 and $\xi_i = M_i$ for Fig. 4. It was taken into
276 account that for $E > 0.2$ MeV/n at $L < 6$ the average (main) charge of helium ions is $Q_i = +2$ (see,
277 e.g., Spjeldvik, 1979). The maximum deviations from the mean values of μ_b/ξ_i and μ_m/ξ_i at energy
278 scale plotted on the red and blue lines with vertical segments; on a logarithmic energy scale, the
279 magnitudes of these segments do not depends on L shell. The isolines of the helium ions fluxes in
280 Figs. 3 and 4 at $L > 2$ pass above the red line almost parallel to it, and average value of the
281 parameter $\gamma \sim 5.5$.

282 Thus, the values of the invariant parameters of the power-law tail of the spatial-energy flux
283 distributions of helium ions in the ERB, obtained from Figs. 3 and 4 are in good agreement with
284 the values of these parameters given in Sect. 2 and obtained in (Kovtyukh, 1999a) by other
285 methods and with other compositions of experimental data. To find the values of the parameters
286 μ_m/ξ_i and μ_b/ξ_i from Figs. 3 and 4, it is necessary to reduce the step between the isolines of the ion
287 fluxes significantly, which leads to large systematic errors in such representation of the data.

288 Figures 5 and 6 show the results for the average stationary fluxes of ions of the CNO group in
289 the space $\{E, L\}$.

290 Figure 5 presented the data of the satellite Explorer-45 (Spjeldvik and Fritz, 1978; Fritz and
291 Spjeldvik, 1981), obtained near maximum of solar activity in 20th solar cycle (1971–1972).

292 Figure 6 presented the data of the satellites ATS-6 (Spjeldvik and Fritz, 1978; Fritz and
293 Spjeldvik, 1981) and ISEE-1 (Hovestadt et al., 1978), obtained near the minimum of the solar
294 activity between 20th and 21th solar cycles (1974–1975, 1977).

295 In Figs. 5 and 6, the red line shows the CNO ions energy values corresponding to the average
296 value of the invariant μ_b/ξ_i , and the blue line corresponds to the average value of the invariant
297 μ_m/ξ_i , which presented in Sect. 2. Here $\xi_i = Q_i$ for Fig. 5 and $\xi_i = M_i$ for Fig. 6. It was taken into
298 account that for $E > 0.1$ MeV/n at $L \sim 3$ –5 the average charge of the CNO group ions is $Q_i = +4$
299 (see, e.g., Spjeldvik and Fritz, 1978). The maximum deviations from the mean values of μ_b/ξ_i and
300 μ_m/ξ_i at energy scale plotted on the red and blue lines with vertical segments; on a logarithmic
301 energy scale, the magnitudes of these segments do not depends on L shell. The isolines of the CNO
302 ions fluxes in Figs. 5 and 6 at $L > 3$ pass almost parallel to each other.

303 However, at the maximum of solar activity their slope on $L > 3$ is significantly less than the
304 slope of the red line, which indicates more significant ionization losses of ions of the CNO group
305 at $L = 3$ –5 compared to these losses for protons and for helium ions. According to these results, at
306 $L \sim 3$ –6 for ions of the CNO group, the average value of the parameter $\gamma \sim 6$.

307 Thus, the values of the invariant parameters of the power-law tail of the spatial-energy flux
308 distributions of the CNO group ions in the ERB, obtained from Figs. 5 and 6, are in good
309 agreement with the values of these parameters given in Sect. 2 and obtained in (Kovtyukh, 1999a)
310 by other methods and with other set of experimental data. To find the values of the parameters
311 μ_m/ξ_i and μ_b/ξ_i from Figs. 5 and 6, it is necessary to reduce the step between the isolines of the ion
312 fluxes significantly, which leads to large systematic errors in such representation of the data.

313 As well as for protons (Figs. 1 and 2), for helium ions (Figs. 3 and 4) and CNO group ions
314 (Figs. 5 and 6), with $E > 0.1$ MeV/n at small L , fluxes during solar minimum more, than during
315 solar maximum. The larger the atomic number Z of the ERB ions, the greater the amplitude of
316 these variations. The solar-cyclic variations of ion fluxes explained by the same mechanism of
317 ionization losses of particles, which proposed for the ERB protons (see above). The main loss



318 mechanism of ions at $E > 0.1$ MeV/n at $L < 3.5$ are Coulomb losses, for which the loss rate
319 increases rapidly with increasing Z of the ions (as Z^2).

320 Figs. 7–9 show the experimental results for differential fluxes of protons and helium ions in the
321 ERB, averaged for quiet geomagnetic conditions and presented in the space $\{L, B/B_0\}$ for different
322 ion energies (as examples). The values of L and B/B_0 are plotted on logarithmic scales. For these
323 results, the lines of equal intensity of ion fluxes are made, and decimal logarithms of the fluxes are
324 shown near each of these line.

325 Figure 7 presents stationary fluxes of protons with $E = 0.4$ MeV, averaged for quiet periods by
326 the data of the satellites Injun-5 (Krimigis, 1970; Venkatesan and Krimigis, 1971; Pizzella and
327 Randall, 1971), Molniya-1 (Panasyuk and Sosnovets, 1973) and GEOS-2 (Wilken et al., 1986).
328 These data obtained near the maxima of solar activity in 20th and 21th solar cycles (1968–1970,
329 1978).

330 Figure 8 presents stationary fluxes of protons with $E = 0.4$ MeV, averaged for quiet periods by
331 the data of the satellites 1964-45A (Mihalov and White, 1966), ISEE-1 (Williams, 1981; Williams
332 and Frank, 1984) and Polar (Walt et al., 2001). These data obtained near the minima of the solar
333 activity between 19th and 20th, between 20th and 21th and between 22th and 23th solar cycles (1964,
334 1977, 1998).

335 From Figs. 7 and 8 it can be seen, that for protons with $E = 0.4$ MeV during solar activity
336 minima fluxes are greater than during solar activity maxima, at the same points in the space $\{L,$
337 $B/B_0\}$; most significantly this discrepancy in proton fluxes is observed at $B/B_0 > 100$ (this is also
338 valid for other proton energies). This effect connected with the solar-cyclic variations of the
339 Earth's upper atmosphere temperature, as for protons of the ERB near the equatorial plane.

340 Figure 9 presents stationary fluxes of helium ions with $E = 0.2$ MeV/n, averaged for quiet
341 periods by the data of satellite Polar (Spjeldvik et al., 1999) which were obtained near the
342 minimum of the solar activity between 22th and 23th solar cycles (1996). For equatorial plane (B/B_0
343 = 1) we used data of satellite Explorer-45 presented in (Fritz and Spjeldvik, 1978, 1979; Spjeldvik
344 and Fritz, 1981).

345 From Figs. 7–9 it seen that with increasing B/B_0 the maximum of the ion fluxes shifts to a
346 higher L . This performed also for other ion energies in the range of 0.1–100 MeV/n and connected
347 with increasing in ionization losses and decreasing in the radial diffusion rate of the ERB ions with
348 increasing B/B_0 .

349 With decreasing altitude of the observation point (at a given L), the concentration of atoms and
350 ionization losses increases, which lead to the formation an altitude dependence of the ion fluxes:
351 with decreasing altitude (at fixed L and E), ion fluxes of the ERB are decreases. With decreasing L ,
352 this dependence enhances and, consequently, at low altitudes (at $h \sim 500$ –1000 km) a maximum of
353 the ERB should be located at larger L compared to the equatorial plane (see Figs. 7–9).

354 When the exosphere is heated, the altitude dependence of a concentration of atoms becomes
355 weaker. Therefore, the difference in the position of the ERB maximum in the equatorial plane and
356 at low altitudes decreases (see Figs. 7 and 8).

357 Since reliable experimental data on helium ions in the ERB are insufficient, the distributions of
358 their fluxes in space $\{L, B/B_0\}$, especially at higher energies, have more or less significant lacunae.
359 The most complete distributions of these ions in the space $\{L, B/B_0\}$ obtained only for $E < 1$
360 MeV/n in the minimum of solar activity. On ions of the CNO group one have even less reliable
361 and complete data. The distributions of these ions in the space $\{L, B/B_0\}$ represent a very
362 variegated picture. However, it is possible to conclude from these incomplete data that for higher
363 helium ion energies, as well as for ions of the CNO group, the pattern of flux distribution in space
364 $\{L, B/B_0\}$ is similar in form to that shown in Figs. 7–9.

365 Patterns of isolines of fluxes of the ERB ions in Figs. 1–9 are almost identical in shape for
366 various ionic components, especially for more complete data of protons and helium ions on $L > 2$,
367 and with increasing E and L the degree of such similarity increases. Some deviations of these



368 isolines from the overall picture for ions with $Z > 2$ are mainly due to the deficiency of
369 experimental data. This similarity has a basis in the unity of the main source (solar wind) and on
370 the unity mechanisms of transfer, acceleration and losses of ERB ions (radial diffusion, betatron
371 acceleration and ionization losses).

372 The absence of ions with $Z \geq 2$ at $L < 2$ (or very low values of these fluxes) is explained the fact
373 that their lifetimes, determined by ionization losses, are much less than the same times for protons.
374 Besides, for protons at $L < 2$ there is an additional source, CRAND (for more on this, see
375 Kovtyukh, 2018).

376 4 Discussion

377 We consider here the main consequences of these results for the physics of the magnetosphere.

378 Since parameters μ_b and γ of the power-law tail of the ion spectra are invariant with respect to
379 L , it can be assumed that this part of the ERB ion spectra formed in the plasma sheet (PS) in the
380 tail of the magnetosphere which is adjacent to the geomagnetic trap. High-energy part of the ion
381 spectra in the PS, at $R \sim 20\text{--}40 R_E$, has approximately the same shape as in the ERB and the
382 average values of the parameters μ_b and γ are close to our estimates of these parameters for the
383 ERB. Moreover, the proton spectra in the ERB are consistent with the PS spectra not only in form
384 but also in absolute fluxes: they can be obtain from the PS ion spectra in result of the simplest
385 adiabatic transformations (for more details see in Kovtyukh, 1999b, 2001).

386 According to the data of the satellites IMP-7, IMP-8 (Sarris et al., 1981; Lui et al., 1981) and
387 also ISEE-1 (Christon et al., 1991) for ion spectra of the PS, the typical substorms not changed of
388 the spectral shape and cause only parallel shift of the spectra along logarithmic axes E and J (for
389 ions with $\mu/\xi_i > 0.5$ keV/nT). These results are point out that the time scales of the processes of
390 formation of the power-law tail of the ion spectra in the PS far exceed the characteristic times of
391 substorms.

392 Invariant parameters γ and μ_b of the power-law tail of the ion spectra reflect, apparently, the
393 most fundamental features of the mechanisms of acceleration of ions in the tail of the
394 magnetosphere. One can try to connect the obtained values of these parameters with the most
395 general presentations on the mechanisms and character of ion acceleration in the PS of the
396 magnetospheric tail.

397 Most likely, the tail of ion energy spectra of the PS formed by statistical mechanisms of the ion
398 acceleration. This supported by many experimental results.

399 Statistical character of these mechanisms reveal itself, in particular, in the fact that the ratios of
400 fluxes (and partial concentrations) of ions with different Z at low and high energies can differ
401 greatly. During wandering in phase space, ions gradually forget their origin, and, therefore, the
402 high-energy tails of the ion spectra do not contain unambiguous information on the partial
403 concentrations of different components of ions in there source (for more details, see Kovtyukh,
404 1999b, 2001).

405 Most likely, the high-energy part of the ion spectra of the PS formed by the mechanisms of
406 acceleration of particles on magnetic irregularities moving relative to each other (Fermi
407 mechanism). If mass of the ions are small compare to masses of the magnetic irregularities in the
408 PS, the average values of the exponent γ of the power-law tail of the spectra should not depend on
409 mass and charge of these ions.

410 Under equilibrium conditions, this parameter determined by the average fraction $\bar{\beta}$ of energetic
411 ions in the total energy density of particles and magnetic irregularities. From the theory developed
412 on these fundament by Ginzburg and Syrovatskii (1964), it follows: $\gamma - 1 \approx (1 - \bar{\beta})^{-1}$. With
413 increasing $\bar{\beta}$ in the interval $0 < \bar{\beta} < 1$, the value γ monotonically increases and $\gamma \rightarrow \infty$ for $\bar{\beta} \rightarrow$
414 1. For real average values $\bar{\beta}$ in the central PS, we get $\gamma \sim 3.5\text{--}7.0$ ($\gamma \sim 4.3$ at $\bar{\beta} \sim 0.7$).



415 Compared with the power-law tail, a short quasi-exponential segment of the ion spectra allows
416 several different interpretations. Remaining within the framework of the most general physical
417 concepts about the mechanisms of acceleration of cosmic plasma particles, the presence of this
418 segment in the ion spectra of the PS and ERB one can explain in the terms of the quasi-particles
419 theory. The structure of the magnetic field of the PS one can represent as quasi-periodic spatial
420 grilles with different periods nested into each other (fractality) and as an energy of ions increases,
421 grilles with more and more long periods gradually fall out from the process of acceleration of ions.
422 Then the upper boundary of the exponential segment of the spectra corresponds to ions for which
423 all small-scale grilles are transparent: ions with $\mu > \mu_b$ detect only the most large-scale grille. The
424 fractal topology of the PS on scales from ~ 0.4 to ~ 8 thousand km reveal itself, for example, in the
425 results of the satellite Geotail (Milovanov et al., 1996).

426 Spectra with a power-law tail and a quasi-exponential segment at lower energies generate when
427 a value $\Delta B / \bar{B}$ for magnetic irregularities is proportional to their size δr and at $\delta r < r_s$ the spectral
428 density of irregularities rapidly decreases with increasing δr , and at $\delta r > r_s$ remains almost
429 unchanged. Apparently, the spectrum of magnetic irregularities in PS with thickness r_s has just
430 such form.

431 Then the lower boundary μ_b of the power-law tail corresponds to the condition $r_s / \rho_i \sim 10$ (ρ_i is
432 the gyroradius of ions), i.e. $\mu_b \sim 0.02(Q_i^2/M_i)B_s r_s^2$ keV/nT, where B_s is the average magnetic field
433 induction in the PS (in nT) and r_s is normalized to the Earth's radius. Believing $B_s \sim 30$ nT and $r_s \sim$
434 $1.3 R_E$, one obtain $\mu_b \sim 1.0 (Q_i^2/M_i)$ keV/nT (for details, see Kovtyukh, 1999b).

435 Thus, the main invariant parameters of the ERB structure one can relate with the average
436 physical properties of the PS.

437 Parameter γ one can relate with the fraction of energetic ions in the total energy density of
438 particles and of a magnetic irregularities in the PS (Kovtyukh, 1999b, 2001).

439 Parameter μ_b one can relate with a thickness of the PS and a magnitude of its magnetic field
440 (Kovtyukh, 1999b, 2001).

441 Parameter μ_0 one can relate with a small-scale structure (spectrum of turbulence) of the PS
442 (Kovtyukh, 1999b, 2001).

443 Parameter μ_m is well corresponded to maximum in the PS ion spectra and in solar wind
444 (Kovtyukh, 1989, 2001).

445 5 Conclusions

446 In this work, it was found that results of the measurements of the stationary fluxes of the main ion
447 components of the ERB (protons, helium ions and ions of the CNO group) line up in the certain
448 regular patterns in the spaces $\{E, L\}$ and $\{L, B/B_0\}$. It is reveal that such patterns is associated with
449 the existence of invariant parameters of the spatial-energy distributions of the ERB ion fluxes and
450 the values of these parameters are determined.

451 Earlier, the results of systematization of the spatial-energy distributions of the main ionic
452 components of the ERB and finding of their invariant parameters were presents in (Kovtyukh,
453 1985a, 1985b, 1999a, 2001). Here, the experimental database is significantly expanded, many
454 modern measurements of the ion fluxes of the ERB have been added, and all results are presented
455 in a more general and visual form.

456 Solar-cyclic (11-year) variations of the spatial-energy distributions of the ERB ion fluxes and
457 their invariant parameters are considered. It is shown that the larger an atomic number Z of the
458 ERB ions, the greater the amplitude of these variations. This is also typical for faster variations in
459 the fluxes of the ERB ions, during geomagnetic storms and other disturbances of the Earth's
460 magnetosphere, which is underlined in the review (Kovtyukh, 2018).



461 The results presented here show that when constructing realistic multicomponent models of the
462 ERB ion fluxes based on limited and incomplete experimental data, the invariant parameters of the ion
463 distributions of the ERB can serve as a pattern of these fluxes distributions.

464 Our drawings have also revealed the localization of “white spots”, especially extensive for ions
465 with $Z \geq 6$, which should be filled on the results of the future experiments on the satellites.

466 The physical mechanisms leading to the formation of the invariant structure of the ERB are
467 considered. It is shown that energy spectra of the ERB ions with $\mu/\xi_i > 0.5$ keV/nT can be
468 generated in the plasma sheet (PS) of the tail of the Earth’s magnetosphere. In the geomagnetic
469 trap these spectra transforms adiabatically, and ions are loss part of their energy by ionization and
470 other loss mechanisms.

471 **Acknowledgements** This work was supported by Russian Foundation for Basic Research RFFI
472 grant No. 17-29-01022.

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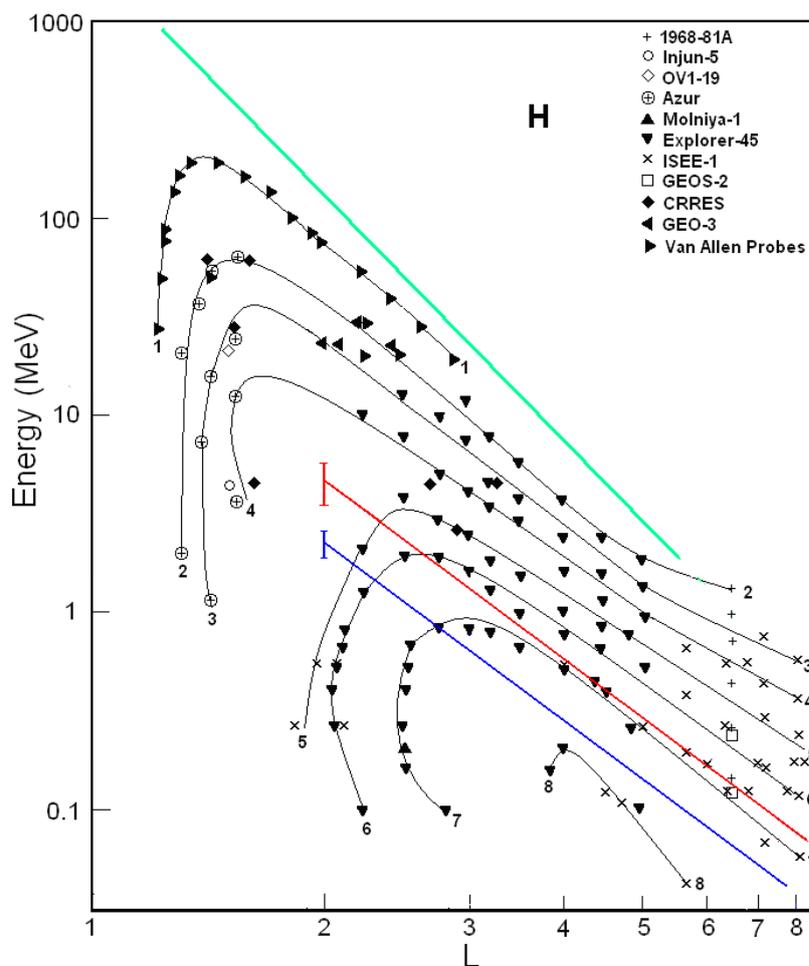
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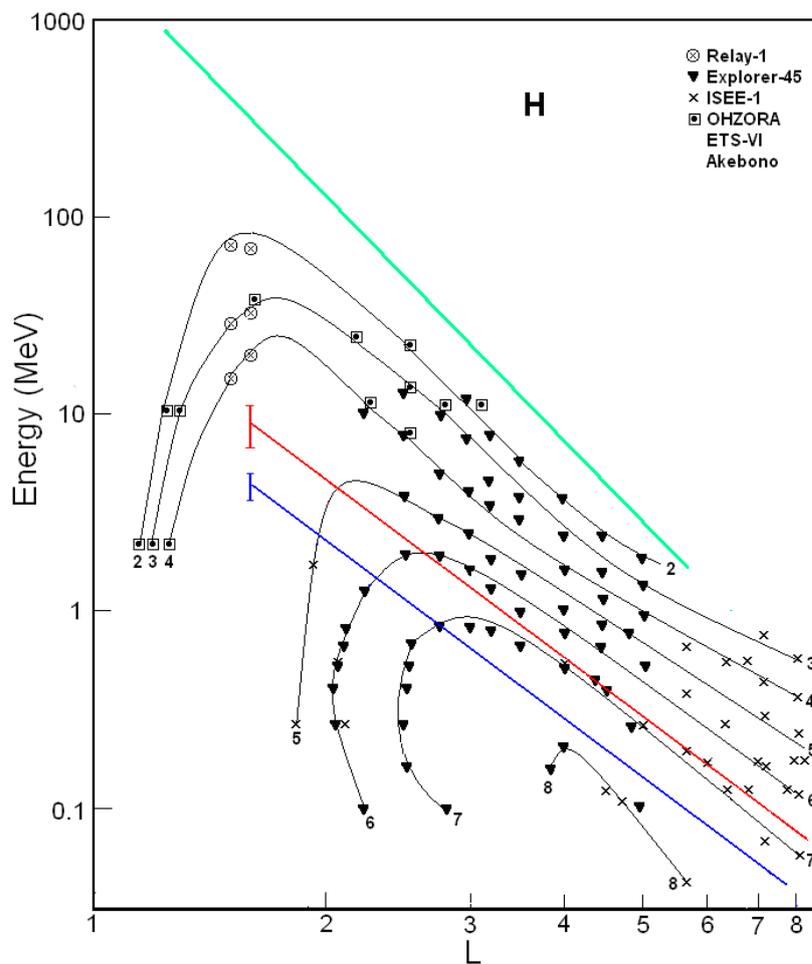
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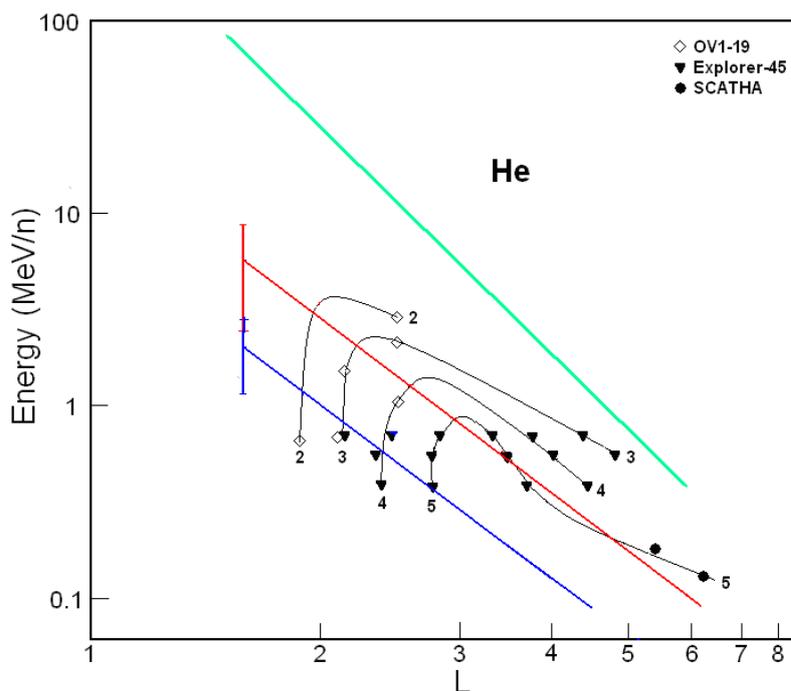
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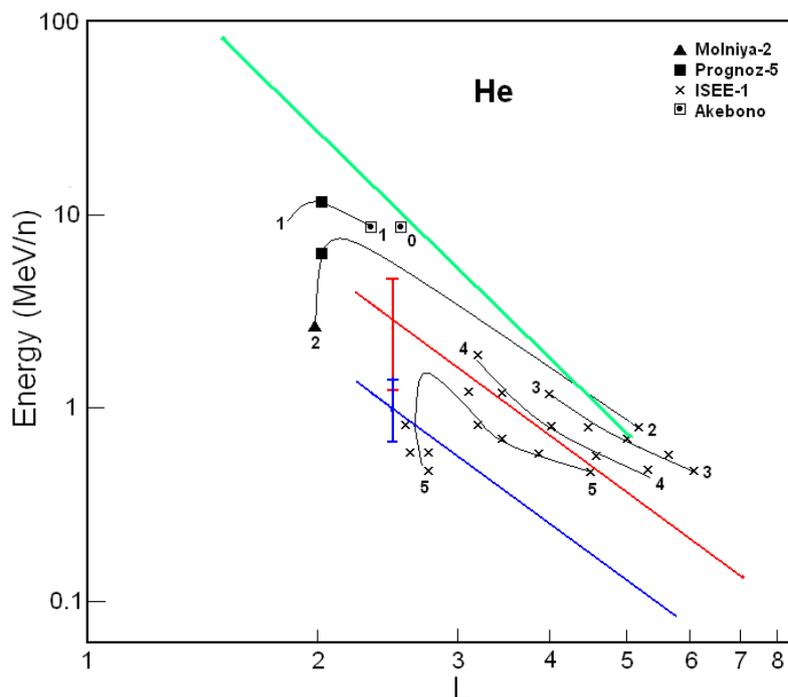
664
 665 **Figure 1.** Proton fluxes in the ERB near maxima of a solar activity. A numbers on the curves are equal the values of a
 666 decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\text{\AA}\text{B})^{-1}$ are differential fluxes of protons with $\alpha_0 \approx 90^\circ$. The data of
 667 different satellites presented by different symbols.



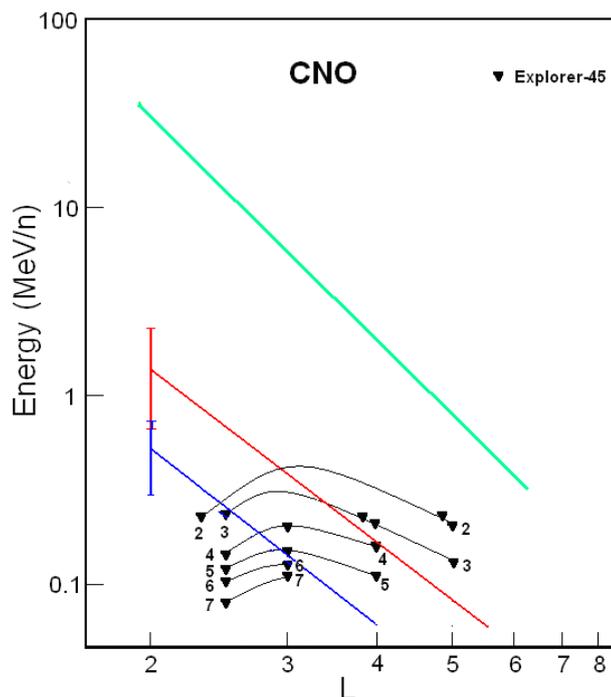
668
669 **Figure 2.** Proton fluxes in the ERB near minima of a solar activity. A numbers on the curves are equal the values of a
670 decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\text{\AA}\text{B})^{-1}$ are differential fluxes of protons with $\alpha_0 \approx 90^\circ$. The data of
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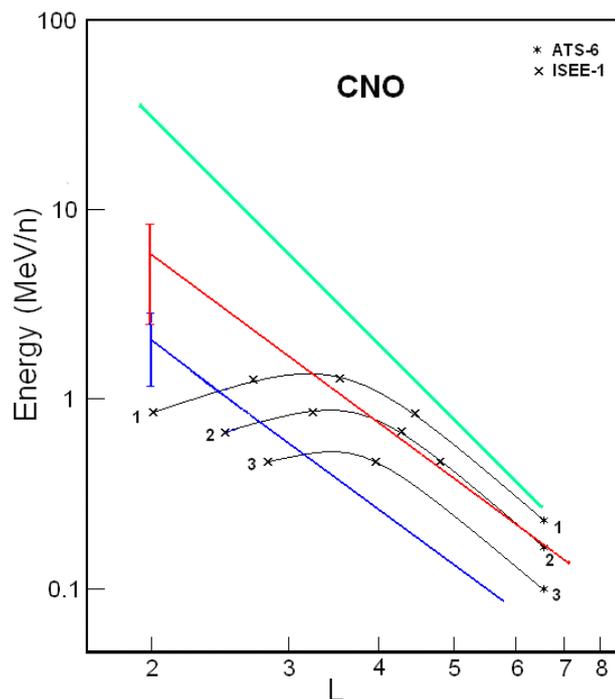
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 673 **Figure 3.** Helium ion fluxes in the ERB near the maxima of a solar activity. A numbers on the curves are equal the
 674 value of a decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\text{\AA}\text{B/n})^{-1}$ are differential flux of protons with $\alpha_0 \approx 90^\circ$ (near
 675 the plane of the geomagnetic equator). The data of different satellites presented by different symbols.



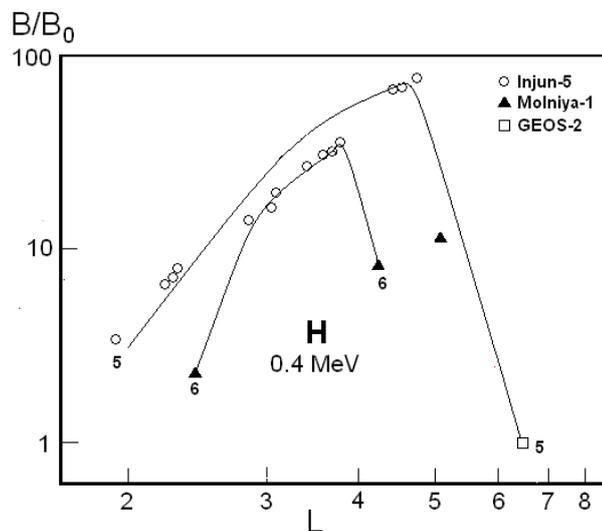
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 677 **Figure 4.** Helium ion fluxes in the ERB near the minima of a solar activity. A numbers on the curves are equal the
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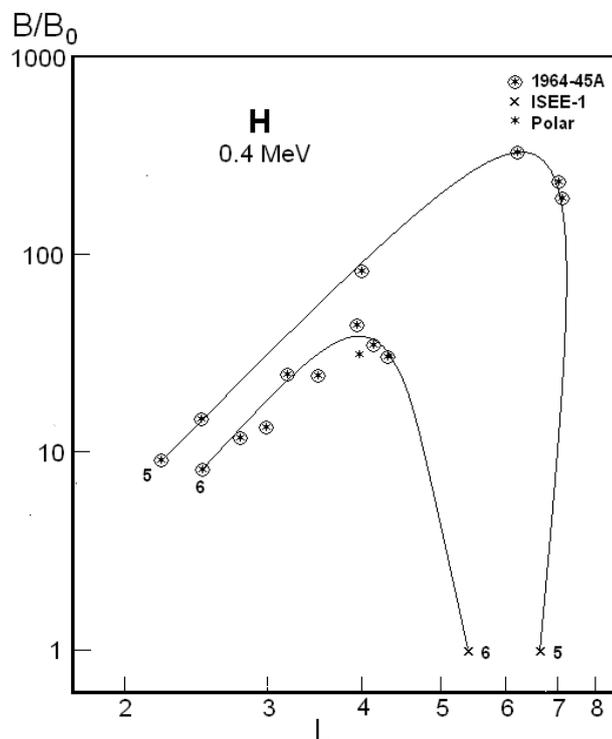
680
 681 **Figure 5.** CNO ion fluxes in the ERB near the maximum of a solar activity. A numbers on the curves are equal the
 682 values of a decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\text{\AA}\text{B/n})^{-1}$ are differential flux of protons with $\alpha_0 \approx 90^\circ$ (near
 683 the plane of the geomagnetic equator). The data of different satellites presented by different symbols.



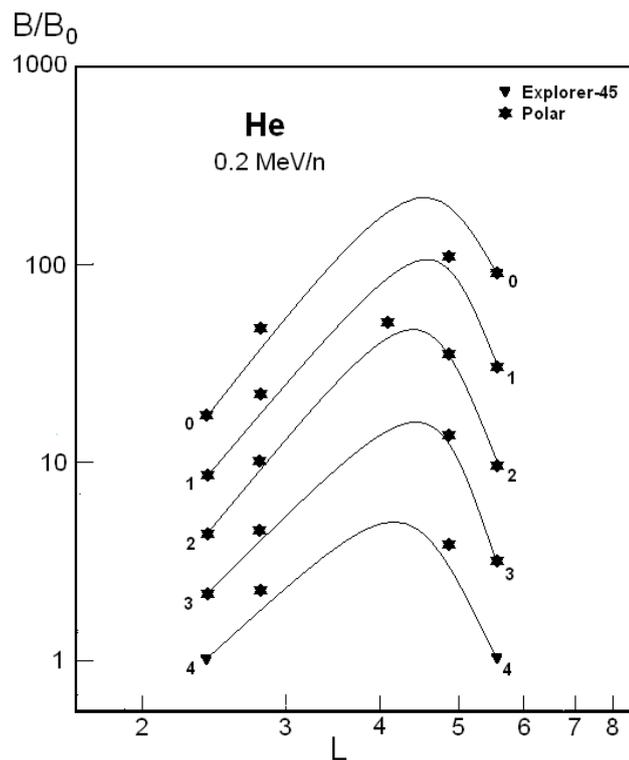
684
 685 **Figure 6.** CNO ion fluxes in the ERB near the minimum of a solar activity. A numbers on the curves are equal the
 686 values of a decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\text{\AA}\text{B/n})^{-1}$ are differential flux of protons with $\alpha_0 \approx 90^\circ$ (near
 687 the plane of the geomagnetic equator). The data of different satellites presented by different symbols.



688
 689 **Figure 7.** Average stationary fluxes of protons with $E = 0.4$ MeV in space $\{L, B/B_0\}$ near the maxima of solar activity.
 690 A numbers on the curves are equal the values of a decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\mathring{\text{A}}\text{B/n})^{-1}$. The data of
 691 different satellites presented by different symbols.



692
 693 **Figure 8.** Average stationary fluxes of protons with $E = 0.4$ MeV in space $\{L, B/B_0\}$ near the minima of solar activity.
 694 A numbers on the curves are equal the values of a decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster M}\mathring{\text{A}}\text{B/n})^{-1}$. The data of
 695 different satellites presented by different symbols.



696
697 **Figure 9.** Average stationary fluxes of helium ions with $E = 0.2 \text{ MeV/n}$ in space $\{L, B/B_0\}$ near the minimum of a
698 solar activity. A numbers on the curves are equal the values of a decimal logarithms of J where J in $(\text{cm}^2 \text{ s ster}$
699 $\text{M}^3 \text{B/n})^{-1}$. The data of different satellites presented by different symbols.