**Answer to Anonymous Referee #2 Interactive comment on "Model of Propagation of VLF Beams in the Waveguide Earth-Ionosphere. Principles of Tensor Impedance Method in Multilayered Gyrotropic Waveguides" by Yuriy Rapoport et al.**

We are very grateful to the Anonymous Referee #2 and are answering directly to each of the main his comments and will answer to each of the minor comments in the final text of the revised paper as well. We believe that the paper becomes really better and more clear, informative and useful for the readers due to the improvements inspired by the Referee.

**The section "5. Influence of the parameters of WGEI on the polarization transformation and losses of the propagating VLF waves" (P21-24 L485-581)** (added especially in response to comments (2), (3) of the Reviewer) and the Figures 8-10 are new, and the corresponding Conclusions are added. In our answers to the Referee's comment, we refer to the improvements in the paper, noting corresponding pages and lines such as f.e. "**P29 L696-703**", respectively, what means: page 31, lines 696-703. The questions of the Referee are revealed below by "black bold" font.

**(1) The interesting goal in a model is to determine what are the main parameters for the increase/decrease of the EM field. Here in this paper we only have a variation of one parameter: the electron density.**

Concerning the perturbations in the electron concentration: we include in the revised paper, besides increase or decrease in electron concentration, **Fig. 5, P19 L460-463**, which has been present already in the previous version of the paper, also parametrization of the perturbation in electron concentration. This parameterization is made of the basis of such parameters, as maximum of the perturbation, region, where it is concentrated and effective width of this distribution by the vertical coordinate, see **P23 L.533-540** and formula (35), **P22 L536.** Then in response to this note of the Referee 2, we added also an influence on the electrodynamics characteristics of the VLF beams propagating in the waveguide Earth-Ion ionosphere (WGEI) of the angle of the inclination of geomagnetic field and carrier frequency on the the beam (**P22-23 L510-531; P 23-24 L538-580**). Then besides just field spatial distributions, as an object of the influence of the parameters, we include two other very interesting and important parameters. One of them is a parameter of the polarization transformation $|E_y/H_y|$, described in new Sect. 5, **P21 L489-506**. This value can be measured and characterizes the effect of the gyrotropy and anisotropy, both volume and surface, described by the tensor surface impedances at the lower and upper boundaries of the waveguide Earth-Ionosphere (WGEI). The other value, the influence on which we analyze, is the complex tensor impedance, in particular real and imaginary parts of its diagonal element $Z_{011}$. In particular, an influence on it of the inclination angle of the geomagnetic field and carrier beam frequency are included into the revised paper. Then we analyze also losses of the VLF beam **(P21 L506-509)**. The details concerning these extra simulations with the influences on the propagation of a beam in the WGEI of different parameters are presented in the Fig. 8**(P22 L512-522),** Fig. 9**(P23 L541-548),** Fig.10**(P24**

**L563-569)** and the texts with explanations included after these Figures in the (new) Sect. 5, revealed in the revised paper by red color.

Note that the Figures 11-13 (**P 25-26**) in the paper after the revision are the same, as Figures 8-10 in the paper before the revision (while the Figures 8-11 (**P22-24**) in the revised paper are new, as it was already mentioned). The Figures 11-13 describe experimentally and (qualitatively) theoretically a common effect of seismogenic changing electron concentration and collision frequency. The last changes are described in our previous model (Rapoport et al. 2006) of the influence on the ionosphere of the seismogenic electrostatic field including the corresponding heating-photochemistry effects. This is rather complicated mechanism which includes a set of parameters in the ionosphere (the number of which is >> 1) such as characteristics of external electrostatic sources placed in the lower atmosphere and mesosphere, photochemistry parameters, electron concentration distribution, photochemistry parameters etc., effect of which is reflected finally in Figs. 12 a- c (**P25-26 L587-598**). In the modeling beam propagation in the WGEI, we used only qualitatively the fact, that as a result of mentioned above seismogenic mechanism, electron concentration $N_e$ and collision frequency $\nu_e$ change simultaneously ($N_e$ and $\nu_e$ increase and decrease, respectively). As a result, the curve 2, comparatively to the curve 1 in Fig. 13 a,b (**P26, L599-606**), reflects an effect of additional losses of VLF beam, which occurs due to simultaneous change of not *one,* but *two* parameters ($N_e$ and $\nu_e$), and, moreover, these parameters change their signs consistently. See also the text in Sect. 6, **P25 L582-584; P26-27 608-626.**

**(2) Here in this paper we only have a variation of one parameter: the electron density. It means that you show something which is evident: when the density increases the electric field decreases.**

As it is already mentioned in item (1), a variations of other parameters are included as well, such as geomagnetic field inclination angle, beam carrier frequency, **value of surface impedance/conductivity of the lower boundary of the WGEI (atmosphere-Earth boundary)**. **Corresponding results are described in more details below (see item (3) ).** But even effects from electron variations are *not* evident! Namely:

(i) The simultaneous effect from change of electron concentration $N_e$ and collision frequency $\nu_e$ is non-trivial. First, as described above, this effect is caused by the consistent change of two different parameters, and consistent change in their signs is described by rather complicated model of seismogenic electrostatic-heating-photochemistry effects, described in particular in (Rapoport et al. 2006; Grimalsky et al. 2003). The consistent change of signs of both $N_e$ and $\nu_e$ causes the proper change in VLF losses/amplitude of the electromagnetic field (curve 2 in Figs. 13 a, b, **P26 L599-606**), which qualitatively corresponds to the effect observed before strong earthquake (Fig. 11, **P25 L585-590**). These results are summarized in the Conclusion **(3), P29 L688-690**.

(ii) As it is seen from Fig. 9 (**P23 L541-548**), the change in the concentration in the lower ionosphere causes rather nontrivial effect on the parameter of polarization transformation |Ey/Hy|. Note that either increase or decrease in the ionosphere plasma concentration have been reported as a result of seismogenic phenomena, tsunamis, particle precipitation in the ionosphere due to wave-particle interaction in the radiation belts (Pulinets et al. 2005; Shinagawa et al. 2013; Arnoldy et al. 1989; Glukhov et al. 1992; Tolstoy et al. 1986) etc. Namely, this effect does not reduces only to increase (Fig. 9 b) or decrease (Fig. 9 c) of the maximum value of the polarization transformation parameter |Ey/Hy|. Note also that the corresponding change of this parameter has rather remarkable absolute values from dozens to thousands percent, as it is seen from the Figs. 9 b, c , Fig. 8 c. It is even more interesting that in the case of decreasing (Fig. 9 a, curve 2) electron concentration, the main maximum of |Ey/Hy| appears in the lower atmosphere (at the altitude around 20 km, Fig. 9 b, curve 3, which corresponds to $\omega = 1.14.\cdot 10^5 s^{-1}$). In the case of increasing electron concentration (Fig. 9a, curve 3) the main maximum of |Ey/Hy| appears near the E region of the ionosphere (at the altitude around 77 km, Fig. 9c). The secondary maximum which is placed, in the absence of the perturbation of the electron concentration, in the lower atmosphere (Fig. 8 c, curves 2, 3, **P22 L512-522**), or mesosphere/ionospheric D region ((Fig. 8_c, curve 1), practically disappears or just is not seen in the present scale, if electron concentration increases (Fig. 9 c, curves 1-3, **P23, L541-548**).

**The new effects mentioned in item (ii) above is described in the revised paper in P23-24 L549-562. These results are summarized in the Conclusion (6), item (ii), P29 L708-714.**

**(3) What is the effect of other parameters as the magnetic field inclination for example? The plasma frequency?**

As it was already mentioned above in the paragraph (1), in response to this comment, we have demonstrated an influence of the change in the angle of the inclination of geomagnetic field and carrier frequency on the electromagnetic characteristics of propagating EM VLF beam. Then besides just field spatial distributions, as an object of the influence of the parameters, we include two other very interesting and important parameters. One of them is polarization transformation. This value can be measured and characterizes the effect of the gyrotropy and anisotropy, both volume and surface, described by the tensor surface impedances at the lower and upper boundaries of the waveguide Earth-Ionosphere (WGEI). The other value, an influence on which we analyze, is the complex tensor impedance, in particular real and imaginary parts of its diagonal element $Z_{011}$. In particular an influence on it of the inclination angle of the geomagnetic field and carrier beam frequency are included into the revised paper. The influence on the losses of the VLF beam carrier frequency is analyzed as well. The following results are obtained, in particular, (and reflected in the new Figs. 8 -10, **P22-24**) basing on this new modeling, performed in the response to the present Reviewer's note.

(i) As it is seen from Figs. 8 a-c (**P22 L512-522**), the altitude dependence of the polarization parameter $|E_y/H_y|$ has two main maxima in the WGEI, the higher of which lies in the gyrotropic region above 70 km, while the other in the isotropic region of the WGEI. As it is seen from Fig. 8 a, b, the value of the larger second maximum increases, while the value of the second maximum decreases and its position shifts to the lower altitudes with increasing frequency. At the higher frequency ($\omega = 1.14 \cdot 10^5 c^{-1}$), the larger maximum of the polarization parameter corresponds to the intermediate value of the angle $\theta = 45^\circ$ (Fig. 8b); for the lower frequency ($\omega = 0.86.\cdot 10^5 c^{-1}$), the largest value of the first (higher) maximum corresponds to the almost vertical direction of the geomagnetic field ( $\theta = 5^{\circ}$ , Fig. 8a). For the intermediate value of the angle ( $\theta = 45^{\circ}$ ), the largest value of the main maximum corresponds to the higher frequency ( $\omega = 1.14 \cdot 10^{5} c^{-1}$ ) in the considered frequency range (Fig. 8c). The total losses increase monotonically with increasing frequency and depend weakly on the value of $\theta$ (Fig. 8d).

**These results are included in the paper after Figure 8 (P22 L512-522) in (P22 L523-531) and summarized in Conclusion (6), item (i), P29 L700-707.**

(ii) Again, as it was already mentioned, as it is seen from Fig. 9 (**P23 L541-548**), the change in the concentration (which determines plasma frequency) in the lower ionosphere causes rather nontrivial effect on the polarization transformation parameter |Ey/Hy |. Note that either increase or decrease in the ionosphere plasma concentration have been reported as a result of seismogenic phenomena, tsunamis, particle precipitation in the ionosphere due to wave-particle interaction in the radiation belts (Pulinets et al. 2005; Shinagawa et al. 2013; Arnoldy et al. 1989; Glukhov et al. 1992; Tolstoy et al. 1986) etc. Namely, this effect does not reduces only to increase (Fig. 9 b) or decrease (Fig. 9 c) of the maximum value of the polarization transformation parameter |Ey/Hy|. Note also that the corresponding change of this parameter has rather remarkable absolute values from dozens to thousands percent, as it is seen from the comparison between Figs. 9 b, c (**P23 L541-548**) and Fig. 8 c, curve 3, **P22, L512-522**. The last curve corresponds to the unperturbed distribution of the ionospheric electron concentration (see curve 1, Fig. 5b, **P19 L460-463**, and curve 1 in Fig. 9). It is even more interesting that in the case of decreasing (Fig. 9 a, curve 2 ) electron concentration, the main maximum of |Ey/Hy | appears in the lower atmosphere (at the altitude around 20 km, Fig. 9 b, curve 3, which corresponds to $\omega = 1.14. \cdot 10^{5} c^{-1}$ ). In the case of increasing electron concentration (Fig. 9 a, curve 3) the main maximum of |Ey/Hy | appears near the E region of the ionosphere (at the altitude around 77 km), while the secondary maximum placed, in the absence of the perturbation of the electron concentration, in the lower atmosphere (Fig. 8, curves 2, 3, **P22, L512-522**), or mesosphere/ionospheric D region ((Fig. 8_new c, curve 1), practically disappears or just is not seen in the present scale, in the case under consideration (Fig. 9 c, curves 1-3, **P23, L541-548**).

**These results are presented in the revised paper after Fig. 9, in P23-24 L549-562, and summarized in Conclusion (6), item (ii), P29-30 L708-725.**

(iii) As it is seen from Fig. 10 (**P24 L563-569)**, the real (a) and imaginary (b) parts of the surface impedance at the upper boundary of the WGEI have a quasiperiodical character with the amplitude of "oscillations" occurring around some effective average values (not shown explicitly in Figs. 10 a, b) decreases with increasing the angle $\theta$ . Even without the determination of the exact average values for each of the curves 1-4 in Figs. 10 a, b, it is seen that corresponding average values of Re($Z_{011}$) and Im($Z_{011}$) , in general, decrease with increasing angle $\theta$ . It is also seen that average values of Re($Z_{011}$) for $\theta$ equal to $5^{\circ}, 30^{\circ}, 45^{\circ}$ and $60^{\circ}$ (curves 1-4 in Fig. 10 a) and Im($Z_{011}$) corresponding to $\theta$ equal to $45^{\circ}$ and $60^{\circ}$ (curves 3, 4 in Fig. 10 b), increase with increasing frequency in the considered frequency range (0.86-1.14)·$10^5$ $s^{-1}$. The average values of Im($Z_{011}$) corresponding to $\theta$ equal to $5^\circ$ and $30^\circ$, change in the frequency range (0.86-1.14)·$10^5$ $c^{-1}$ non-monotonically, having maximum values around frequency (1-1.1)·$10^5$ $s^{-1}$.

**These results are included in the revised paper after Figure 10 (P24 L563-569), in (L24 L571-580) and summarized in Conclusion (6), item (iii), P29-30 L713-723.**

(iv) It is interesting to note that the value of finite impedance at the lower (Earth-atmosphere) boundary of the WGEI make a quite observable influence on the minimum value of the polarization transformation parameter near the E region of the ionosphere (curves 1, 2 in the Fig. 10 c, **P24 L562-568**). Namely, the decrease of surface impedance *Z* at the lower boundary (Earth-atmosphere) of the WGEI in two orders causes the increase of the corresponding minimum value of |*Ey/Hy*| in ~ 100% (compare minima in the curves 1 and 2 around *z*=70 km).

**These results are presented in the end of the text after the caption to Figure 10, P34, L22-26, and summarized in Conclusion (7), item (iv), P30 L722-725.**

Concerning the plasma frequency ($\omega_{pe,i}$) it is proportional to the square root of the electron concentration $N_e$ and included into complex conductivity and complex tensor $\hat{\varepsilon}$. These values determine also effective tensor of surface impedance. Therefore all of the electrodynamics characteristics both presented in the initial version of the paper and added and reflected in the new Figs. 8-10, **P22-24,** made in the response to the Reviewer's notes, are influenced by these volume tensor $\hat{\varepsilon}$ and surface impedance $Z_{0ij}(i,j=1,2)$ and therefore by the altitude distributions of electron concentration $n$. For example, change in the altitude distribution $n$ (Fig. 5b, **P19 L460-463**) causes change in $\hat{\varepsilon}$ (Figs. 5 c, d) and surface tensor impedance $\hat{Z}_0$ (Table 1, **P19-20 L470-471**). As a result, spatial distributions of the electromagnetic field change (Fig. 6, **P20 L472-475**; Fig. 7, **P20 L476-479**). Due to changes in electron concentration (and therefore $\omega_{pe,i}$), transformation polarization parameter |*Ey/Hy*| and its altitude distribution change very non-trivially (Fig. 9, **P23 L541-548**). There are only couple of examples, but in any of the other results obtained in the paper, an influence of $n$ and, respectively, $\omega_p$ are reflected. We do not emphasize separately $\omega_{pe,i}$ because they are determined unambiguously by $n$, the influence of which on the electrodynamics characteristics of the VLF beam is investigated rather in details, and the masses of the corresponding particles, while the latter are assumed to be given within the framework of the approximations adopted in this article.

**(4) Why the calculation is stopped at 80 km ?**

The proposed new tensor impedance method for modeling propagation of electromagnetic beams (TIMEB) and the developed model allows and we really did the simulations of (all) the electromagnetic field components both inside the WGEI ($0<z<L_z$, $L_z$=85 km) and above the WGEI ($L_z<z<L_{max}$, $L_{max}$=300 km). Nevertheless the

detailed calculations of the field distribution outside the WGEI will be not presented in this paper and will be published elsewhere. This paper is devoted, besides the new method in general, also to the propagation of the beam *inside* the WGEI, i.e. in the range of altitudes . $0<z<L_z$ , $L_z$=85 km. The calculations of the field above the WGEI are performed in this context only to establish /confirm the present approximation of the propagation of the beam *inside* the WGEI . In other words the results of the field calculations above the WGEI, in the region $L_z<z<L_{max}$,, $L_{max}$=300 km confirms that the region $0<z<L_z$ , $L_z$=85 km is really a good waveguide for the VLF field accounting for the effects of the gyrotropy and anisotropy for the plasma-like media placed in the inclined geomagnetic field. This fact is really confirmed by the calculations of (all) field components performed for the range of the altitudes from 0 to 300 km. Nevertheless because, as it was already mentioned above, this paper is devoted to the beam propagation in the WGEI, only final qualitative conclusion based on the calculation above the WGEI is presented in the paper. Namely, in the response to the present Reviewer's question, the following text is added into the paper **(P27 L627-634):**

"The TIMEB is a new method of modeling characteristics of the WGEI. The results of beam propagation in WGEI modeling presented above include the range of altitudes inside the WGEI (see Figs. 4-7). Nevertheless, the TIMEB method described by Eqs. (15)-(19), (22-24), (27), (30) and allows to determine all field components in the range of altitudes $0 \le z \le L_{\max}$, where $L_{\max} = 300$ km . The structure and behavior of these eigenmodes in the WGEI and leakage waves will be a subject of separate papers. We present here only the final qualitative result of the simulations. In the range $L_z \le z \le L_{\max}$, where $L_z = 85$ km is the upper boundary of the effective WGEI, all field components are (1) at least one order of altitude less than the corresponding maximal field value in the WGEI and (2) field components have the oscillating character along $z$ coordinate and describe the modes, leaking from the WGEI."

**These results are summarized in the paragraph (7) of the Conclusions, P30 L726-728.**

To the details concerning the waves leaking from the WGEI and comparison of the theoretical simulations to the results of the corresponding observations, the special and separate paper will be devoted. And all the calculations for the field in the altitude range above the WGEI will be suitable for a direct inclusion only in this paper.

**(5) In Figure 4 why Ey is oscillating along Z ?**

**The following note is made in the revised paper in the response to this question of the Reviewer, see . P27 L635-643:**

"Let us make a note also on the dependences of the field components in the WGEI on the vertical coordinate ($z$). The initial distribution of the electromagnetic field with altitude $z$ (Fig. 4a) is determined by the boundary conditions of the beam (see Eq. (32)). The field component includes higher eigenmodes of the WGEI. The higher-order modes experienced quite large losses and practically disappear after beam propagation on 1000 km distance. This determines the change in altitude ($z$) and transverse ($y$) distributions of the beam field during propagation along the WGEI. In particular, at the distance $x$=600 km from the beam input (Figs. 4b, c), the few lowest modes of the WGEI along $z$ and $y$ coordinates still persist. At distance $x$=1000 km (Fig. 4d, c; Fig. 6e, f; and Fig. 7a, b), only the main mode persists in the z direction. Note, the described field structure correspond to real WGEI with losses. The gyrotropy and anisotropy causes the volume effects and surface impedance, in distinction to the ideal planar metallized waveguide with isotropic filling (Collin, 2001). “

**Minor points:**

**The English is not fluent and there are many mistakes (or typos) which can be easily corrected with a word processor**

The English language is polished with many relevant grammatical and stylistic improvements that did not stand out in the text of the reviewed article due to their significant number. The collective of authors in particular those who are working in USA and UK and working abroad for a long time did this.

**Page 3 line 21 To Be Corrected**

This line is written in the revised paper (**P3 L75-76**) in the form: “Some other details on the distinctions from the previously published models are given below in Sect. 3.”

Just in case I include below also the lines neighboring to the corresponding line in the revised paper (**P3 L73-81**):

**“**This combined approach, finally, creates the possibility to interpret adequately a data of both ground-based and satellite detection on the EM wave/beam propagating in the WGEI and those, which experienced a leakage from the WGEI into the upper ionosphere and magnetosphere. Some other details on the distinctions from the previously published models are given below in Sect. 3.

The methods of effective boundary conditions such as effective impedance conditions (Tretyakov, 2003; Senior and Volakis, 1995; Kurushin and Nefedov, 1983) are well-known and can be used, in particular, for the layered metal-dielectric, metamaterial and gyrotropic active layered and waveguiding media of different types (Tretyakov, 2003; Senior and Volakis, 1995; Kurushin and Nefedov, 1983; Collin, 2001; Wait, 1996) including plasma-like solid state (Ruibys, and Tolutis, 1983) and space plasma (Wait, 1996) media.. **”**

As we see now, this peace of this text (incl. line (**P3 L75-76**)) in the revised paper is correct.

**Page 3 line 25 Wait –** done, and the name “**Wait**” is written right and revealed using the red color font everywhere in the revised paper, see **P3 L80, 81, 102; P8 L196; P35 L911**.

**(I have not checked the references but I have seen that Ruibie & Tolutue is not correct) – this reference is** improved, as "Ruibys, and Tolutis" everywhere in the revised paper, see **P3 L80, 82, 102**; **P35 L878**.

**Page 4 line 10 – waves:** changed to "electromagnetic waves", **P3 L101**.

**Page 4 line 11 LAIM appear before and then must be explained before –** "LAIM" is explained now in the second line of the Introduction, see **P1 L33**.

**Legend of Figure 1 is too long. A part must be in the text (it is also true for other figures).**

As it was recommended, the caption to Figure 1 is reduced. Namely the **(modified for the revised paper)** phrases: "In Fig. 1, $\theta$ is the angle between the directions of the vertical axis $z$ and geomagnetic field $\vec{H}_0$. Note that theta $\theta$ angle is complementary to the angle of inclination of the geomagnetic field. Geomagnetic field $\vec{H}_0$ is directed along $z'$ axis, lies in the plane $xz$, while the planes $x'z'$ and $xz$ coincide with each other."

are transferred from the end of the caption to Figure 1 to the text of the article before Figure 1, see **P4-5 L135-138**. The captions to the remaining figures includes only the data necessary to identify these figures, and to distinguish among themselves the different curves in each figure and are necessary. The information included in the captions is fundamentally necessary to provide readers with the opportunity to quickly find out what the meaning of each of the Figures separately is, as well as all the Figures in general. As for the description of the figures in the text of the article, it is included for each figure separately and when comparing the physical effect between themselves, illustrated by various figures or groups of figures. At the same time, a description is also given for the corresponding figures in the necessary proportion while the basic physical effects, illustrated by the corresponding figures are described. These physical effects are mainly then included in the Conclusions.

**Of course I have not checked the correctness of all equations but I have seen an error in the first equation (equation (1)) for the ion plasma frequency.**

The mass of electrons $m_e$ (present in the formula (1) (**P6, L167**) for the square of plasma frequency) is replaced by the mass of ions $m_i$. Now the formulas (1) for electron and ion plasma concentrations in the accepted approximation of the three-component plasma-like ionosphere (including electron, effective one type of ion and effective one type of neutral components) and quasineutrality, are right. The phrase describing this approximation is included into the text of the paper, Subsection 3.1.1, see **P6 L170-175**.

**Page 6 line 5 and line 17 the sign inside exp() is different.**

Yes, this is right- there are two parts of the argument of the phase multiplier ~exp$(i\omega t - ik_x x)$, which really have different signs, see **P5 L162**, **P6 L179** in revised paper.

**Page 7 the values of BETAij are not clear. What parameters they contain ?**

Please look at the line placed 3 lines above the upper formula from Eqs. (1), namely (**P5 L163)**. It is written there: $\hat{\beta} = \hat{\varepsilon}^{-1}$, or $\vec{E} = \hat{\beta}\vec{D}$ (the arguments of $\hat{\varepsilon}, \hat{\beta}$ are omitted here, but included in the paper. Therefore the tensor $\hat{\beta}$, inverse respectively to $\hat{\varepsilon}$ and depends on the same parameters as $\hat{\varepsilon}$, which is described in Sect. 3.1, in particular using formulas (1) . Therefore $\hat{\beta}$ depends on the same parameters as the tensor $\hat{\varepsilon}'$, described by formulas (1), with components, the altitude distributions of which is illustrated in Figs. 3 (**P17-18 L435-439)** and 5c, d (**P19 L460-463**) , and by the angle $\theta$ and corresponding standard rotation matrices, mentioned in (**P6 L173**), the fifth line after Eq. (1) (and not included explicitly). Please note that due to chosen subject- layered anisotropy and gyrotropy inhomogeneous plasma-like Earth-Atmosphere-Ionosphere media - and respectively to do "what is necessary and how it is necessary", we are forced to choice combined analytical-numerical approach. Namely, all what is possible, we are doing analytically and all other- numerically. In particular the tensor $\hat{\beta}$ is obtained, using proper formulas for reversal tensors (they are standard and not included in the paper), from the tensor $\hat{\varepsilon}$.

**Page 9 line 10 respectively two time –** yes, this phrase is improved now as follows, see **P9 230-231:**

" $k_x^{'}$ and $k_z^{'}$ are the transverse and longitudinal components of wave number relative to geomagnetic field."

**Page 9 line 15 relation - The matrix at the end of equation (14) seems strange. The left lower element is not 1-? –** thank you very much, the typos in this formula is improved, see **P9 L238 (eq. (14))**. Namely, the upper right element is (-1-i). Just in case, this matrix has been obtained analytically by means of few different approaches independently, with the same result.

**Page 13 another parameter DELTA appears here. Is the DELTA in equation (11) similar to the DELTA in equation (24) ?** - thank you, there are different values. To distinguish between them, the corresponding value in equation (24) is re-denoted now as $\Delta_0$, see **P13 L336**.

**Title 3.5 too long.-** Yes, the title of the Subsect. 3.5 is shortened as follows **(P16 L397)**:
**"3.5 VLF Waveguide Modes and Reflection from the VLF Waveguide Upper Effective Boundary ."**

**Page 16 a lot of typos, discharges, demonstrating, speaking, present, presentation. – improved, namely:**

The first two phrases in the beginning of Chapter 3.5, namely

"Our model, in general, needs the consideration of the excitations of the waveguide modes by means of current sources such as dipole-like VLF radio source and lightning discahrges. Then, we will present the results of the reflection of the waves incident on the upper boundary (*z=Lz*) of the effective WGEI demonsrtrating that this structure has indeed good enough waveguiding properties."

Are replaced by **(P16 L398-401)**:

"In general, our model needs the consideration of the waveguide modes excitations by a current source such as dipole-like VLF radio source and lightning discharge. Then, the reflection of the waves incident on the upper boundary ($z$=$L_z$) of the effective WGEI can be considered. There will be possible to demonstrate that this structure has indeed good enough waveguiding properties"

As a result of this replacement:

**"discharges" –** replaced by "discharge" **(P16, L399)**
**"demonsrtrating" –**removed and the word "demonstrate" **(P16 L400)** is used in this piece of text in the revised paper.

**Then the following improvement are done in this text, in the first paragraph of Sect. 3.5, p. 16:**

**Shortly speaking -** removed
**Present; presentation –** are improved **(P16 L401,403)**.
**Page 17 line 1 these - corresponding piece of text (P16, 17 L409-413) is rephrased and the word ("these ") is included in P17 L413.**

**Page 17 line 9 why Figures 3_2 and 4_3 –** (the same concerns line 8) – improved as Fig. 3, Fig. 4 – see in the revised paper **P17 L420, 421.**

**In Figure 3 it is difficult to understand the contain of the panels b) to g) - Page 19 line 13 figure 5 –** Improved. Namely, quality of all panels in Figs. 3 and Fig. 5 is improved (widths of the lines increased, sizes of the letters and numbers in the Figures increased). Therefore the contain of all panels in Figs. 3, 5 should become understandable now, see **P17-18, L435-436; P19 L460**.

**Besides of that, after extra cross-checking the set of improvements have been done which only clarified the content of the paper and has not changed its physical sense. In particular the following extra improvements have been included in the revised paper.**

**Ssign in square root in the first of formula (9) is improved –** there was before mistakenly, line 17, p. 8:

$$\kappa_{1,2}{}^2 = \frac{\beta_{11}+\beta_{22}}{2} \pm \left( (\frac{\beta_{11}+\beta_{22}}{2})^2 + \beta_{12}\beta_{21} \right)^{1/2}$$

There became after the improvement in the revised paper, **P8 L214**:

$$\kappa_{1,2}{}^2 = \frac{\beta_{11}+\beta_{22}}{2} \pm \left( (\frac{\beta_{11}-\beta_{22}}{2})^2 + \beta_{12}\beta_{21} \right)^{1/2}$$

**Note that besides improving Figs. 3 (P17-18 L435-439), 5 (P19 L460-463), In accordance with Reviewer's requirements, the new Figs. 8, 9, 10 (P22 L512-522; P23 L541-548; P24 L563-569) are prepared, and then Figs. 11-14 from the revised paper got now the new numbers (11-14 mentioned above). The numbers of these figs. in the previous version of the paper were 8-10.**

**In Eq. 27 (P14 L345) the value $\Delta$ is re-denoted as $\Delta_y$ (in P14 L346, placed after Eq. (27)) to distinguish it from the similar value $\Delta$ included in Eq. 30b (P14 L358) or, more accurately speaking in the Eqs. for $\gamma_{12},...,\tilde{\beta}_{11}...\Delta$ written in (P14 L360) *after* Eq. (30b).**

**The value included into the first of Eqs. (30b) (P14 L358) has not been determined in the previous version of the paper. In the revised paper, the value $\tilde{\varepsilon}_{22}$ is included into the set of Eqs. in the (P14 L360) placed after formulas (30b).**

**The impedance *Z* and corresponding impedance elements $Z_{ij}$** (I,j=1,2) **are re-denouted as $Z_0$ and $Z_{0ij}$, respectively everywhere in the paper, to distinguish impedance from the vertical coordinate *Z*. This concerns, f.e. formulas 22 (P11-12 L 289-292) for the elements $Z_{ij}$** (I,j=1,2) **of the impedance tensor; caption to Fig. 10 (P24 L568, 569); in the Table 1 (P19-20 L470-471) with the values of tensor impedance components etc.; extra column is added in the Table 1 with the proper titles of the corresponding lines of this table.**

**The Reference to** Yaxin et al., 2012 in previous version of the paper is rewritten as (Yu et al. 2012) **in (P2 L66) in the revised paper; the reference to** (Artemyev et al., 2015), missed previously, is added in the revised paper in **(P2 L59); the reference to (**Yiğit et al., 2016), missed in the previous version of the paper is added in the revised paper in **(P2 L38)**.

**Abstact and Conclusion 1 are improved stylistically.**