

Final author comments on the manuscript “Invariants of the Spatial-Energy Structure and Modeling of the Earth’s Ion Radiation Belts” by Alexander S. Kovtyukh for Anonymous Referee #2

I am very grateful to Referee #2 for very helpful comments on the manuscript. All these comments are taken into account in the revised manuscript.

General comments

The review written by the author and published last year in the journal Space Science Reviews (SSR) concluded that there is a need for the development of radiation belt heavy ion empirical models. The submitted paper gathers available heavy ion measurements obtained all over the space age and tries to pave the way for the development of such models. To do so, invariant parameters (that are constant over a given range of L) are detailed and available measurements are shown on Figures and discussed in detail. The solar cycle variation of heavy ion fluxes is for the first time explored in the submitted article. Finally, the presented measurement database is important to explore the physical mechanisms that govern the heavy ion radiation belts, what is done by the author in sections 3 and 4. Comparative lessons with what is known for the protons are drawn.

There is no doubt that the work presented here is important and may contribute to advances in our understanding and prediction of the heavy ion radiation belts. The submitted article may therefore, according to the reviewer, be ultimately published after several clarifications.

(1) The article needs to first be edited for English, in order to make it easily understandable so that it would have an impact on the work of others.

(2) I corrected the text of the manuscript.

(3) I edited the manuscript. Many words and sentences are clarified. New paragraphs added and some paragraphs moved.

(1) There viewer is then wondering: what is new in this article, compared in particular with there view published in SSR last year (see specific comments)?

(2) A new view of known experimental data. There are considered a wider ranges of L and E and not only the outer, but also the inner belt. The comparison and analysis of data on ions with $Z > 1$ obtained in years near the minima and maxima of solar activity made for the first time. New methods proposed here allows progress in the problem of constructing empirical models of heavy ions (data for which is clearly not enough).

(3) New results obtained in this work are highlighted (lines 14-27, 172-181, 199-200, 306-309, 247-354, 471-477, 479-484, 489-502, and Figs. 1-9).

(1) I recommend the article to be revised and reviewed again to see if, after English proof and clarifications, the article would be suitable for publication in Annales Geophysicae.

(2-3) Manuscript is completely revised. Many details of this work are clarified.

Specific comments

(1) Section 2: would it be possible to clarify if the invariant parameter values given here come from previous publications or are the outputs of the new study? The values reported here are very important. If the values have now been recomputed or updated, would it be possible to have them highlighted in a table, for instance?

(2) The invariants of the ERB structure were obtained for $L > 3$ in the works of the author 1984-1999 (see references in the manuscript). Here are considered more complete data of the satellites and for protons with $E > 10$ MeV invariants μ_b and γ were traced to $L \sim 2$. Of course, I compared the values of these invariants with the results of experiments that were published after 2000, but this had no effect on the average values of the invariants and their variances. From the experimental radial profiles of the fluxes for different ion energy the values of these invariants were obtained most precisely.

(3) I revised Section 2, clarify and expand it. It is explained how the invariants of the structure of the ERB ion fluxes were obtained (lines 97-102, 118-119, 138-147, 165-170, 288-295, 317-318, 337-346).

(1) A possible use from other researchers would be to compare them with what has been observed by now two orbiters around Jupiter, as these orbiters performed numerous observations of trapped heavy ions (helium, oxygen, sulfur).

(2) The distributions of ions in the belts of other planets (Jupiter, Saturn) have such invariants also and their values correspond to the mechanisms discussed in Section 5 (for the magnetic fields of these planets).

(3) Remark about radiation belts of Jupiter and Saturn is given (lines 471-477).

(1) Would it be possible to remind, with maybe one sentence, the criterion used to select quiet periods over which the measurements are averaged?

(2) The absence of storms and substorms and $K_p < 2-3$ were chosen as the criterion for the quiet magnetosphere.

(3) The criterion of the quiet magnetosphere given (lines 138-139, 190-191, 226, 297, 326).

(1) The measurements are averaged near solar cycle minimum and near solar cycle maximum. Are the measurements very dispersed around this average in each case or are the standard deviations small compared to the shown averages? A comment may be added in the main text about this.

(2) The values of scatter of the structure invariants connected mainly with instrumental errors and with the inevitable methodical errors. For many experiments, the scatter of these values is much smaller the average results given in Section 2. In my analysis 1984-2001 the solar-cyclic variations were considered in detail only for the parameter ξ , which varied greatly during the solar cycle.

(3) Questions related to the scatter of the values of structural invariants are clarified (lines 155-160). Questions related to solar-cyclic variations in the values of structural invariants are discussed (lines 161-170, 239-240).

(1) The dataset of heavy ion measurements is limited, but is it large enough to conclude if there is any observable Magnetic Local Time asymmetry in the heavy ion radiation belts, in particular at the lowest considered kinetic energies?

(2) At $L > 5-6$ even in quiet periods there is a dependency of ion fluxes on MLT. But for quiet periods on the data of the geosynchronous satellite Gorizont (and using averaged empirical model of the magnetic field at the GSO) it was established that in this region no the dependence of structural invariants on MLT.

(3) The question of the dependence of ion fluxes and structural invariants of the ERB on MLT at $L > 5-6$ is discussed (lines 215-218).

(1) Section 4: What are the new conclusions on the physics of the heavy ion radiation belts? If you confirm what has already been reported in previous publications, would it be possible to add a sentence to state it? Otherwise, new findings may be more highlighted in this section and in the conclusion.

(2) For the first time, the role of fluctuations of the thickness of the plasma sheet of the magnetospheric tail in the formation of the power-law tail of the ion spectra of the ERB is highlighted. A hypothesis about analogous structure invariants of the radiation belts of other planets is suggested.

(3) These findings are highlighted in Section 5 and in the Conclusion (lines 448-449, 466, 471-477, 505-509).

(1) Lines 453-454: "Here, the experimental database is significantly expanded, many modern measurements of the ion fluxes of the ERB have been added", what are the modern heavy ion measurements added since the article published by the author in 2001? For the protons, one can see the GEO-3 and Van Allen Probe observations, however there does not seem to be any "modern" measurement of heavier ions.

(2) In my publications of 1984-2001 considered only region of the ERB at $L > 3$ and ions with $E < 10$ MeV. Here I considered a wider range of L (from 1.2 to 8) and E (up to 200 MeV), i.e. is considered also the inner belt. Due to this, we succeeded in tracing the invariants corresponding to the power-law tail of the proton spectra up to $L \sim 2$. Figures 8 and 9 present data of the satellite Polar for protons and helium ions, which I had not previously considered because of the significant deviation of this satellite's orbit from the equatorial plane.

(3) I describing what additional experimental results, compared with my publications of 1984-2001, were used in this work (lines 172-173, 288-295, 363-367).

Technical corrections

(1) The article needs to be edited for English.

(2) I corrected the text of the manuscript.

(3) I edited the manuscript.

(1) It would help the reading to explain in the figure captions what the colored lines refer to, even if it is explained in the main text.

(2) Additional explanations in the captions for Figs. 1-6 are added.

(3) It is made (lines 718-721, 724-727, 731-733, 737-739, 743-745, 749-751).

(1) In the main text, would it be possible to clarify what the maximum deviations shown by the colored vertical segments are: are they based on energy spectra measured by all the satellites, or only a subset?

(2) The maximum deviations of the colored lines correspond to the dispersions of the parameters given in Section 2. For many experiments, especially with heavy ions, the values of these invariants are determined much more accurately not by the spectra, but by the radial profiles of the ion fluxes for different pairs of energy channels.

(3) It is made (lines 138-147, 159-160, 191-192, 201-214).

(1) Would it also be possible to clarify the meaning of the following statement “on a logarithmic energy scale, the magnitudes of these segments do not depend on L shell”? Does it mean that the size of the segments changes a little bit with L , but not enough to be clearly seen when plotted with a logarithmic scale?

(2) For particles moving in the equatorial plane, as in Fig. 1-6, the first adiabatic invariant is $E/B(L)$. The ratios of the upper and lower values for each invariant do not depend on L . Consequently, the difference of the logarithms of these values is also independent on L . Therefore, the vertical segments on the colored lines can be shifted along the corresponding lines without changing their sizes on the energy scale. This is also true for magnetic traps of non-dipole type, i.e., in our case, for large L .

(3) It is explained (lines 205-209).

(1) Section 3: this section is quite long, would it be possible to add subsection titles to help the reader? You may have a subsection on the protons in the (E, L) space that would start after line 170, one on the helium ions in (E, L) space that would start after line 259, one on the CNO ions in (E, L) space starting after line 287, and finally one on the protons and helium ions in the $(L, B/B_0)$ space starting after line 319.

(2) Section 3 is divided into subsections (for different ion components). Figures 7-9 and the associated text is detached in Section 4.

(3) As a result, the number of sections increased from 5 to 6 (lines 171, 225, 296, 325, 362).

I am very grateful to Referee #2 for very helpful comments.

With grand regard,
Alexander S. Kovtyukh