Reply to Referee_1

I would like to thank the Referee_1 for the questions and suggestions that helped me improve the manuscript. Below I will provide answers to your questions.

Referee 1:

This study challenges the standard ionospheric electrodynamics and proposes the new definition of the linear growth rate of the Rayleigh-Taylor instability (RTI) in the equatorial ionosphere. The commonly used linear growth rate of RTI in the equatorial ionosphere is written as $g/(L\upsilon_{in})$. Author noticed the problem that the growth rate is going to be infinity when υ_{in} goes to zero, whereas the growth rate of RTI in the collision-less plasma should have finite value. The obtained result seems to connect the theoretical gap between collisional and collision-less plasma naturally and build a seamless instability theory from the ionosphere to magnetosphere. However, the assumption to derive the old expression is usually valid in the ionosphere, and the difference between the old and new growth rate is negligible. Therefore, the title and the conclusion of the paper are misleading. I recommend that author should not focus on the equatorial ionospheric F layer, but on filling the theoretical gap between collisional and collision-less plasma, which may be an interesting topic.

Reply: I think the assumptions to derive the old expression by Kelley (2009) is physically unreasonable for several reasons. 1. The assumption that $\nabla \cdot J = 0$ and the growth of RTI is due to charge accumulation is contradicted. From the Gauss law, $\partial\rho/\partial t + \nabla \cdot J = 0$ we know that when $\nabla \cdot J = 0$, $\partial\rho/\partial t = 0$. It does not say $\rho = 0$, however, in the initial state of RTI $\rho = 0$, $\nabla \cdot J = 0$ means ρ will always equals to zero. 2. The assumption that the growth of RTI is due to charge accumulation is invalid. It is shown quantitatively in this manuscript that the contribution of charge accumulation to the growth of RTI is ignorable. Qualitatively, the divergence of current create charge accumulation, the associated electric field tries to amplify the initial perturbation. At the same time, the electric field drives a current with reduce the charge accumulation. The net result may be charge accumulation is so small that the contribution to the growth of RTI can be neglected. 3. Base on the above assumptions, the full expression of the linear growth rate in the ionosphere F region calculated by Kelley (2009) (Chapter 4.2) is $\gamma = g/(Lv_{in})$, it did not say how the growth rate changes with the wavenumber k. In real circumstances, the linear growth rate is a function of the wavenumber k.

Since the old growth rate is not a function of the wavenumber k, in the manuscript I compared the old growth rate with the maximum of the new growth rate. As shown in figure 4 of the manuscript, the different between the old and maximum of the new growth rate is negligible when v_{in} is in the range $[10^{-1}\text{-}10^{1}]$, the corresponding altitude range is [200-500] km (Kelley, 2009). When v_{in} is in the range [10^{-2}-10^{-1}], the corresponding altitude range is [500-700] km, the difference is large. In real circumstances, when the wavenumber k is small, the difference between the old and new growth rate maybe large.

When studying the linear growth rate of plasma in magnetic field, I found the deriving process of the linear growth rate of RTI in ionosphere by Kelley (2009) is quite different form the normal ways. Generally, the linear growth rate for collision-less plasma ($v_{in} = 0$) is derived by linearize the momentum equation, and later the boundary conditions are required to solve the related equations, the resultant linear growth rate is a function of the wavenumber k. However, the deriving process of the linear growth rate of RTI in ionosphere ($v_{in} > 0$) was done by linearize the particle continuity equation and no boundary conditions was used, the resultant linear growth rate is not a function of the wavenumber k. It should be noted that in previous deriving process of linear growth rate for collision-less plasma in magnetic field, $\nabla \cdot J = 0$ was used and the resultant growth rate is $\gamma = \sqrt{\frac{g}{L}}$, which is the same as that of neutral fluid. The process to derive the linear growth rate in collisional plasma is simply add the collisional term in the related equations. The difference between the old and new linear growth rate is negligible in low altitude ionosphere F region. However, during the old deriving process, the only constrain is $v_{in} > 0$, as long as v_{in} is not zero, the result should apply. However, the old linear growth rate, tends to infinite, when v_{in} is very small, which is physically unacceptably. The new linear growth rate when v_{in} tends to zero, automatically reduced to that of collision-less may indicate that the physics between collisional and collision-less plasma RTI is the same. In conclusion, in this manuscript, I give an accurate expression of the linear growth rate of RTI, and shows that the growth of RTI is not due to the charge accumulation.

Author mentioned in page 2 that “when current continuity equation applied, there will be no perturbation electric field due to charge accumulation.” It is not correct description. From the Gauss law, $\partial\rho/\partial t + \nabla \cdot J = 0$ is derived. It means $\partial\rho/\partial t = 0$ when the current continuity is satisfied. It does not say $\rho = 0$. Due to very small charge accumulation, electrostatic polarization field is set up. The charge accumulation is so small that the current continuity equation is applied in the electrodynamics in the ionosphere. Authors should estimate quantitatively the amount of charge accumulation produced during the Rayleigh-Taylor instability process. See Chapter 2.3 of Kelley (2009). Very small charge accumulation could produce large electric field. I think the new point in this paper is the inclusion of $\partial E/\partial t$ term in Equation (15). It is very small in the ionosphere, and is going to be important when the ratio of Alfvén speed to the speed of light becomes large.

Reply: When current continuity is satisfied, $\partial\rho/\partial t = 0$. Yes, it does not say $\rho = 0$. However, $\partial\rho/\partial t = 0$ means ρ remains constant. If in the initial state $\rho = 0$, ρ will be constantly zero and there will be no charge accumulation. If in the initial state

$\rho = C$, where C is some constant greater than zero, ρ will equals to C in later times. So, we can say that when initially $\rho = 0$, when current continuity equation satisfied, there will be no perturbation electric field due to charge accumulation. When initially $\rho = C$, when current continuity equation satisfied, there will be no perturbation electric field due to additional charge accumulation. In the description of Rayleigh-Taylor instability (RTI) in equatorial ionosphere by Kelley (2009) (Chapter 4.2 "Since the current is in the g×B direction, which is strictly horizontal, J_x will be large when n is large and small when n is small. There is thus a divergence, and charge will pile up on the edges of the small initial perturbation."), in the initial state $\rho = 0$ and later ρ increases, which means $\partial\rho/\partial t \neq 0$ or equivalently $\nabla \cdot J \neq 0$ during the growth of RTI. If one attribute the growth of RTI in equatorial ionosphere to charge accumulation such as Kelley (2009), or want to study the contribution of charge accumulation to the growth of RTI, $\nabla \cdot J = 0$ should not be used during the calculation of the linear growth rate of RTI.

It is usually accepted that when $\nabla \cdot J \neq 0$, charge density will creates an electric field that forces the divergence to zero, so $\nabla \cdot J = 0$ was used. However, $\nabla \cdot J = 0$ means $\nabla \cdot J$ is strictly equals to zero, which indicate that ρ will remain constant. Which is not the case in most circumstances. In steady state $\nabla \cdot J = 0$ can be applied, in unsteady state the constraint $\nabla \cdot J = 0$ is too strict. In unsteady state the electric field due to charge accumulation tries to force the divergence to zero but failed, the net effect is to keep $\nabla \cdot J$ small but not strictly equals to zero. The RTI process is obviously not in a steady state, so $\partial\rho/\partial t + \nabla \cdot J = 0$ should be used. However, as shown in the manuscript, the process of RTI involves charge accumulation, but the effect of charge accumulation to the growth of the RTI is negligible in equatorial ionosphere. So using $\nabla \cdot J = 0$ when deriving the linear growth rate of RTI in the equatorial ionosphere is safe, but simultaneously using the current continuity equation and perturbation electric field equation is inaccurate. Also, the physical description that the growth of RTI is due to charge accumulation is inaccurate. When $\nabla \cdot J = 0$ is used, the contribution of charge accumulation to the growth of RTI is totally neglected.

The inclusion of $\partial E/\partial t$ term in Equation (15) is possible is due to that fact that $\nabla \cdot J$ is not strictly zero. If $\nabla \cdot J = 0$, take the divergence of equation $\frac{\partial \boldsymbol{E}}{\partial t} = -4\pi \boldsymbol{J} + c\nabla \times \boldsymbol{B}$ we get $\nabla \cdot \frac{\partial \boldsymbol{E}}{\partial t} = 0$, if **E** is created by charge accumulation, $\frac{\partial \boldsymbol{E}}{\partial t} = 0$. Or if $\frac{\partial \boldsymbol{E}}{\partial t} = 0$, take the divergence of $\frac{\partial \boldsymbol{E}}{\partial t} = -4\pi \boldsymbol{J} + c\nabla \times \boldsymbol{B}$, we get $\nabla \cdot J = 0$.

During RTI process, due to the divergence of the current density, charge pile up on the edges of the small initial perturbation which create perturbation electric field, the electric field tries to amplify the initial small perturbation, and at the same time, the perturbation electric field tries to forces the divergence of the current density to zero. It seems that there will be not much charge accumulation and the effect of the associated electric field is limited. It is hard to estimate quantitatively the amount of charge accumulation produced during the RTI process. However, in the manuscript I estimate quantitatively the contribution of the charge accumulation to the growth of RTI. The results shows that the contribution of the charge accumulation to the growth of RTI is related to the ratio of Alfvén speed to the light speed. In equatorial ionosphere, this ratio is very small, the contribution of charge accumulation to the growth of RTI can be neglected. Using $\nabla \cdot J = 0$ when deriving the linear growth rate in equatorial ionosphere is safe, but deriving process is questionable and the description of the RTI process by Kelley (2009) was inaccurate.

In order to compare the old and new growth rate intuitively, the new growth rate should be written in the following way.

$$\gamma = \frac{\left(\sqrt{\frac{g}{L}+\frac{v_{in}^2}{4}}-\frac{v_{in}}{2}\right)\left(\sqrt{\frac{g}{L}+\frac{v_{in}^2}{4}}+\frac{v_{in}}{2}\right)}{\left(\sqrt{\frac{g}{L}+\frac{v_{in}^2}{4}}+\frac{v_{in}}{2}\right)}$$

$$= \frac{g}{L}\frac{1}{\sqrt{\frac{g}{L}+\frac{v_{in}^2}{4}}+\frac{v_{in}}{2}} = \frac{g}{Lv_{in}}\frac{1}{\sqrt{\frac{g}{Lv_{in}^2}+\frac{1}{4}}+\frac{1}{2}}$$

Then it can be easier to understand when the new terms become significant. When v_{in}^2 is significantly larger than g/L, which is usually satisfied in the ionosphere, the growth rate turns to be the traditional expression g/(Lv_{in}). In Figures 2 and 3, the estimated growth rate is plotted with regard to the normalized parameters. What altitude do these parameters correspond to? If the new growth rate should be applied in the ionosphere, substitute the typical values of collision frequency and Alfvén speed of the ionosphere, and show how the growth rate is modified.

Reply: Yes, write the new growth rate in the above form is intriguing, I will use the above form in the manuscript. In Figures 2 and 3, I just want to show how the growth rate of changes with the ratio of Alfvén speed to light speed and the collision frequency. In Figure 4 I showed the variation of the maximum growth rate with collision frequency with typical values in the ionosphere F layer. See from Figure 1, v_{in} in the range of 10^{-3}-10^{1}, the corresponding altitude is around 200- 900 km. Seen from figure 4 in the manuscript only in the low altitude F region the difference is negligible. The Alfvén speed is too small in the ionosphere, also, even if Alfvén speed is large, for the maximum growth rate (the wavenumber tends to infinity), the Alfvén speed term will be vanished, and the effect of charge accumulation is negligible.

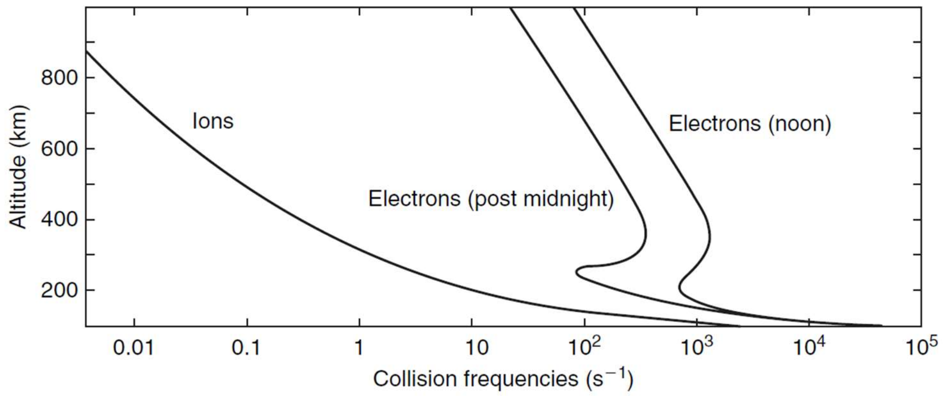

Figure 2.3 Typical electron neutral plus electron ion collision frequency along with the ion-neutral collision frequency at a high sunspot number.

Figure 1. Figure 2.3 in Chapter 2.2 of Kelley (2009)

Reference

Kelley, M. C.: The Earth's ionosphere: plasma physics and electrodynamics, Academic press, 2009.

Sincerely,

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