Reply to referee comments

Multi-channel coupling of decay instability in three-dimensional low-beta plasma

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We thank very much the reviewer for reading our manuscript and raising helpful questions and valuable suggestions. Here we give our reply. The changes in the manuscript are marked in boldface.

Reviewer:
1(a). Authors conclude that: "The overall decay process is not controlled by the field-aligned decay but by the dispersion relation of the participating waves which drives the oblique decay to share identical parallel wavenumbers with those attained in the parallel decay" My understanding of their statement is that the growth rate of the oblique modes is not directly related to (i.e., a simple function of) the growth rate of the parallel mode. Is this interpretation correct?

Reply:
In order to answer the reviewer's question, we introduce the 3-wave coupling equations in more details,

$$k_{\parallel}^{-} = k_0 - k_{\parallel} \tag{1}$$

$$k_{\perp}^{-} = -k_{\perp} \tag{2}$$

$$\omega^{-} = \omega_0 - \omega \tag{3}$$

where $(k_0, \omega_0 = k_0 V_A)$, $(k_{\parallel}^{-}, k_{\perp}^{-}, \omega^{-} = -k_{\parallel}^{-} V_A)$, and $(k_{\parallel}, k_{\perp}, \omega)$ are the wavenumbers, frequencies, and dispersion relations characterizing the Alfvén pump, Alfvén daughter, and ion acoustic waves, respectively. Eqs. (1)-(3) and the assumption $\omega \sim k_{\parallel} c_s$ (available in low β plasma) provide after some elementary algebra, the parallel wavenumber of the oblique ion acoustic wave,

$$k_{\parallel} = \frac{2k_0}{1 + c_s/V_A} \equiv k(0) \tag{4}$$

where k(0) is the wavenumber of the sound daughter driven by the field aligned decay, see e.g., Spangler et al., (1997).

The above rough evaluation is validated both by the analytic analysis in the MHD framework and by the hybrid simulation applied in the present study. In the analytic analysis, the evolution of the daughter waves is determined by the interaction with the field-aligned pump wave by constructing a system of quasi-linear equations which exhibits wave-wave couplings generating obliquely-propagating waves as the daughter component. The growth rate of the oblique modes has generally a maximum value at the parallel wavenumber prescribed by the maximum growth rate of the field-aligned decay instability and depends on the propagation angle.

Changes in the manuscript:
We rearrange the related comments in the former manuscript and introduce new comments as below,
Page 8, Line 34 to Page 9, Line 8:
" This result can be easily understood by introducing the 3-wave coupling equations and the dispersion relations,

$$k_{\parallel}^{-} = k_0 - k_{\parallel}$$

$$k_{\perp}^{-} = -k_{\perp}$$

$\omega^- = \omega_0 - \omega$

where $(k_0, \omega_0 = k_0 V_A)$, $(k_{\|}^-, k_{\perp}^-, \omega^- = -k_{\|}^- V_A)$, and $(k_{\|}, k_{\perp}, \omega)$ are the wavenumbers, frequencies, and dispersion relations characterizing the Alfvén pump, Alfvén daughter, and ion acoustic waves, respectively."

Page 9, Line 12 - Line 15:

"Eqs. (3)-(5), the dispersion relations and some elementary algebra provide the parallel wavenumber of the oblique ion acoustic wave,

$$k_{\|} = \frac{2k_0}{1 + c_s/V_A} \equiv k(0)$$

where k(0) is wavenumber of the sound daughter driven by the field aligned decay, see e.g., Spangler et al. (1997)."

Page 9, Line 17 - Line 23:

" The above rough evaluation is validated both by the analytic analysis in the MHD framework and by the hybrid simulation used in the present study. In the analytic analysis, the evolution of the daughter waves is determined by the interaction with the field-aligned pump wave by constructing a system of quasi-linear equations which exhibits wave-wave couplings generating obliquely-propagating waves as the daughter component. The growth rate of the oblique modes has generally a maximum value at the parallel wavenumber prescribed by the maximum growth rate of the field-aligned decay instability and depends on the propagation angle."

Reviewer:

1(b). However, it should be still highlighted that the field aligned decay remains in any case the fastest one, as shown in Fig.4. Also, this dynamics is different from the case of an oblique pump wave propagating at some theta angle with respect to B_0, where the oblique mode's decay rate gamma is found to scale cos(theta), so controlled by the $k_{||}$ projection of the initial oblique k (Del Zanna GRL 2001). This suggests then that there are 2 possible ways of generating oblique modes from the parametric decay: 1) from a purely parallel mother wave, as in this study; 2) from an oblique pump wave (e.g. Matteini et al. GRL 2010). In both cases the oblique modes grow at a rate that is smaller than the parallel decay;

Reply:

Agreed. We changed the section 3 accordingly.

Changes in the manuscript:

Page 9, Line 24 - Line 30:

"However, we still point out that the field-aligned decay remains in any case the fastest one, as shown in Fig. 4. Also, this dynamics is different from the case of an oblique pump wave propagating at a given theta angle with respect to B_0, where the oblique mode's decay rate gamma is found to scale $\cos\theta_{kB}$, so controlled by the $k_{||}$ projection of the initial oblique wavevector (Del Zanna GRL 2001). This suggests then that there are two possible ways of generating oblique modes from the parametric decay: "(1)" from a purely parallel mother wave, as in this study; "(2)" from an oblique pump wave (e.g. Matteini et al. GRL 2010). In both cases the oblique modes grow at a rate that is smaller than the parallel decay."

Reviewer:

1(c). ; it would be interesting then to establish which configuration would be more efficient in driving a transverse broadband modulation for, e.g., a given amplitude of the mother wave. Can authors discuss this possible competition in more detail?

Reply:

In our study the central role is played by the conservation of frequencies and the dispersion relations of the components involved in the 3-wave coupling. Matteini et al. GRL (2010a) show that in their analysis, the central role in driving the transversal spectrum is played by the conservation of the momentum and the non-zero perpendicular projection of the pump wavevector. An obliquely-propagating (pump) wave is expected to be more compressive and it can generate a broad-band spectrum of compressive fluctuations. Thus, the configuration with an oblique pump wave should be more efficient in driving oblique modes, even though the daughter waves in our study are driven more early after a short time of nonlinear evolution. On the other hand, one should not ignore the major differences in the involved setups, e.g., the different polarization of the pump wave and different β_e values (see discussion bellow). Besides the plasma beta, the wavenumber of the pump wave may play a significant role in the process of driving the transverse broadband modulation. In both studies, the medium is still weak dispersive ($k_0 V_A/\Omega_p \sim 0.2$). In a more dispersive medium such that one used by Verscharen et al. (2012), the dispersion relations probably cannot construct a transversal spectrum of waves for the decay of a field-aligned Alfvén wave.

Changes in the manuscript:
Page 9, Line 30 to Page 10, Line 7:
“In our study the central role is played by the conservation of frequencies and the dispersion relations of the components involved in the 3-wave coupling. Matteini et al. GRL 2010a show that in their analysis, the central role in driving the transversal spectrum is played by the conservation of the momentum and the non-zero perpendicular projection of the pump wavevector. An obliquely-propagating (pump) wave is expected to be more compressive and it can generate a broad-band spectrum of compressive fluctuations. Thus, the configuration with an oblique pump wave should be more efficient in driving oblique modes, even though the daughter waves in our study are driven more early after a short time of nonlinear evolution. On the other hand, one should not ignore the major differences in the involved setups, e.g., the different polarization of the pump wave and different β_e values (see discussion bellow). Besides the plasma beta, the wavenumber of the pump wave may play a significant role in the process of driving the transverse broadband modulation. In both studies, the medium is still weak dispersive ($k_0 V_A/\Omega_p \sim 0.2$). In a more dispersive medium such that one used by Verscharen et al. (2012), the dispersion relations probably cannot construct a transversal spectrum of waves for the decay of a field-aligned Alfvén wave.”

Reviewer:
2. About Figure 3. Bottom panels show the spectral pattern of both daughter waves (sound and Alfvén). Different peaks are labelled according to their 3-wave coupling described in Fig.1. However, there is no indication about the parallel A^0 and S^0 modes (present in the top panels). On the other hand, the most powerful signatures in the bottom panels are not labeled; should one conclude that they are the daughters labeled with "0" in Fig. 1? If so, why those modes have a non-zero $k_\perp$ in spectra of Fig. 4 ($k_\perp \sim 0.03$)? Does it mean that the parallel daughters of the top panels are not exactly field-aligned and have a finite $k_\perp$? This could be an interesting result. In any case, I think it would be useful if authors could extend the axis of the lower panels in order to include the $k_\perp = 0$ condition and show the reader the exact location of the "0" modes and if they lie on the $k_{||}$ axis as expected.

Reply:
No, the most powerful signatures in the bottom panels of Fig. 3 are not the daughters labeled with "0" in Fig. 1. By labeling several modes in the bottom panels we suggested several possible 3-wave coupling processes while the mentioned modes cannot fulfill the same vector triangle with the pump wave. The daughter modes labeled with "0" are exact parallel-propagating waves ($k_\perp$=0). We did not show them in the bottom panels because the vertical axes have a logarithmic scale in order to delimitate the labels $A_2^{(\dots)}$ and $S_2^{(\dots)}$.

We follow the suggestion of the reviewer to include $k_\perp = 0$ and extend the axes for both positive and negative values of the perpendicular wavenumbers. We also flip the sign of frequency to comply the conventional wave-wave coupling diagram (parallelogram diagram) in the k-omega domain shown in Fig. 1. In this way the left-handed polarization of the involved waves is conventionally represented for positive values of the frequency.

Changes in the manuscript:
Figure 3 is updated.

Reviewer:
3. What is the exact amplitude of the initial wave? This can be approximately inferred from Fig.2, however the information

should appear clearly in the text. Indeed, the daughter and sound waves saturate at a surprisingly low level ($\delta b \sim \delta\rho \sim 0.01$). Do the authors have an explanation for this? Does it depend on the - quite high - amplitude of the mother wave? or is it for a different reason (see next comment)?

Reply:
The magnetic-field amplitude of the pump wave (normalized to the value of the background magnetic field) has a value of 0.2. Thank you very much for pointing out this lacking information in the original manuscript.

We note that the overall background level of the 1-D reduced density spectrum of density fluctuation $\delta\rho(k_{\|})$ is significantly lower than the 1-D reduced density spectrum obtained from an equivalent 2-D simulation. However, the amplitude of the driven daughter mode related to the background level is approximatively the same between the 3-D and 2-D setups. The same kind of result is obtained for the 1-D reduced spectrum of the magnetic field fluctuations.

No, we do not think that the large amplitude of the Alfvén pump wave could have a significant role here. The analytic analyses, see e.g., Viñas and Goldstein (1991b), show that the growth rate of the decay instability increases as the amplitude of the pump wave is getting larger values, and a weaker slope is observed at about $\eta = \delta B/B_0 > 0.4$ for the parallel propagation case. Naively speaking, one may expect that the amplitude of the daughter modes should be larger as well, as long as there is no saturation of the instability with respect to the pump wave amplitude.

Changes in the manuscript:
Page 4, Line 9 - Line 10:
"The magnetic-field amplitude of the Alfvén pump wave (normalized to the value of the background magnetic field) has a value of 0.2."
Page 4, Line 23 - Line 25:
"The low level of the amplitude of the magnetic field and density daughter modes in Fig. 2 could be a consequence of the lower level of the fluctuation background developed in the 3-D system with respect to that one developed in a two-dimensional system with equivalent characteristics."

Reviewer:
4. This study is performed with an hybrid model, retaining ion kinetics, but through the manuscript no aspects about ion dynamics are mentioned or discussed. For example the fact that the instability saturates when the pump wave is still dominant could be a consequence of the kinetic nature of the plasma. Unlike MHD, where the instability can saturate only through the steepening of the excited sound waves, in hybrid it saturates via particle trapping and phase-space modulation (e.g. Matteini et al. JGR 2010). A consequence of this dynamics is a significant perturbation of the ion VDF, leading also to the generation of field-aligned beams (e.g. Araneda et al. 2008). I think it would be useful to add some information about the evolution of the particle VDF during the process, and possibly show it in a figure to also exploit the advantage of the hybrid model over fluid ones.

Reply:
We thank very much the reviewer for the suggestion.
In low beta plasmas, the decay instability dominates over the other parametric instabilities. Plasma beta values for both ions and electrons play also an important role in the dynamics of the parametric decay. For instance, β_e is responsible for activating different saturation mechanisms. Matteini et al. (2010) JGR reported that for cold fluid electrons ($\beta_e \sim 0$), the MHD saturation mechanism is recovered. At larger values (e.g., $\beta_e \sim 0.1$), the trapping and beam formation is in use.
The electron plasma beta β_e in our simulation has an intermediate value of 0.01. Information on the particles has not been saved from this simulation but particle data are available from a former 3-D simulation performed at a lower resolution at time $t\Omega_p = 300$ and at the final time $t\Omega_p = 600$ (Comişel et al. 2018). The parallel distribution in the phase space $z - v_{\|}$ (not shown) at a time still close to the linear phase of the instability ($t\Omega_p = 300$) suggests that particles are confined and accelerated in different regions along the parallel axis to the mean magnetic field. The phase-space modulation can be a signature for the instability saturation. There is no evidence of field-aligned velocity beam at the latest time of the simulation. The accelerated particles are thermalized via pitch-angle diffusion by the developed oblique modes.

Changes in the manuscript:
Page 10, Line 8 - Line 20:
“Unlike MHD simulations, where the instability can saturate only through the steepening of the excited sound waves, in hybrid it saturates via particle trapping and phase-space modulation (e.g. Matteini et al. JGR 2010). A consequence of this dynamics is a significant perturbation of the ion velocity distribution function, leading also to the generation of field-aligned beams (e.g. Araneda et al. 2008). In low beta plasmas, the decay instability dominates over the other parametric instabilities. Plasma beta values for both ions and electrons play also an important rule in the dynamics of the parametric decay. For instance, β_e is responsible for activating different saturation mechanisms. Matteini et al. (2010) JGR reported that for cold fluid electrons ($\beta_e \sim 0$), the MHD saturation mechanism is recovered. At larger values (e.g., $\beta_e \sim 0.1$), the trapping and beam formation is in use. The electron plasma β_e in our simulation has an intermediate value of 0.01. The parallel distribution in the phase space $z - v_{||}$ (not shown) at a time still close to the linear phase of the instability ($t\Omega_p = 300$) suggests that particles are confined and accelerated in different regions along the parallel axis to the mean magnetic field. The phase-space modulation can be a signature for the instability saturation. There is no evidence of field-aligned velocity beam at the latest time of the simulation. The accelerated particles are thermalized via pitch-angle diffusion by the developed oblique modes.”
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