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Interactive comment on "Scaling laws in Hall-inertial range turbulence" *by* Yasuhito Narita et al.

Yasuhito Narita et al.

yasuhito.narita@oeaw.ac.at

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Thank you very much for taking time for reading the manuscript and raising helpful comments. Reply to Referee-2 comments are as follows. Please refer to the full version of the manuscript revision with the reply comments uploaded as a supplementary pdf file to the reply to the Referee-1 report.

1. *Narita et al. propose a phenomenological model of turbulence for ion kinetic scaleson the basis of Hall-MHD. The paper is clearly written and coherent and appropriate for Annales Geophysicae. However, several relevant references are missing, including some that arrive to similar results using the same formalism (Hall-MHD). I would highly encourage the authors to write a more detailed*

discussion and introduction incorporating theoretical and observational results relavant to their work and to describe limitations of their model.

1. Contrary to what the authors are saying on L5, there are now strong indication of kinetic Alfven waves in the ion kinetic range, e.g. Roberts et al. GRL, 45, 2018. The authors should include this reference.

Reply:

- We agree that Roberts et al. (GRL, 2018) indicate the kinetic Alfvén mode in the magnetosheath region from the wave analysis using the fluctuation sense such as the Alfven ratio or the ratio of the ion density to the magnetic field, not from the dispersion relation analysis. Narita et al. (2016) shows, on the other hand, that there is a freuqency scattering in the observationally-determined dispersion relation with an indication to kinetic-drift mirror mode.
- We added the following text. (page 2, line 16–19).
"The study by Roberts et al. (2018) indicates the existence of the kinetic Alfvén mode in the magnetosheath region from the wave analysis for the fluctuations in the MMS data such as the Alfven ratio. No dispersion analysis is performed. On the other hand, the study by Narita et al. (2016) exhibits a frequency scattering in the observationally-determined dispersion relation with an indication of a kinetic-drift mirror mode."

2. *Regarding the limits of Hall-MHD to describe ion kinetic scale turbulence, there is plenty in the literature available. Hall MHD is valid in the limit where the electron temperature is much greater than the ions temperature and when the inverse of the linear transit time for an ion is much smaller than the turbulent frequency and the inverse ofthe linear transit time for an electron, respectively. Thus, in the*

instance where thetemperature of the ion is finite, phase-mixing and damping of modes ought to be taken into account. Perhaps a good recent reference that can be added and discussed is that of Howes et al., Nonlinear Processes of Geophysics, 16, 2009.

Reply:

- Agreed. We added the following text on page 2, line 27–31.

  "Limitations of Hall MHD have been discussed, for example, by Howes (2009). The concept of Hall turbulence is valid in the limit where the electron temperature is much greater than the ions temperature and when the inverse of the linear transit time for an ion is much smaller than the turbulent frequency and the inverse of the linear transit time for an electron, respectively. Thus, in the instance where the temperature of the ions is finite, phase-mixing and damping of modes ought to be taken into account. This causes deviations from Hall MHD."

3. *Schekochihin et al., ApJ Supplement, (2007) provides a detailed description of ion-scale turbulence for weakly collisional plasmas through the use of gyro kinetic. Gyrokinetic is a reduced anisotropic limit of Hall-MHD with comparable results to that of the authors. However, Gyrokinetic, unlike Hall-MHD, incorporates phase-mixing due to Landau damping (not cyclotron-resonance). Can the authors incorporate in the discussion of a comparison of their results with that of Schekochihin et al..*

   Reply:

   - Agreed. Done. Schekochihin et al. (2007) should read Schekochihin et al.

     (2009). We added the following text (page 13, line 6–12).

     "Schekochihin et al. (2009) provide a detailed description of ion-scale turbulence for weakly collisional plasmas through in a gyro-kinetic treatment. Gyro-kinetic theory is a reduced anisotropic limit of Hall-MHD with comparable results to that of the authors. However, the gyrokinetic theory, unlike Hall-MHD, incorporates phase-mixing due to Landau damping (not cyclotron-resonance). In weak turbulence of energy-cascading kinetic Alfvén waves gyro-kinetic theory predicts inertial-range energy spectra (in the perpendicular wavenumber domain) with spectral slopes $k_\perp^{-1/3}$ for the electric and $k_\perp^{-7/3}$ the magnetic fields, and spectral density slopes $k_\perp^{-7/3}$. These are identical to the compressive magnetic field fluctuations obtained here."

4. *Chen et al., ApJ, 122, 2017, among many others, report magnetic energy spectra that are steeper for ion kinetic range. Can the authors incorporate a more detailed discussion incorporating observational evidence that are quantitatively different from their theory? Perhaps differences between Hall-MHD turbulent estimates and observationscan be used to quantify the contributions of kinetic physics at the ion scale?*

   Reply:

   - The same question was raised by Referee-1. We added the following text (page 11, line 17–22).

     " While the presented model is qualitatively similar to previous observations in that the magnetic energy spectra become steeper in the kinetic range, observed slopes are often steeper than $-7/3$, for example, as in Stawarz et al. (2016), CHen and Boldyrev (2017), and Breuillard et al. (2018). It should be noted that the theory predicts the energy spectra in the wave vector domain and the observations have often access to the spectra in the frequency domain. Possible reasons for the difference in the spectral slope between the theory and the observations include the presence of dispersive waves and the non-Gaussian frequency broadening in the random sweeping effect."

5. *Alexandrova et al.[Small scale energy cascade of the solar wind turbulence] arrive to a similar scaling as that of the authors using Hall-MHD. Can the authors differentiate their work from that of Alexandrova et al.*

Reply:

- We added the following text (page 11, line 32 – page 12, line 2).

  "The data analysis motivated model of Alexandrova et al. (2008) introduces an ad hoc measure α of the compression distinguishing between the incompressible ($\alpha = 0$) and isotropic compressible ($|\alpha| = 1$) cases. It maps the spectral slope of the magnetic field energy from $k^{-7/3}$ in the incompressible case to $(-7 + 6\alpha)/3$ in the compressible case. Our physically motivated Hall MHD model differs from that of Alexandrova et al. (2018) in that the slope $-7/3$ is obtained for the compressible field fluctuations."

Other changes

1. We added the following text (page 8, line 4–16).

"It is interesting to compare the density spectrum with $k_\perp^{5/3}$ for the Hall-scaling (Eq. 27) with the Kolmogorov-Poisson density spectrum with the $k_\perp^{1/3}$-scaling obtained earlier (Treumann et al., 2019, Eq. 24) for non-Hall turbulence. The ratio of the two expressions is

$$\frac{\mathcal{E}^H_{\text{dens}}}{\mathcal{E}^K_{\text{dens}}} \sim \left(\frac{V_A}{c}\right)^2 \frac{c_m^2}{c_K} k_\perp^{4/3}. \tag{28}$$

It still depends on the unknown constant of proportionality c_m which must be determined otherwise. However, the deformation of the spectral scaling caused by the Hall turbulence is stronger than in the non-Hall case. Its contribution might thus become important, even though numerically its contribution to the density variation is smaller than that of the Kolmogorov-Poisson spectrum, because $V_A \ll c$. The difference in the spectral slopes of $k_\perp^{4/3}$, indicates that the Hall density spectrum becomes increasingly more effective at larger wave numbers.

The Hall magnetic energy spectrum is steeper than the Kolmogorov-type one with wave number ratio

$$\frac{\mathcal{E}^H_{\text{mag}}}{\mathcal{E}^K_{\text{mag}}} \sim k_\perp^{-2/3} \tag{29}$$

Finally, the ratio of the kinetic power spectra yields a flatter Hall kinetic energy spectrum than Kolmogorov:

$$\frac{\mathcal{E}^H_{\text{kin}}}{\mathcal{E}^K_{\text{kin}}} \sim k_\perp^{4/3}. \tag{30}$$

"

2. The following literatures were added to the manuscript based on the referee comments and our reply comments.

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