1. **Interactive comment on "Invariants of the Spatial-Energy Structure and Modeling of the Earth's Ion Radiation Belts" by Alexander S. Kovtyukh**

Anonymous Referee #1

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General comments

This paper presents a parametric model for the density of various ion species in the Earth radiation belts. This model describes the global structure of the radiation belts for protons, helium, and for ions of the CNO group. Based on extensive satellite data, the parameters of the models have been fitted, independently for the proton populations and for the other ions. The validity of the model is discussed, species by species, by comparison with in-flight data. The solar cycle dependency is presented. Finally, a physical interpretation of the model is detailed.

Overall, the paper is difficult to understand. Firstly, the English is very poor (see technical corrections hereafter), and difficult to grasp. Secondly, the model is not appropriately explained, as the author refers to previously published literature, which could not be accessed by this reviewer. Thirdly, as described in the specific comments, the figures are not very clear and supportive of the arguments developed in this paper.

I am very grateful to the Referee for these comments: they help clarify the essence of my work and I, of course, will make all the necessary changes in the revised manuscript.

In many places of this manuscript, starting with the title, the word "model" is found and in almost all cases this word has the most general meaning here: any observation or measurement is a physical model (modeled by instruments or a brain). But in fact, no empirical or mathematical models are used in this manuscript, and what is presented in Fig. 1-9, these are collections of experimental data.

Associations with models (in the narrow sense) also invoke the word "parameters".

What presented here, in section 2, and called the invariant parameters of ERB are not the parameters of mathematical or empirical models of ERB. These parameters obtained directly from the results of various experiments, from the figures of the corresponding articles. They were found for each energy spectrum and for each dependence from L of the ions fluxes, and only then these parameters were averaged (separately for each ion component).

These invariants exist only in the region where the transport (radial diffusion) of ions dominates their losses. With increasing B/B_0, the rate of radial diffusion of ions decreases, and the rate of their loss increases rapidly. Therefore, on small L and for large B/B_0, these invariants are not applicable to ERB and in Figs. 7-9 they are not presented (these figures are given only for completeness).

Author's papers in journals Cosmic Res. and Geomagn. Aeron. (in English) can be found in any major university library. But in order to make sure that there are such invariants and in the correctness of the values given here, it is not necessary to read these articles. It is enough to open several articles where there are experimental spectra or radial profiles of ion fluxes with $E > 0.1$ MeV for $L > 3$, obtained in quiet periods near the equatorial plane.

I will take into account all these remarks and the words "model" and "parameter" will be saved only where it says about specific ERB models.

I checked the descriptions of the figures and added them with the necessary explanations.

I agree that I don't speak English well enough and gratefully accept any corrections to the text of the manuscript.

Specific comments

Section 2 of this paper presents the model parameters and their measured values, but not the model itself, which is only suggested by the description of the parameters. A detailed and self-sufficient description of the model should be given in this section.

On section 3, numerous similar figures are presented. It is not clear how these figures were obtained from the data. In particular, these figures present iso-lines on power of tens, with most satellite data points placed on the iso-lines, which suggest some interpolation was done on the satellite data. Section 3 should detail how this figures were made.

The conclusions of the comparison of figures 1 to 4 proposed at line 269 are not clearly apparent in the figures. Similarly, the low-altitude effects described at line 352 cannot be clearly seen on the figures, because the transformation B/Beq to altitude is not straightforward (for instance, the 1000 km altitude line could be drawn on figure 7-9 to support the arguments of this paragraph).

A reference should be provided at line 347 for the dependency of the radial diffusion rate on B/Beq.

A figure supporting the information at line 362-364 about the CNO group data could be provided.

The invariants presented in Section 2 are not tied to any particular model, but they can be used in works on the creation of both empirical and mathematical models of spatial-energy distributions of ERB ions. In particular, they were tested in many my works on modeling the pitch-angle distributions of protons, the empirical model of ion fluxes for region of the geosynchronous orbit (GSO), the daily course of ion fluxes on the GSO and in other problems.

Points on fig. 1-9 were obtained from experimental radial profiles of differential ion fluxes. It was taken into account that, for different authors, these fluxes have different dimensions. For example, for ions with $Z > 1$, these fluxes are given in $(\text{cm}^2 \text{ s ster MeV/n})^{-1}$ or in $(\text{cm}^2 \text{ s ster MeV})^{-1}$; in the latter case the same ion flux will have a value of Z times less.

Iso-lines in Fig. 1-9 are envelopes of experimental points (they are constructed by the method of the χ^2).

It was important to choose a form of representation (space of variables) in which the results of different experiments (with different sets of the energy chennels) one could accommodate organically and without resorting to interpolations and extrapolations of the data. As such representation was chosen the space $\{E. L\}$. All other representations, such as radial profiles of ion fluxes for different energies, suggest interpolation and extrapolation of an experimental data; this is greatly complicates the solution of our problem and introduces large systematic errors and arbitrariness in the choice of curves approximation.

To present the data obtained outside the equatorial plane, it was natural to use the space $\{B/B_0, L\}$ for ions of different energies. Here I had to resort to interpolation of data and Figs. 7–9 are less complete and accurate compared to Figs. 1–6.

For comparison Figs. 1-2 with Figs. 3-4, specific values of proton and helium ion fluxes are given (lines 269-271).

For Figs. 7-9 interpretation can be simplified, and then it is not necessary to put on these figures the dependences $B/B_0(L)$ for fixed heights.

For the dependency of the radial diffusion rate on B/B_0 a reference are added (at line 347).

There are not enough data for CNO group ions and they are very fragmentary; build or them figures like to Fig. 7-9 while is impossible. One can only make assumptions on the basis of available data and general considerations. I changed the sentence at lines 362-364, made it more careful.

Technical corrections

Numerous English errors have been found, for instance on lines 21, 24, 34, 48, 63, 67, 69, 79, 118, 132, 134, 137, 161, 163, 166, 173, 185, 193, 201, 215, 220, 226, 239,271, 290, 314, 338, 379, 387, 389, 398, 403, 469.

In the figure legends, the MeV unit is displayed in Cyrillic. Moreover, the model lines on the figures are not described in the legends.

I would correct the errors in the revised version of the manuscript.

All dimensions of the MeV unit displayed in Cyrillic replaced by on Latino.

In the caption to Figs. 1-6 added explanations to color lines. In Figs. 1–6, the color lines are show dependencies on L of the ion energies corresponding to the structure invariants ($E \propto \mu L^{-3}$) and also the maximum energy of ions, which can be trapped in the ERB($E \propto L^{-4}$).