Response to Reviewer

We would like to thank the reviewer for carefully reading our manuscript and providing constructive comments. We appreciate that the reviewer introduce recent progress on related topics. We particularly notice the issue concerning the derivation of Equations 5-8 in our manuscript, which could indeed be critical to the scientific content. We have examined every equations in the manuscript.

In the following, we present a detailed derivation of equations 5-8 in our manuscript. Before elaborating the equations, we start with the wave electric field in the original drift-resonance theory by Southwood and Kivelson, 1981. Then, we demonstrate how different terms were introduced into the theory to obtain a wave electric field model described by Equation 8. Finally, we show that the wave electric field given by Equation 8 corresponds to the magnetic vector potential given by Equations 5-7. We hope that our clarification can address the reviewer's concern and be helpful to potential readers of our manuscript. Besides, we provide a one-on-one response to the comments of the reviewer.

1. Derivation of Equations 5-8 in the manuscript

The drift-resonance theory developed by Southwood and Kivelson was basically an integral of the particle energy change along the unperturbed particle drift trajectory (in a dipole background magnetic field). The wave electric field was originally given in the simplest form by:

$$\vec{E} = E_\varphi \hat{e}_\varphi$$

(Eq. 1)

and

$$E_\varphi = E_0 \exp[i(m\varphi - \omega t)]$$

(Eq. 2)

which describes an oscillation of constant amplitude E_0 at the angular frequency ω.

Zhou et al., 2005 & 2006 introduce the term $\exp\left[-\frac{t^2}{\tau^2}\right]$ to describe the growth and damping of the wave amplitude. The wave electric field now becomes:

$$E_\varphi = E_0 \exp\left[-\frac{t^2}{\tau^2}\right] \exp[i(m\varphi - \omega t)]$$

(Eq. 3)

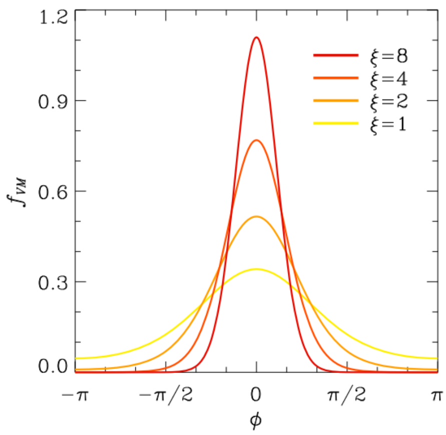
The amplitude of the wave increases from 0 at t=-∞, reaches its maximum E_0 at t=0, and decreases to 0 at t=+∞. The rate of growth and damping is described by the time scale τ.

If we further allow the wave to grow at the time scale τ_+, reach its maximum amplitude at t_0, and damp at the time scale τ_-, Eq. 3 rewrites as:

$$E_\varphi = E_0 \exp\left[-\frac{(t-t_0)^2}{{\tau_\pm}^2}\right]\exp[i(m\varphi - \omega t)]$$

(Eq. 4)

Li et al., 2017 introduced the term $\frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)}$ to describe the azimuthal localization of the wave. The figure below illustrates the value of $f_{VM} = \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)}$ for different concentration parameter ξ, where the x-Axis denotes $\phi=\varphi-\varphi_0$.



With the distribution of the wave amplitude in the azimuthal direction described by $\frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)}$, the wave electric field becomes:

$$E_\varphi = E_0 \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)} \exp\left[-\frac{(t-t_0)^2}{{\tau_\pm}^2}\right]\exp[i(m\varphi - \omega t)]$$

(Eq. 5)

The amplitude of the wave electric field also depends on radial distance to the Earth's center. We use a general term G(r) to describe the radial dependence:

$$E_\varphi = E_0 \mathrm{G(r)} \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)} \exp\left[-\frac{(t-t_0)^2}{{\tau_\pm}^2}\right]\exp[i(m\varphi - \omega t)]$$

(Eq. 6)

Since we are dealing with two monochromatic wave in our manuscript, we allow each of the monochromatic wave to have its own initial phase, by adding θ_0 in the oscillation term $\exp[i(m\varphi - \omega t)]$. Also, for the consistency of the variable names, the constant factor E_0 is replaced by A_0. Now we obtain the wave electric field of each monochromatic ULF wave in the form shown in Equation 8 in our manuscript:

$$E_{\varphi} = A_0 \mathrm{G(r)} \frac{\exp[\xi \cos(\varphi - \varphi_0)]}{2\pi I_0(\xi)} \exp\left[-\frac{(t-t_0)^2}{{\tau_{\pm}}^2}\right] \exp[i(m\varphi - \omega t + \theta)]$$

(Eq. 7)

Then, we will demonstrate that such wave electric field corresponds to magnetic vector potential described by Equations 5-7 in our manuscript.

Since $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial(\nabla\times\vec{A})}{\partial t} = \nabla \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$, a magnetic vector potential defined by $\vec{A} = \int_{-\infty}^{t} \vec{E} dt$ guarantee that the corresponding $\vec{E}$ and $\vec{B} = \nabla \times \vec{A}$ follows the Faraday's law.

In a general form, the integral $\int_{-\infty}^{t} \exp\left[-\frac{(x-A)^2}{B^2}\right] \exp[i(C-Dx)]dx$ yields:

$$\int_{-\infty}^{t} \exp\left[-\frac{(x-A)^2}{B^2}\right] \exp[i(C-Dx)]dx \\ = \frac{\sqrt{\pi}}{2} B \exp\left[-\frac{1}{4}B^2D^2 + (C-AD)i\right]\left(\mathrm{erf}\left[-\frac{t-A}{B} + \frac{BD}{2}i\right] + 1\right)$$

(Eq. 8)

where A, B, C, and D are parameters independent of variable x.

From Eq. 8, it is easy to obtain:

$$\int_{t_0}^{t} \exp\left[-\frac{(x-A)^2}{B^2}\right] \exp[i(C-Dx)]dx \\ = \frac{\sqrt{\pi}}{2} B \exp\left[-\frac{B^2D^2}{4}\right]\left(\mathrm{erf}\left[-\frac{t-A}{B} + \frac{BD}{2}i\right] - \mathrm{erf}\left[-\frac{t_0-A}{B} + \frac{BD}{2}i\right]\right) \exp[i(C-AD)]$$

(Eq. 9)

Let A=t_0, B=τ, C=$m\varphi+\theta_0$, and D=ω, using Equations 8 and 9, we have:

$$\int_{t_0}^{t} \exp\left[-\frac{(t-t_0)^2}{{\tau_{\pm}}^2}\right] \exp[i(m\varphi - \omega t + \theta_0)]dt \\ = \frac{\sqrt{\pi}}{2} \tau_{\pm} \exp\left[-\frac{{\tau_{\pm}}^2\omega^2}{4}\right] \cdot \left(\mathrm{erf}\left[-\frac{t-t_0}{\tau_{\pm}} + \frac{\tau_{\pm}\omega}{2}i\right] - \mathrm{erf}\left[\frac{\tau_{\pm}\omega}{2}i\right]\right) \exp[i(m\varphi - \omega t_0 + \theta_0)]$$

(Eq. 10)

and

$$\int_{-\infty}^{t_0} \exp\left[-\frac{(t-t_0)^2}{{\tau_{+}}^2}\right] \exp[i(m\varphi - \omega t + \theta_0)]dt \\ = \frac{\sqrt{\pi}}{2} \tau_{+} \exp\left[-\frac{{\tau_{+}}^2\omega^2}{4}\right] \cdot \left(\mathrm{erf}\left[\frac{\tau_{+}\omega}{2}i\right] + 1\right) \exp[i(m\varphi - \omega t_0 + \theta_0)]$$

(Eq. 11)

Using Equations 10 and 11, we have:

$$\int_{-\infty}^{t_0} E_\varphi dt = \int_{-\infty}^{t} A_0 \mathrm{G(r)} \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)} \exp\left[-\frac{(t-t_0)^2}{{\tau_\pm}^2}\right] \exp[i(m\varphi-\omega t+\theta)]dt$$

$$= A_0 \mathrm{G(r)} \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)} \int_{-\infty}^{t} \exp\left[-\frac{(t-t_0)^2}{{\tau_\pm}^2}\right] \exp[i(m\varphi-\omega t+\theta)]dt$$

$$= A_0 \mathrm{G(r)} \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)} \left(\begin{array}{c} \int_{-\infty}^{t_0} \exp\left[-\frac{(t-t_0)^2}{{\tau_+}^2}\right] \exp[i(m\varphi-\omega t+\theta_0)]dt \\ + \int_{t_0}^{t} \exp\left[-\frac{(t-t_0)^2}{{\tau_\pm}^2}\right] \exp[i(m\varphi-\omega t+\theta)]dt \end{array} \right)$$

$$= A_0 \mathrm{G(r)} \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)}$$

$$\cdot \left\{ \begin{array}{c} \frac{\sqrt{\pi}}{2}\tau_+ \exp\left[-\frac{{\tau_+}^2\omega^2}{4}\right] \cdot \left(\mathrm{erf}\left[\frac{\tau_+\omega}{2}i\right]+1\right) \exp[i(m\varphi-\omega t_0+\theta_0)] \\ + \frac{\sqrt{\pi}}{2}\tau_\pm \exp\left[-\frac{{\tau_\pm}^2\omega^2}{4}\right] \cdot \left(\mathrm{erf}\left[-\frac{t-t_0}{\tau_\pm}+\frac{\tau_\pm\omega}{2}i\right] - \mathrm{erf}\left[\frac{\tau_\pm\omega}{2}i\right]\right) \exp[i(m\varphi-\omega t_0+\theta_0)] \end{array} \right\}$$

(Eq. 12)

Though the expression seems a little lengthy, the only term in the final result of Equation 12 which depends on time is shown in red.

We define F as the terms in the curly brackets in Equation 12:

$$F = \frac{\sqrt{\pi}}{2}\tau_\pm \exp\left[-\frac{{\tau_\pm}^2\omega^2}{4}\right] \cdot \mathrm{erf}\left[-\frac{t-t_0}{\tau_\pm}+\frac{\tau_\pm\omega}{2}i\right] \exp[i(m\varphi-\omega t_0+\theta_0)] + C$$

(Eq. 13)

where C is defined as the rest terms in the curly brackets in Equation 12 which are independent of time.

Using Equations 12 and 13, we define:

$$A = \int_{-\infty}^{t_0} E_\varphi dt = A_0 \mathrm{G(r)} \frac{\exp[\xi\cos(\varphi-\varphi_0)]}{2\pi I_0(\xi)} F$$

(Eq. 14)

A, F, and C defined here are identical to those in Equations 5-7 in our manuscript (for one monochromatic wave).

So far, we have demonstrated the meaning of the different terms in our wave electric field, and have proved the derivation of equations 5-8 in our manuscript.

We hope that the demonstration here can help to clarify the equations in our manuscript. We leave the one-on-one response to the reviewer's comments concerning our derivation to the next section, together with our responses to other comments.

2. Response to reviewer's comments

In this section, we present one-on-one responses to reviewer's comments. The comments of the reviewer will be shown in blue and our responses will be shown in black.

Line 7: "We adopt the calculation scheme therein to derive the electron energy change in a multi-period ULF wave field"
Are there other kinds of schemes to explain this energy change in ULF wave field? If yes, the authors are suggested to briefly talk about at least one of them and explain why they choose the specific scheme.

The calculation scheme here refers to the integral of energy change along the unperturbed particle orbit developed by Southwood and Kivelson, 1981 to study the drift-resonance between charged particles and ULF waves.
Li et al., 2018 employed a pendulum equation to study the particle energy change with the perturbed orbit taken into account. Also, lots of numerical simulations were conducted to study the particle energy change in a ULF wave field.
However, as Line 7 refers to the Abstract, it would be better to stay focused on the contents of the present manuscript. A brief description of other schemes might be included in the introduction section.

Line 23: "..., called the resonance energy, at which the particles would experience a stable electric field during their drift motion, ..."
What about magnetic field experienced by particles?
Also, "stable" might not be an accurate or even a necessary word here. The unstable force could also affect particles as long as the resonance condition is met.

We agree with the reviewer that "stable" is not the proper word here. We will replace it by "steady", which means that the sign of the electric field experienced by the particle does not change. The particle would definitely experience a wave magnetic field. (Otherwise, it violets Faraday's law.) However, since Lorentz force is always perpendicular to particle velocity, it does no work on the particle.

Line 39: "In addition, ULF waves in the magnetosphere have been found to be asymmetrically distributed (e.g. Takahashi et al., 1985; Liu et al., 2009), whereas a symmetric ULF wave field is assumed in the conventional drift-resonance theory."
Mentioning recent works that address the distribution of the ULF waves is suggested. For example, Barani, M., Tu, W., Sarris, T., Pham, K., & Redmon, R. J. (2019). Estimating the azimuthal mode structure of ULF waves based on multiple GOES satellite observations. Journal of Geophysical Research: Space Physics, 124, 5009–5026. Figure 5 (a & b) as well as figures 6 of the mentioned paper and the explanations therein clearly show that the wave can be azimuthally localized. It is also suggested that for the sake of clarity authors mention which "symmetry" they are talking about; the symmetry mentioned in the Figure 2 of Southwood and Kivelson (1981) or they are generally talking about the homogeneity of ULF amplitude (power) pulsations in the azimuthal direction? If the latter is meant, bringing the word "symmetry" could be miss-leading.

We thank the reviewer for introducing recent papers on related topics. We will include these recent findings in our revised introduction section.
We agree with the reviewer that "symmetric" is not the proper word here. We will rephrase as "uniform wave amplitude".

Line 63: "The electron flux data in this study are obtained by the BeiDa Imaging Electron Spectrometer (BD-IES) onboard a 55 inclined geosynchronous orbit (IGSO) spacecraft of China"
Since the study is conducted for equatorially mirroring electrons (~0 degree pitch angle), the following questions must be addressed in section 2 of the manuscript: Did the authors project (map) the data to the equatorial plane? If yes, which scheme/methods they applied. If not, how this 55-degree inclination would affect/alter the results.
Which level of the data is used? What is the spin period and the spin axis direction of the spacecraft? Is the de-spun data used for the analysis in this manuscript?

We did not project the data. The magnetic latitude of the spacecraft when the flux modulation was observed was about 30 degree. We will justify the way we used the electron flux data in the response to comments on Line 112 (concerning the issue that we used omnidirectional flux but perform the numerical calculation with equatorially mirroring electron).
The spacecraft with BD-IES onboard is not a spinning spacecraft. (See Zong et al., 2018 https://doi.org/10.1002/2017SW001708 for more information about the instrument and spacecraft)

Line 86: The residual flux, defined as $(J-J_{Avg})/J_{Avg}$, represents the flux variation normalized to the background flux so that the relative change of the particle flux caused by the waves can be quantitatively compared across different energy channels.
As the authors correctly stated later, the J_{Avg} is not necessarily $J_{background}$. So, this might be misleading to name J_{Avg} as background flux.

Agree.
"$(J-J_{Avg})/J_{Avg}$" will be replaced by "$(J-J_0)/J_0$" where J_0 represents the background trend of the flux.

Line 93: "... to display the wavelet power spectrum."
Some basic information about the way the authors conduct the Wavelet analysis is highly suggested and should be addressed such as: Name of the mother wavelet, the cadence of the data used as the input to the wavelet computation functions, the scale (frequency) range selected for the wavelet analysis.
The reviewer (for his reference) is interested in learning about the scale (frequency) spacing in the output of the wavelet function. (In other words, what is the y axis spacing/resolution between the data points in Figure 2 and which shading method was used in the color-coded visualization in that figure?)

We use Morlet wavelet, with $\omega_0=6$. The cadence of the input for the wavelet analysis is 10 seconds, the same as the cadence of the data product of BD-IES. More specifically, the only difference between the original electron flux data and the input for the wavelet analysis is that the bad data are removed by a linear interpolation of the valid data in the nearest neighbor.

The wavelet power is defined as the product of wavelet transform multiplied by its conjugate (Torrence and Compo, 1998). Therefore, given a frequency (scale), the corresponding wavelet power can be calculated. To create Figure 2, we calculated the wavelet power at 201 frequencies that corresponds to periods evenly distributed in the range from 100 sec to 500 sec. We change the opacity of the part of the figure using Adobe illustrator to obtain the shading effect.

Line 94: "As the wavelet power is proportional to the square of the oscillation amplitude … ." Please pay attention that this is the case for any power hopefully regardless of the spectral analysis methods such as FFT, WFFT, Wavelet, … .

Agree. We specify wavelet power here, as it is the analysis method we use.

Line 100: "Besides, the electron flux modulation exhibits a dispersive characteristic." What kind of dispersive? In other words, dispersive with respect which quantity?

We will rephrase as "energy dispersive characteristic", which means the modulation was observed first at higher energy and then at lower energy.

Line 112: "For a symmetric background magnetic field, the unperturbed drift orbit of an equatorial mirroring particle can be given by … "

Here in the model the authors look at ~90-degree pitch angle (or equatorially mirroring) electrons while in the measured flux data omni-directional particles' differential flux was analyzed. How this discrepancy can be addressed? Further information/reasoning or references seems to be needed to make sense of this omni-directional choice specially for those whose primary research focus is not looking at particle flux/phase space density data.

Here, we justify our usage of the BD-IES data.

As the time scale of the problem we study (in the order of hundreds of seconds) is much longer than the bounce period of the particle (in the order of a second), we assume that any inhomogeneity would be quickly dispersed along the bounce path of the particle. Therefore, the measurement of electron flux off the equator at energy W and pitch angle α corresponds the electron flux at the equator at energy W and equatorial pitch angle α_{Eq}.

As demonstrated in the introduction section of our manuscript, the key parameter for a particle interacting with a monochromatic ULF wave via drift-resonance is its angular drift velocity ω_d. In a dipole field, ω_d is in proportion to $W(0.35+0.15\sin\alpha_{Eq})$. Therefore, the phase difference between the flux modulations is larger at different energies than at different pitch angles. Thus, we can still see pattern of flux modulation caused by drift-resonance in the omnidirectional fluxes at different energies.

e.g. Li et al., 2017a, 2017b used BD-IES data in the same way.

Line 123: "Therefore, particle flux modulation caused by drift-resonance would present a characteristic 180 phase shift across the resonant energy."

It is strongly suggested that the authors bring some key previous works observing this phenomenon.

In the introduction section, we have mentioned quite a few papers observing this characteristic phenomenon (e.g. Line 34)

Line 129: "Then, when the wave starts damping, the phase shift would keep growing as the drift velocities of the particles depend on their energies"
Clarifying the connection between dependency of particles' drift frequency on their energy and growth of the phase shift in δW after the onset of wave damping is suggested here even by adding one/few sentences.

The connection is quantitatively described by Equation 14 in Zhou et al., 2016, doi:10.1002/2016JA022447, elaborated in text in the paragraph following Equation 14, and illustrated in their Figure 2. We will add a detailed reference here.

Line 135: "While the characteristic particle signatures of drift-resonance predicted by these prevailing theories have been proved by recent spacecraft observations, the particle energy change therein is derived in an incomplete way"
References on recent spacecraft observations that proved the mentioned drift-resonance driven behavior of particles flux should be added.

As this comment is related to the comments on Line 39, please see our replies there.

Line 140: "The Betatron acceleration caused by the curl of the wave electric field, denoted by $\mu/\gamma \partial B/\partial t$ is omitted in those drift-resonance theories.
Again, referring to some of the main papers (in addition to Southwood and Kivelson (1981)) on those theories is needed.

Zhou et al., 2015, 2016, Li et al.,2017b mentioned in the previous paragraph (Line 125-134) are all among papers of "those theories".

Line 141: "One might neglect this energy change, because the magnetic field of fundamental mode waves has a node at the equator. Especially in the case of a purely poloidal wave, the perpendicular component of the wave magnetic field B_r can be identically zero in the equatorial plane"

We would like to clarify first that E_φ, B_r, and B_z give the poloidal mode wave; E_r and B_φ give the toroidal wave. To describe a poloidal wave, one can first define B_r and B_z, then derives E_φ according to Faraday's law. Alternatively, one can first define E_φ, then use Faraday's law to derive B_r and B_z.

The reviewer's immediate understanding is that this reasoning might not be necessarily valid for all ULF wave cases. For example, for broadband ULF pulsations which is different from the field line resonance oscillations we do not necessarily have a node in the equatorial plane.

By saying "can be zero", we mean that it is possible to be zero, not that it is always zero.
Nevertheless, we will remove these two sentences, since they possibly lead to misunderstandings.

Looking at equation $dW/dt = q\boldsymbol{E}\cdot\boldsymbol{u} + \mu/\gamma \partial B/\partial t$, we can see that vanishing/small B_r has no effect on making the $\partial B/\partial t$ term zero. Since it is the change of B in time that determines the second term, not the B itself. If the authors meant $\partial B_r/\partial t$ (not B_r itself) in the text, the explanation/reasoning should be provided since in the fundamental mode (Figure 2 panel (a) in Southwood and Kivelson (1981)) the temporal change of B in the equatorial plane is actually maximum.

$\partial B/\partial t$, as it appears here, means $\partial B/\partial t$, not $\partial B_r/\partial t$ guessed by the reviewer.
In the paragraph from Lines 135-153, we are arguing that $\partial B/\partial t$ can not be neglected in the general case. After the sentence in Line 141 quoted by the reviewer, we immediately state in the next sentence that the neglect of this energy change is not correct.

Line 143: "However, even then, there would still be a non-negligible change of magnetic field magnitude, because there should be a parallel wave magnetic field B_z according to [the] Faraday's law."
Correct, although we do not have to assume that there is always an E_φ. Since $\partial E_r/\partial \varphi$ can also give non-zero B_z.

Since we focus on poloidal wave, we study E_φ, B_r, and B_z, though E_φ in an actual case can be zero.

Line 146: "Note that, for poloidal waves, $\nabla \times E$ is controlled by $\partial E/\partial r$ since E is in the azimuthal direction. Consequently, the particle energy change would be greatly influenced by the radial gradient of wave electric field amplitude, although the particle drifts at a constant L shell in the unperturbed orbit approximation."
If the reviewer understood correctly, poloidal component for dipole field at equatorial plane means Br, and following Faraday's law, $\partial B_r/\partial t = -(\nabla \times E)_r = -1/r \partial E_z/\partial \varphi + r \partial E_\varphi/\partial z$. Therefore, the reviewer does not see the effect of $\partial E/\partial r$ for poloidal component of pulsations, and more explanation by the authors is necessary here.
Also, the readers should notice that only in a pure dipole field, the poloidal means B_r at equatorial plane. However, in the more general case of non-dipole or extension of the study to non-zero magnetic latitudes, it does not have to only be B_r.

The poloidal wave has components of E_φ, B_r, and B_z. Following Faraday's law,
$\partial B_r/\partial t = -(\nabla \times E)_r = -1/r \partial E_z/\partial \varphi + \partial E_\varphi/\partial z = \partial E_\varphi/\partial z$

$\partial B_z/\partial t = -(\nabla \times E)_z = -1/r \partial (rE_\varphi)/\partial r + 1/r \partial E_r/\partial \varphi = -1/r \partial (rE_\varphi)/\partial r$

Therefore, $\partial E_\varphi/\partial r$ affects B_z.

Line 153: "The background field is given by $B_0 = \cdots = B_E/r^3 e_z$ where ..."
In the denominator, there should be L instead of r. It is suggested the authors review all of the derivations and make sure that this typo/mistake was not present in their calculations.

Agree. "r" here should be replaced by "r/R_E" which is identical to "L".

Line 157: "The poloidal ULF wave fields can be given in a general form by $\boldsymbol{E}_1 = -\partial A/\partial t e_\varphi \triangleq$ ..."
It is worth noticing that in a general case there is an electric potential term. So, the general form for an electric field can be $\boldsymbol{E}_1 = -\nabla\phi - \partial \boldsymbol{A}/\partial t$. Therefore, it would be very useful if the authors could shortly talk (or bring references) about why $-\nabla\phi$ is neglected here and how

likely the electrons face a free electric potential field in the magnetosphere during their drift around Earth.

We agree with the reviewer that "general form" might not be the proper word here.
As we demonstrated in the previous section, the wave electric field in Southwood and Kivelson type theories of drift-resonance is defined somewhat arbitrarily. In such cases, the wave magnetic field can not be arbitrarily defined. For a given wave electric field, we can solve the equation **E**=-∂**A**/ ∂t to obtain A, and then define the wave magnetic field as **B**=**∇**×**A**. This scheme is general in the sense that it is not restricted to a specific form of the wave electric field. Nevertheless, "general" is not a proper word here. We will avoid using it in this context.

Line 158: "For fundamental mode waves, it is reasonable to further assume that the amplitude of the wave does not vary in the vicinity of equator (i.e. $\partial \boldsymbol{A}/\partial z = 0$)"
Please look at the reviewer's comment under Line 141.

Our response is also presented above.

Line 162:
Relation (4) in the manuscript is neat. It is very good that the authors quantitatively explain the correction while considering a more general case although yet limited to the odd (and probably not even) modes of field line resonances.

We agree that Relation 4 may not apply to even modes.

Line 184 Equation (6):
In the exponential, there we see $t_{0,n}$ while it cannot be a constant. Having a constant time in the exponential means F_n would not give a wave behavior. The reviewer's understanding is that it should be t instead. Otherwise, authors are required to explain how the wave behavior is represented in the mentioned equation.

As we demonstrated in the previous section. The derivation of Equation 6 in the manuscript is correct. The term that varies with time is $\text{erf}\left[-\frac{t-t_0}{\tau_\pm}+\frac{\tau_\pm\omega}{2}i\right]$. The error function erf(z) is a monotonically increasing function when the argument z is a real number. However, when the argument is a complex number, the error function can give a wave behavior. (See e.g. https://mathworld.wolfram.com/Erf.html for reference)

Line 190 Right hand side of Equation (8):
If we take time derivative of $\boldsymbol{A}$, we will get two terms: one is exactly what we see in Equation (8), another is proportional to $i\omega_n$ multiplied by the error function. Explanation of why we should not get the second term (or if we get, why that term must be neglected) is necessary here.
The reviewer stops here and would let the authors provide their reasonings, explanations, and corrections since the final results and conclusion might depend on these corrections

Related to the comments on Line 184 Equation 6, the term $\exp[i(m_n\varphi - \omega_n t_{0,n} + \theta_{0,n})]$ is correct in the form it appears. $t_{0,n}$ should not be replaced by t. Therefore, the "second term" does not exist.

Comments on Figures

Figure 1:
What are the narrow white-color vertical lines in the spectrogram (a) panel?
The reviewer suggests bringing the MLT and MLAT information to the readers' attention as captions under panel (d) or by briefly addressing them in the text. There, no information of the solar wind (such as solar wind P_{dyn}, IMF Bz, as well as geomagnetic Dst, and AE) is given, and the readers would not be able to track any relation between the plots and the geomagnetic and solar wind indices. Following or not following the mentioned suggestions here is not critical and would not affect the reviewer's final decision.

We agree with the reviewer that information about the spacecraft position should be provided. Spacecraft position will be provided in the revised figure.
Since we focus on the wave-particle interaction (instead of issues such as the excitation of the wave or the source of the particle), the solar wind, IMF condition, and the geomagnetic index may not be crucial here.

White color in Fig 1a refers to "no valid data". For example, the instrument recorded bad data at UT~10:32. In panels c and d, the bad data appear as breakpoints in the time series of the electron flux. Accordingly, there is a white line in panel a.

Figure 2 and line 82:
Is the dimension of the plots amplitude squared, or amplitude squared over frequency? If it is just the power (former case), there should be an explanation on how the power is deduced (i.e. is it an averaged value or summed over scale (frequency) range from ... to ...) since typically the spectral analysis codes give the analysis in terms of power density not power.

Figure 2 shows the wavelet power, defined as the product of wavelet transform multiplied by its conjugate (See Torrence and Compo, 1998 for reference)

Minor comments

Line 11: "wave electron field"
It should be wave electric field.

Agree. The typo will be corrected.

Line 52: "First, we revisit the origin drift-resonance theory..."
Do the authors mean the origin of the or they mean original?

Origin will be corrected by original.

Line 54: "We show that the Betatron acceleration caused by the curl of the wave electric field, which is omitted in these theories, is comparable with the energy change caused by the poloidal electric field along the drift trajectory of the particle"
Before Betatron acceleration there should be energy change due to.

If "Betatron acceleration" refers to the acceleration process, "energy change due to" should be added. If "Betatron acceleration" refers to the result that particle are accelerated by the

Betatron acceleration process, “energy change due to” might not be added.

Line 72: “The IGSO spacecraft with BDIES onboard passes through the radiation belt twice per orbit. Figures 1a and 1b show the electron flux in a full pass of the spacecraft through the radiation belt in the format of spectrogram and series plot respectively.”
It is suggested that before the words radiation belt there should be outer.
plot should be plots
In the spectrogram, did you use any kind of interpolation or smoothing?

Agree. It should be outer radiation belt. “plot” should be “plots”.
No interpolation or smoothing is used in the spectrogram

Line 95: “..., the upper limit of the colorbar for each energy channel is chosen to be the square of the mean value of the electron flux in the selected interval from 10:15 UT to 11:15 UT and the widths of the colorbars are consistently set to be 2.”
Which Figure the manuscript is pointing to? Dimension of the value should be mentioned: for example, is it power or power over frequency?
Did you detrend the signals? If yes, what was the detrending method and width of the moving average scheme?

“colorbar” in Line 95 refers to Figure 2 as we are discussing the wavelet power in the context.
Figure 2 shows the wavelet power, defined as the product of wavelet transform multiplied by its conjugate (See Torrence and Compo, 1998 for reference).
No detrending is applied to the data before wavelet analysis. As we demonstrate in Line 93, we use the choice of colorbars to achieve comparison across energy channels instead of detrending the data.

Line 112:“For a symmetric background magnetic field, the unperturbed drift orbit of an equatorial mirroring particle can be given by ... “
It should be equatorially mirroring particles.

Agree. “equatorial mirroring particle” is not grammatically correct. It should be “equatorially mirroring particle”

Line 143: “However, even then, there would still be a non-negligible change of magnetic field magnitude, because there should be a parallel wave magnetic field B_z according to the Faraday’s law.”
the should be deleted.

Agree. The redundant “the” will be deleted

Line 165: redundant the

Agree. The redundant “the” will be deleted

Line 165: “For the empirical electric field model denoted by $E_\varphi \propto \exp[\sigma r]$ (e.g. Perry et al., 2005; Ozeke et al., 2014), ...”
Which paragraph or equation number in the aforementioned references the authors are specifically pointing to?

As σ is set to be a positive value, the authors should bring a proper reference or explanation on the limitation of the exponential behavior with positive power in E_φ.

Equation 19 in Perry et al., 2005; Equation 10 in Ozeke et al., 2014.

Perry et al., 2005 and Ozeke et al., 2014, as they stand here, are the reference.

Line 171: "According to Li et al. (2017b), this dispersive characteristic implies that the ULF waves were azimuthally confined…"
Again, referring to a direct measure/evidence of this azimuthally confined ULF waves power (such as Barani et al. (2019)) would strengthen the authors' argument on the validity of defining an envelope in the azimuthal direction.

The direct evidence of azimuthally confined ULF waves will be included in the introduction section. By referring to Li et al., (2017b) here, we want to demonstrate that such particle characteristic (also observed in the present study) can be explained by azimuthally localized wave.

Line 178: "The constant factor $A_{0,}$ denotes the amplitude of the wave. The second term $G(r)$ describe of wave amplitude in the radial direction"
If defining two different amplitudes for a wave is not common, the reviewer suggests using a different language for the sake of clarity avoiding two-amplitude language.
We will avoid wording such as "two-amplitude of one wave". We will rephrase as "For each monochromatic wave, we define its amplitude as $A_{0,n}$."
describe of should be describes
Agree. Will be corrected.

Line 178: "The third term $H(\varphi) =$… is a von Mises function, …"
As H_n here is not just function of n and φ, it would be more inclusive/accurate to use the notation $H_n(\varphi|\varphi_0, \xi_n)$ instead of $H_n(\varphi)$.

Agree. Will be revised

Line 182: "…is the zeroth-order modified Bessel function."
It should be modified Bessel function of the first kind.

Agree. Will be corrected.

Line 182: "The von Mises distribution is an analogue of the normal distribution."
The reviewer suggests adding words like in the rotational/periodic scheme at the end of this sentence.

Agree. "in the periodic scheme" will be added.