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Interactive comment on "Scaling laws in Hall-inertial range turbulence" *by* Yasuhito Narita et al.

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Thank you very much for taking time for reading the manuscript and raising helpful comments. Reply to Referee-1 comments are as follows. We add the full version of the manuscript revision and the reply comments as a supplementary pdf file.

1. *General comments*

   The paper, "Scaling laws in Hall-inertial range turbulence", discusses the behavior of the Hall term from generalized Ohm's law in the context of turbulent systems. The authors focus on two-dimensional turbulence and develop phenomenological scaling laws for the wavenumber spectra of the magnetic field,

   electric field and particle number density between ion and electron scales. The authors find differences in the spectral behavior associated with the parallel (compressive) and perpendicular (incompressive) components of the magnetic fluctuations with respect to the mean magnetic field and qualitative similarities between the presented model and previous observations are noted. I find the paper to be well written and believe the analysis provides important insight into the role of the Hall term in turbulent plasmas, which may be useful for interpreting future high-resolution measurements of space plasma turbulence. My main comment is that further discussion of the differences between the presented modeland previous observations could be provided, which would give useful insight on the limitations of the presented model. Additionally, I have noted some statements which could use clarification and additional references that may be relevant.

   Reply:

   - Thank you very much for the positive evaluation of our work.

2. *Specific Comments*

   Pg. 3, line 19 – 22: I am unclear how the compressible and incompressible components of the magnetic field are obtained from $\delta B_H \cdot \delta J_H$ *and* $\delta B_H \times (\delta E_H \times B)$. *Additionally, how is it possible for the incompressible component of B to be anti-parallel to* δE_H *if* δE_H *is in the* $k_\perp$ *direction? Wouldn't this imply that the magnetic field has a divergence?*

   Reply:

   - Right. Thank you for noticing this point. The fluctuating magnetic field is projected by referring to the mean magnetic field, not to the Hall current. Otherwise the product j dot B makes the current helicity, which is beyond the scope of the manuscript. The Hall current δj_H is used when estimating the fluctuating electric field in our theory (in the lowest-order sense). The fluctuating magnetic field is expressed by the scaling law.

- We modify the first part of section 2.2 as follows (page 3, line 29 –page 4 line 10).

  "The magnetic field fluctuations δB_H in Hall turbulence have two components, one compressive component $\delta B_\parallel$ parallel to the mean field B and the other perpendicular component $\delta B_\perp$. It is convenient to introduce an orthogonal coordinate system with base vectors e_1 and e_2 perpendicular to the mean field, and $e_\parallel$ along the mean field. Moreover, we are free to choose the direction of the perpendicular wave vector, letting e_1 refer to $k_\perp$. The Hall magnetic field has no divergence, so it must be perpendicular to k. This yields

  $$\delta B = (0, \delta B_\perp, \delta B_\parallel). \tag{3}$$

  The fluctuation of the Hall electric field is given by

  $$\delta E_H = \frac{1}{en} \delta j_H \times B - \frac{\delta n}{n} E. \tag{4}$$

  The last term on the right containing the fluctuations in density and their contribution to δE_H is important only in the stationary frame where $E \neq 0$. Using Ampère's law $\mu_0 \delta j = \nabla \times \delta B$ (from here on suppressing the index H on the fluctuations when dealing exclusively with Hall fluctuations in Hall MHD) yields

  $$\delta E = \frac{1}{en\mu_0} B \times (\nabla \times \delta B). \tag{5}$$

  "

3. *Section 4: Some further discussion of differences between the presented theory and previous observations and the possible explanations for these differences may be useful.*

   For example:

   – While the presented model is qualitatively similar to previous observations in that the magnetic energy spectra become steeper in the kinetic range, observed slopes are often steeper than -7/3 [for example as in Stawarz et al. JGR, 121, 11021, 2016; Chen et al. ApJ, 842, 122, 2017; Breuillard et al. ApJ, 859, 127,2018].

   Reply:

   - Agreed. We added the following text (page 11, line 17–22).

     "While the presented model is qualitatively similar to previous observations in that the magnetic energy spectra become steeper in the kinetic range, observed slopes are often steeper than $-7/3$, for example, as in Stawarz et al. (2016), Chen and Boldyrev (2017), and Breuillard et al. (2018). It should be noted that the theory predicts the energy spectra in the wave vector domain and the observations have often access to the spectra in the frequency domain. Possible reasons for the difference in the spectral slope between the theory and the observations include the presence of dispersive waves and the non-Gaussian frequency broadening in the random sweeping effect."

4. *– Would the presented theory predict that the fluctuations in the parallel component of the magnetic field should dominate the spectrum? Some observations*

[for example Stawarz et al. JGR, 121, 11021, 2016 and Chen et al. ApJ, 842,122, 2017] seem to show the perpendicular component dominating into the kinetic range.

Reply:

- No, not necessarily. The theory does not immediately predict that the parallel component of the magnetic field should dominate the energy spectrum because fluctuations in the parallel and perpendicular components of the magnetic field are modeled independently. Our theory predicts that the electric field (or the Hall effect, naively speaking) associated with the parallel component of the magnetic field should dominate the spectrum (Equations 5–7). The magnetic energy spectrum can be dominated either by the parallel fluctuations or by the perpendicular fluctuations. But a naive estimate indicates that the parallel fluctuating component may dominate if both compressive and incompressible fluctuations are excited by the electron flow.

- We added a subsection "Parallel vs. perpendicular components of the magnetic field" and discuss the competition between the parallel and perpendicular components of the magnetic field (page 12, line 13–26).

  "Some observations (Stawarz et al., 2016; Chen and Boldyrev, 2017) indicate the dominance of the perpendicular magnetic field component in the kinetic range. Our scaling laws are derived separately for the parallel one. It predicts that the Hall electric field associated with the parallel component of the magnetic field should dominate the electric spectrum (Eqs. 5–7). The magnetic energy spectrum can be dominated by either parallel or perpendicular fluctuations. However note that the scaling contains the undetermined numerical constant c_m which determines the absolute value.

  The parallel fluctuating component dominates if both compressive and incompressible fluctuations are excited by the electron flow. The normalised perpendicular component of the magnetic field is smaller than the parallel component according to $\delta\tilde{B}_{\parallel} \sim |\delta\tilde{B}_{\perp}|^2$. This follows from the electron flow velocity $\tilde{v}_{\perp} \sim \tilde{k}_{\perp}\delta\tilde{B}_{\parallel}$ and the association to the perpendicular component $\tilde{v}_{\perp} \sim \tilde{k}_{\perp}|\delta\tilde{B}_{\perp}|^2$. In Hall MHD the flow velocity is $E \times B$ passive being subject to the magnetic and electric fields.

  The relative contribution between the parallel and perpendicular components of the magnetic field depends on the length scales. Using Eq. (21) and Eq. (33) yields

  $$\frac{\delta B_{\parallel}}{\delta B_{\perp}} \propto \tilde{k}^{-1/6}. \qquad (6)$$

  Therefore, the contribution of the parallel component of the magnetic field is reduced with increasing wavenumber."

5. – *How does the predicted increasing density fluctuation spectra mesh with the observed decreasing density spectra in the kinetic range, for example as recently observed with MMS by Breuillard et al. [ApJ, 859, 127, 2018] and Chen et al.[ApJ, 842, 122, 2017]?*

Reply:

- We added the following text (page 13, line 6–12).

  "The increasing sense of the smaller-scale (or higher-frequency) density spectrum is indeed found using the Spektr-R spacecraft data in the solar wind (Šafránková et al., 2013). Treumann et al. (2019) provide a theoretical explanation of the density spectrum bump using the convected fluid model which the present theory extends the the inclusion of Hall dynamics. In the magnetosheath, to date no such increase is observed in the electron density spectrum based on the spacecraft data. Figure 3 in Breuillard et al. (2018) shows a flattening of the density spectrum at spacecraft-frame frequencies of 10 Hz or higher. Chen and Boldyrev (2017, Fig 4, bottom panel) shows that the density has about the same fluctuation power as the magnetic field at lower frequencies, indicating similar density and magnetic spectral slopes, with density spectrum estimated for electrons and inert ions. Theoretically information about the density spectrum can also be obtained either making use of the continuity equation or the quasi-static approximation (Cohen and Kulsrud, 1974; Narita and Hada, 2018)."

6. *Matteini et al. [MNRAS, 466, 945, 2017] and Franci et al. [ApJ, 812, 21, 2015] may also be relevant references to discuss, as they also consider the role of the Hall term in generating a linear ratio between the electric and magnetic field.*

   Reply:

   - Agreed. We added the following text (page 12, line 8–9).

     "...as considered earlier in Cluster data analysis (Matteini et al., 2017) and hybrid plasma simulation (Franci et al., 2015)"

7. *Technical Corrections*

Abstract and Section 2: While the 2D nature of the turbulence in the solar wind is mentioned in the introduction, I think it would be useful to explicitly state in the Abstract and in Section 2 that a 2D geometry (i.e. no parallel component of the wavevectors) is being considered in this paper.

Reply:

- Done. We added the sentence in abstract and section 1 (not section 2). (page 1, line 6–7; page 2, line 24–26).

  "In the present paper we consider a two-dimensional geometry with no wave vector component parallel to the magnetic field as is appropriate in Hall-MHD."

  "We consider a two-dimensional geometry which has no parallel wave vector component. The full expression for the Hall electric field contains also parallel wave vector components Treumann et al. (2019) which in Hall MHD are neglected."

8. *Pg. 1, line 21: For a 500 km/s flow speed, shouldn't the frequency range be 0.5to 5 Hz?*

   Reply:

   - Done. (page 1, line 22).

9. *Pg. 4, line 5: I think the right-hand-side of eq. 5 should be positive instead ofnegative.*

Reply:

- Right! Thank you! Corrected. (page 3, equations 3–11).

10. *Pg. 5, line 15: A citation for the author's previous publication which is being referred to would be helpful.*

    Reply:
    Done, "Treumamnn et al., 2019)" (page 5, line 26).

11. *Pg. 7, line 10: I think that "revoke" should be "invoke".*

    Reply:

    - Done. (page 7, line 18).

12. *Pg. 7, line 19–21: Can the authors provide a reference for the statement in the last sentence in Section 3.1.*

    Reply:

    - We added a reference to Šafránková et al. (ApJ, 2015). (page 8, line 2).

Please also note the supplement to this comment:
https://www.ann-geophys-discuss.net/angeo-2019-69/angeo-2019-69-AC1-supplement.pdf

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