Reply to Referee_2

I would like to thank the Referee_2 for the questions and suggestions that helped me improve the manuscript. Below I will provide answers to your questions.

Referee_2:

Specific comments:

1. On the effect of the background electric field and the neutral wind on the RTI: The conclusion that the background electric field and neutral wind do not impact the RTI is not substantiated in my view. Perhaps I did not follow the derivation presented in section 2 well, but it appears to me that the background electric and neutral wind were not considered. In particular, line 94 states that the background electric field was set to zero, and the neutral wind, which is present in equation (1), is no longer present following the linearization in equation (9) and it's not clear to me why this is the case; was there a reference frame change to that of the neutral wind or was it also set to zero? In either case, the author should clarify this point and either justify this conclusion (by better explaining the derivation in section 2) or remove this conclusion from the manuscript.

Reply: For the sake of simplicity, the background electric field $\mathbf{E}$ and neutral wind $\boldsymbol{V_n}$ are set to zero during the derivation process, and $\mathbf{E}$ and $\boldsymbol{V_n}$ did not appear in the expression of linear growth rate of RTI. It should be noted that $\boldsymbol{V_n}$ and $\mathbf{E}$ also do not appear in the expression of the linear growth rate of RTI calculated by Kelley (2009, Chapter 4.2.1). When deriving the generalized linear growth rate of RTI, Kelley (2009, Chapter 4.2.2) thought that the fundamental destabilizing source of RTI is the current and the background electric field and neutral wind will drive a current, $\boldsymbol{J} = \boldsymbol{\sigma} \cdot \boldsymbol{E}'$ where $\boldsymbol{E}' = \boldsymbol{E_0} + \boldsymbol{U} \times \boldsymbol{B}$. However, as discussed in the manuscript that the effect of charge accumulation due to the divergence of the current can be neglected, and base on the above statement, it was considered that the background electric field and neutral wind do not impact the linear growth rate of RTI. Since, the fundamental destabilizing source of RTI was gravity, the generalizing process should be related to gravity-like forces.

In the deriving process in the manuscript, in the initial equilibrium state, the plasma is assumed stationary, that is $\boldsymbol{V^0} = 0$, $\boldsymbol{V} = \boldsymbol{V^1}$, and $\boldsymbol{V_n} = 0$. Since electric filed did not appear in the momentum equation, electric field will not appear in the expression of the linear growth rate of RTI. If a constant background electric field $\boldsymbol{E^0}$ is present, the plasma will moves with a constant velocity $\boldsymbol{V^0}$, with $\boldsymbol{V^0}$ and $\boldsymbol{E^0}$ related by the expression $c\boldsymbol{E^0} + \boldsymbol{V^0} \times \boldsymbol{B}^0 = 0$. To study the effect of background electric field and neutral wind on the linear growth rate of RTI, the initial equilibrium state condition should be changed. The initial equilibrium state condition is that the plasma moves with $\boldsymbol{V^0}$, $\boldsymbol{V} = \boldsymbol{V^0} + \boldsymbol{V^1}$ and $\boldsymbol{V_n} \neq 0$.

The momentum equation is

$$\frac{\partial(\rho \boldsymbol{V})}{\partial t} = \frac{1}{c}\boldsymbol{J}\times\boldsymbol{B} + \rho\boldsymbol{g} - \nabla p - \rho\upsilon_{in}(\boldsymbol{V}-\boldsymbol{V}_n) \quad (1)$$

In initial equilibrium state, the left term Eq.(1) vanish, the $\boldsymbol{J}\times\boldsymbol{B}$ force and other forces are balanced, we have

$$\frac{1}{c}\boldsymbol{J}^0\times\boldsymbol{B}^0 + \rho^0\boldsymbol{g} - \nabla p^0 - \rho^0\upsilon_{in}(\boldsymbol{V}^{\boldsymbol{0}}-\boldsymbol{V}_n) = 0 \quad (2)$$

Linearizing Eq.(1), we have

$$\rho^0\frac{\partial \boldsymbol{V}^{\boldsymbol{1}}}{\partial t} + \boldsymbol{V}^{\boldsymbol{0}}\frac{\partial \rho^1}{\partial t} = \frac{1}{c}\boldsymbol{J}^0\times\boldsymbol{B}^1 + \frac{1}{c}\boldsymbol{J}^1\times\boldsymbol{B}^0 + \frac{1}{c}\boldsymbol{J}^0\times\boldsymbol{B}^0 + \rho^1\boldsymbol{g} + \rho^0\boldsymbol{g} - \nabla p^1 - \nabla p^0 - \rho^0\upsilon_{in}\boldsymbol{V}^{\boldsymbol{1}} - \rho^0\upsilon_{in}\boldsymbol{V}^{\boldsymbol{0}} - \rho^1\upsilon_{in}\boldsymbol{V}^{\boldsymbol{0}} - \rho^1\upsilon_{in}(-\boldsymbol{V}_n) - \rho^0\upsilon_{in}(-\boldsymbol{V}_n) \quad (3)$$

Substituting Eq.(2) into Eq.(3) we get

$$\rho^0\frac{\partial \boldsymbol{V}^{\boldsymbol{1}}}{\partial t} + \boldsymbol{V}^{\boldsymbol{0}}\frac{\partial \rho^1}{\partial t} = \frac{1}{c}\boldsymbol{J}^0\times\boldsymbol{B}^1 + \frac{1}{c}\boldsymbol{J}^1\times\boldsymbol{B}^0 + \rho^1\boldsymbol{g} - \nabla p^1 - \rho^0\upsilon_{in}\boldsymbol{V}^{\boldsymbol{1}} - \rho^1\upsilon_{in}\boldsymbol{V}^{\boldsymbol{0}} - \rho^1\upsilon_{in}(-\boldsymbol{V}_n) \quad (4)$$

Assuming perturbations in the form

$$\psi \propto \psi(y)e^{i(kx-\omega t)} \quad (5)$$

where ω is the frequency of the perturbation, k is the wave number.

For simplicity, assume $\boldsymbol{V}^{\boldsymbol{0}}=(0,\ V^0, 0)$, and $\boldsymbol{V}_n=(V_n, 0, 0)$. Which correspond to zonal background electric field and zonal neutral wind in the equatorial ionospheric F layer.

$\boldsymbol{z}\cdot\nabla\times$ Eq. (4) yields

$$-i\omega\left(ik\rho^0V_y - \frac{\partial}{\partial y}(\rho^0V_x) + ik\rho^1V^0\right) = -ik\rho^1g - \frac{1}{c}(\nabla\cdot\boldsymbol{J}^1)B^0 - \upsilon_{in}\left(ik\rho^0V_y - \frac{\partial}{\partial y}(\rho^0V_x) + ik\rho^1V^0\right) \quad (6)$$

Ignore the effect of charge accumulation and following the deriving process in the manuscript, we can get the maximum linear growth rate

$$\gamma = \sqrt{\frac{g+\upsilon_{in}V^0}{L} + \frac{{\upsilon_{in}}^2}{4}} - \frac{\upsilon_{in}}{2} \quad (7)$$

From above expression, we can see that g was replaced by $g^* = g + \upsilon_{in}V^0$. When y component of $\boldsymbol{V}_n$ is zero and $\boldsymbol{V}^{\boldsymbol{0}}$ has only x component, $\upsilon_{in}V^0$ is a gravity-like force. Since, the fundamental destabilizing source of RTI was gravity, the generalized maximum linear growth rate should be

$$\gamma = \sqrt{\frac{g+\upsilon_{in}(V_y^0-V_{ny})}{L} + \frac{{\upsilon_{in}}^2}{4}} - \frac{\upsilon_{in}}{2} \quad (8)$$

$\boldsymbol{V}^{\boldsymbol{0}}$ is related with background electric field $\boldsymbol{E}^{\boldsymbol{0}}$ with $c\boldsymbol{E}^{\boldsymbol{0}} + \boldsymbol{V}^{\boldsymbol{0}}\times\boldsymbol{B}^0 = 0$, the generalized linear growth rate now is

$$\gamma = \sqrt{\frac{g+\upsilon_{in}\left(\frac{cE_x^0}{B^0}-V_{ny}\right)}{L} + \frac{{\upsilon_{in}}^2}{4}} - \frac{\upsilon_{in}}{2} \quad (9)$$

The generalized linear growth rate is related to background electric field in x direction and neutral wind velocity in y direction. In equatorial ionospheric F region, zonal neutral wind velocity has no effect on the linear growth rate of RTI, eastward (westward) background electric field increase (decrease) the linear growth rate of RTI.

2. Exponential decay of plasma density with altitude: Equation (18) states that the initial plasma density has an exponential dependence on y. How does this assumption impact the updated description of the RTI shown in figure 5, where the plasma density is represented as a step function from n2=0 to n1? Should this formula for the plasma density be included in this schematic; i.e., should this formula be included for n1 to further complete this description? Further, is this description for L the same as other formulations; i.e., gradient scale length L in Kelley (2009)?

Reply: The linear growth rate of RTI for both plasma density that has an exponential dependence on y and plasma density that is a step function in y direction has been derived previously. Goldston and Rutherford (1995, Chapter 19.1) calculated the linear growth rate of RTI for plasma density that has an exponential dependence on y. The linear growth rate was

$$\gamma = \left(\frac{g}{L}\frac{h^2k^2}{h^2k^2+m^2\pi^2+h^2/4L^2}\right)^{\frac{1}{2}} \tag{10}$$

The linear growth rate for plasma density that is a step function in y direction previously calculated by Chandrasekhar (2013) was

$$\gamma = \left(g\,\frac{\rho_2-\rho_1}{\rho_2+\rho_1}k\right)^{\frac{1}{2}} \tag{11}$$

Where ρ_1 and ρ_2 are plasma density below and above the interface.

Following the deriving process of Chandrasekhar (2013), it can be easily shown that the linear growth rate for plasma density that is a step function in y direction is

$$\gamma = \sqrt{g\,\frac{\rho_2-\rho_1}{\rho_2+\rho_1}k + \frac{{v_{in}}^2}{4}} - \frac{v_{in}}{2} \tag{12}$$

The definition of density gradient scale length in Kelley (2009) is

$$L = \left(\frac{1}{\rho_0}\frac{\partial\rho_0}{\partial y}\right)^{-1} \tag{13}$$

The plasma density profile in the manuscript is

$$\rho^0(y) = \rho^0 e^{\frac{y}{L}} \tag{14}$$

It can be easily see that L in the density profile is gradient scale length the same as that in Kelley (2009).

3. General readability of derivation: It is suggested that the author attempt to improve the readability of the derivation in section 2; e.g., in lines 116-120, there are references the “the above relation” and it’s not clear to the reader which formula is being discussed, and some of the English is not entirely clear. For example, “the exact relation between E and V … is not simply cE+VxB = 0…” yet this relation is used to obtain equation (16). This section needs to be clarified.

Reply: I rewrited this section.” The exact relation between E and v in collisional plasma is not simply $c\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B} = 0$ (Vasyliunas, 2005). In collisional plasma, with a given E, the plasma will moves with velocity small than the velocity calculated from the relation $c\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B} = 0$. For simplicity we will use the relation $c\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B} = 0$ and note that the contribution of charge accumulation to the growth of RTI is maximized. From the relation $c\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B} = 0$ we get

$E_x = -\frac{1}{c} V_y B^0$ (16) "

4. Local RTI versus flux-tube integrated RTI: It is clear to this reader that the formulation in the current manuscript is focused on the "local" RTI, as opposed to the flux-tube integrated RTI, which was derived by Sultan (1996), which is arguably more "accurate" than any formulation of the local RTI, which includes only a 2-D description of the phenomenon that ignores aspects like interhemispherical asymmetries in physical parameters. While requesting the author to expand their formulation from 2-D to 3-D is clearly outside of the scope of this work, it is suggested to the author to include some mention of this previous work in their introduction and to include some comments on the differences between these approaches in their manuscript. Such additions would help readers better place this work in the context of previous theoretical works.
Reply: It should be noted that the work by Sultan (1996) was mentioned in the introduction (line 53). I added a paragraph in the discussion. " It should be noted that only the linear growth rate of local RTI was discussed up to now. However, in real circumstances, the linear growth rate of flux-tube integrated RTI should be calculated. Sultan (1996) calculated the linear growth rate of flux-tube integrated RTI, however, the derivation is based on the old physical description in which the fundamental destabilizing source of RTI is the current. The linear growth rate tends to infinity when the collision frequency approaches zero. A more accurate linear growth rate of flux-tube integrated RTI base on the new physical description should be calculated. "

Technical corrections:

1. Line 14: "calculations" instead of "calculation"
Reply: Done.
2. Line 24: "this ratio" instead of "the ratio"
Reply: Done
3. Lines 25-26: "previous physical description: : : is wrong" is quite strong language. I would suggest that perhaps "is inaccurate" is a better choice.
Reply: Done.
4. Line 45: Remove second "in"
Reply: Done
5. Line 53: I think the author means "From" from "Form"
Reply: Done.
6. Line 57: "contradicts" not "contradict"
Reply: Done
7. Lines 60-61: "equatorial plasma bubble" and "EPB" to plural
Reply: Done
8. Line 63: "when the current continuity is applied"
Reply: Done.
9. Line 65: "growth rate he calculated" I suggest removing personal references from the manuscript
Reply: Done

10. Line 71: “RTI are”

Reply: Done

11. Line 73: “F layer is depicted”

Reply: Done

12. Lines 116 and 118: is “v” supposed to be capitalized?

Reply: Done

13. Line 159: “g=10” appears twice here

Reply: Done.

14. Line 167: This language needs to be smoothened.

Reply: This sentence was smoothened. “Ignore the $V_A{}^2/c^2$ term in Eq.(25), the growth rate is”

15. Line 169: “As k increases..”

Reply: Done

16. Line 183: The term “gamma subscript K” for Kelley’s growth rate formula appears for the first time here, but does not appear to be used throughout; e.g., line 232 and 59-60. I suggest making this consistent throughout the manuscript

Reply: Done.

17. Line 194: “Taking the divergence”

Reply: Done.

18. Line 199: “field has significant”

Reply: Done.

19. Line 204: “discussed in section 3.2”

Reply: Done.

20. Lines 207-208: Should this sentence be two separate sentences? “: : :accumulation. In order: : :”

Reply: Done.

21. Line 237: “by including the effects”

Reply: Done.

22. Line 238: Sentence beginning with “He think” needs to be reworded; also theprevious sentence needs a period. Further, remove personal references.

Reply: This sentence was reworded. “When deriving the generalized linear growth rate of RTI, Kelley (2009) thought that the fundamental destabilizing source of RTI is the current, the background electric field and neutral wind will drive a current, $\boldsymbol{J} = \boldsymbol{\sigma} \cdot \boldsymbol{E}'$ where $\boldsymbol{E}' = \boldsymbol{E_0} + \boldsymbol{U} \times \boldsymbol{B}$, and contribute to the growth of RTI.”

Reference

Kelley, M. C.: The Earth's ionosphere: plasma physics and electrodynamics, Academic press, 2009.

Goldston, R. J., & Rutherford, P. H. (1995). Introduction to plasma physics. CRC Press.

Chandrasekhar, S.: Hydrodynamic and Hydromagnetic Stability, Dover Publications., 2013.

Sincerely,

Kangkang Liu