

We thank referee #1 for careful reading and valuable comments on the manuscript. Accordingly, we have modified the text. All the modifications and changes are shown in the revised manuscript in red font. Our responses to the referee's comments are listed below.

### **Response to Referee #1**

#### **COMMENTS TO THE AUTHOR:**

Referee #1: Interactive comment on "Validation and application of optimal ionospheric shell height model for single-site TEC estimation" by Jiaqi Zhao and Chen Zhou

Comments:

The manuscript is well written and clear. This paper models the optimal thin layer altitude as 40th order Fourier series. The optimal altitude coefficients are estimated at a reference station and used for estimating the thin shell altitude of the near stations. This approach is very useful for precisely estimating the TEC and DCB in not IGS platform.

#### Reply:

We thank the referee for the encouraging evaluation on our study, which has driven us into a deeper investigation.

It would be interesting to see what are the performances in DCB estimation if the number of coefficients of the Fourier series changes, it would be interesting to see what is the minim number of coefficients required.

#### Reply:

We thank the referee for this valuable comment. In our study, the order of Fourier

series is preliminarily set to 40. For one station, the outstanding frequencies of optimal thin layer altitude are only a few, it is possible to reduce the number of coefficients. Maybe less coefficient, clearer physical relationship.

There is in addition a typo comment, on page 7 line 132, you should insert the IPP acronym that has not been specified before.

Reply:

We thanks the referee for careful reading and pointing out this mistake. We have corrected it. Please see page 7 line 135 in the revised manuscript.

Dear referee #2,

We are grateful for your careful reading and valuable comments on the manuscript. Accordingly, we have modified the text. All the modifications and changes are shown in the revised manuscript in red font. Our responses are listed below.

## **Response to Referee #2**

### **General comments**

This article presents a novel approach to estimate GPS permanent stations DCBs in a radius less than 2000 km from a mid-latitude IGS station. It is a validation of a technique recently developed by the same authors. An optimum ionospheric shell height is estimated using the assumption that the IGS DCBs represent reliable values. This study covers a complete solar cycle for the estimation of the ionospheric shell height at a reference station and one year for the tests with additional stations. I think that the manuscript in its present form lacks of necessary discussion on the limitations of the assumptions made in this work and that a number of points need to be explained deeper. I therefore suggest major revisions.

### **Specific comments**

The ionospheric height for the reference station GOLD appear to be very high: in average it is 712 km. By an ionospheric point of view, the shell height should correspond to the height of the ionosphere barycenter, i.e. higher than hmF2 of about 100-150 km. There is a long-lasting debate on the operational shell height to use for the thin shell approximation of the ionosphere and the authors recall many of the publications discussing this problem. While it is true that some authors allow altitudes as high as 1200 km, care should be taken to understand if the obtained shell height are reliable. In this work an elevation mask of 15° has been used. Under many conditions this elevation mask could be too low and introduce a large uncertainty on the optimum shell height (see for instance the discussions of Rama Rao et al. 2006, recall that under some conditions they even obtained unphysical negative shell

height).

Reply:

We thank for your important remark. We agree with that the shell height should correspond to the height of the ionosphere barycentre by an ionospheric point of view. While for accurate TEC and DCB estimation, because of VTEC model error and mapping function error and so on, optimal shell height is different with ionosphere barycentre. Actually for different VTEC model, the optimal shell height is also different. Lu et al. (2017) did the similar work by using another ionospheric shell height estimation method. In our manuscript, the optimal shell height is also affected by the accuracy of reference values of DCB. The optimal shell height is more like a modification of the mapping function for the selected VTEC model, and have relationship with solar activity. We believe that optimal shell height and ionosphere barycentre could be closer with the improvement of the VTEC model and mapping function.

Reference

Lu W, Ma G, Wang X, Wan Q, Li J (2017) Evaluation of ionospheric height assumption for single station GPS-TEC derivation. *Advances in Space Research* 60(2):286-294

It is not clear why the technique proposed for ionospheric shell height estimation cannot be implemented to isolated GNSS receivers not belonging to IGS stations (line 107).

Reply:

We thank for your carefully reading and helpful comment. The optimal ionospheric shell height is calculated from IGS DCB values. DCB is normally not released by non-IGS stations, which means ionospheric shell height cannot be calculated by using this method. However, if we could get the long-term observations and reference values of DCB from non-IGS station, this technique could also work. We have deleted this mistake. Please see page 6 line 107-109 in the revised manuscript.

A Fourier model of the shell height is constructed for GOLD and PTBB for a complete solar cycle between 2003 and 2013. This model does not include any input regarding solar activity. It is well known that the current solar cycle is considerably less strong than the previous one. The ionosphere development has also been substantially lower. Thus it is also expected the optimum shell height should follow a different pattern. A discussion on this point is essential for the correct understanding of this work.

Reply:

We thank for your constructive suggestion. We total agree with the reviewer that solar activity is the dominant factor for ionospheric variability. However, other factors such as atmospheric variability and human activity can also cause ionospheric disturbance. In this study, we do not consider all physical factors explicitly. However, we try to include all the factors by utilizing empirical modeling with data. The Fourier model is a preliminary result. Evaluations on different models will be investigated and compared in the following work.

line 52: the work of Sardón et al. (1994) was not oriented towards real-time ionospheric VTEC, but to develop a technique of prediction of DCBs under adverse conditions (antispoofing, ionospheric disturbances).

Reply:

We appreciate the reviewer for this helpful comment. We have accordingly made the revision. Please see Lines 52-54 in the revised manuscript.

line 77: specify that the Nava et al. (2007) technique uses multiple stations to obtain a “coinciding pierce point”.

Reply:

We thank for the reviewer for providing this suggestion. We have accordingly made the revision. Please see Lines 77 in the revised manuscript.

line 125-126: the polynomial model is referred to Lanyi and Roth (1988). However the expression used in this article does not correspond to the one used by those authors.

Reply:

We appreciate the reviewer for pointing out this mistake. We have replaced this reference with (Wild, 1994; Komjathy, 1997). Please see Lines 128-129 in the revised manuscript.

Line 132: does the regional center of the model correspond to the location of the receiver?

Reply:

Yes. The regional center of the model is the location of the receiver.

line 134: it is not clear why 9 VTEC models are applied per day. It should be specified that a VTEC model is generated over 3 hours of time.

Reply:

We have accordingly made the revision. Please see Lines 137-139 in the revised manuscript.

line 166: I suggest to indicate explicitly that the 40/L corresponds to a period of 100 days.

Reply:

We thank for this helpful suggestion. We have accordingly write it explicitly. Please see Lines 170-171 in the revised manuscript.

line 178: why only stations providing P1 code measurements of pseudorange were used? Will the result be significantly different if any station would have been selected regardless of the measured code?

Reply:

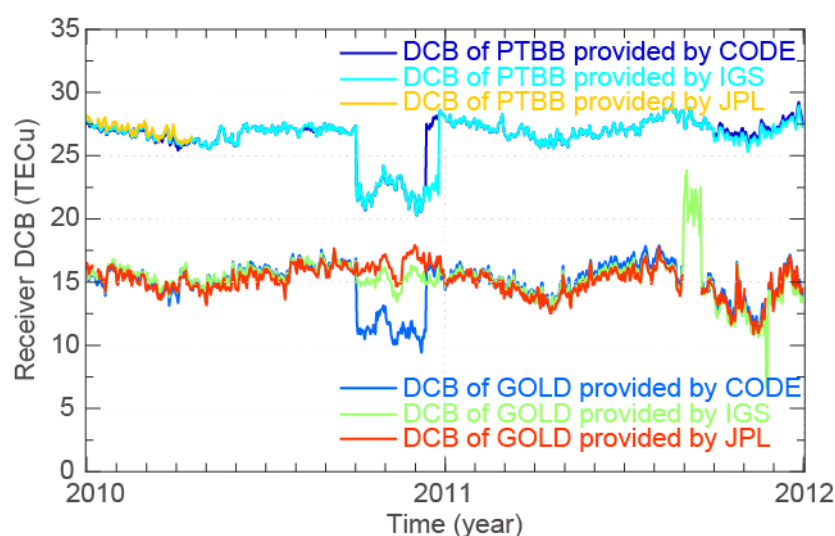
We thank for this comment. CODE also provides the DCB of P1-C1, but only for satellites. And we are not sure whether the receiver DCB is C1-P2 bias in CODE DCB

file, for the station providing C1 code but no P1 code. So we use the DCB of P1-P2 for reference. Accordingly, we just select stations with P1 code.

On figure 2 an anomaly appears at the end of 2010, where a gap (or values outside the vertical axis limit ?) appears on the estimated shell heights. In this article there is not a discussion about this strange behavior, but in the previous article (Zhao and Zhou 2018), figure 3 shows that all stations have simultaneously anomalous DCBs during a few months. I suggest to make a deeper investigation on why this happen, but clearly these DCBs values are not reliable. Some hypotheses: an error in CODE processing chains; an error in the receivers firmware that affect the time estimate; some error at GPS system level. The impact to the results of this article concern the Fourier model to represent the whole solar cycle behavior of the shell height, but should not affect the station comparisons of 2014.

Reply:

We appreciate the reviewer for raising this important comment. We totally agree with the reviewer that the anomaly at the end of 2010 could be an error in CODE processing procedures. We have followed the reviewer's suggestion that discussion on the data gap have been added in the revised manuscript. Please see Line 217-219.



**Fig.1** Receiver's DCB released by CODE, IGS and JPL from 2010 to 2011

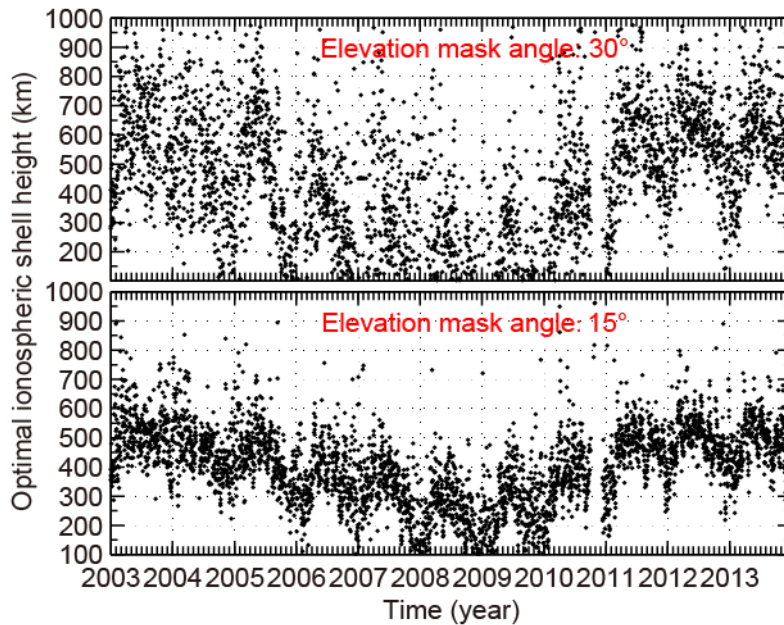
Figure 1 plots the DCB of receiver provided by different analysis center. At the end of 2010, for CODE, PTBB and GOLD both appear anomaly; for IGS, only PTBB is anomalous; the DCB of GOLD provided by JPL seems continuous. JPL doesn't release the DCB of PTBB after April 2010. It seems that the DCB provided by JPL is more reliable, in terms of stability.

Additional comments on Figure 2: -the spreading of daily shell height values is extremely large (>200 km) with strong variations from one day to the other. How this spreading is affected by the choice of elevation mask angle? -If there is such a high variability, what is the benefit of using a Fourier model up to order 40? A much lower order could provide comparable results. On the other hand, the fast variability is not achievable with this model. -both stations show the limits of the proposed approach: the distributions on the right panels present each a missing tail, suggesting that the imposed shell height limits are not adequate. For GOLD station we could expect shell heights higher than 1000 km and for PTBB shell heights lower than 100 km, which are unphysical, because outside the ionosphere

Reply:

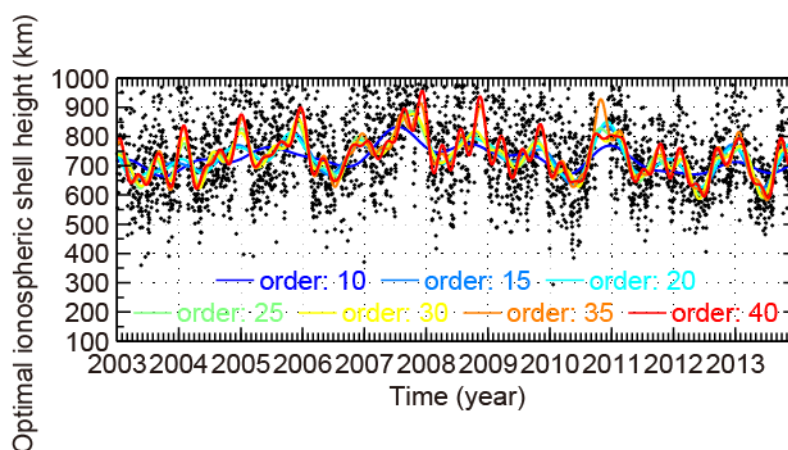
We thank for this constructive suggestion. We set the elevation cut-off angle as 15° and 30°, Figure 2 shows their results. When elevation mask angle is set as 30°, the spreading is larger, compare to 15°. But their optimal ionospheric shell heights have similar fluctuation frequencies. When other stations around apply the model, their elevation mask angles must to be same with the reference station.



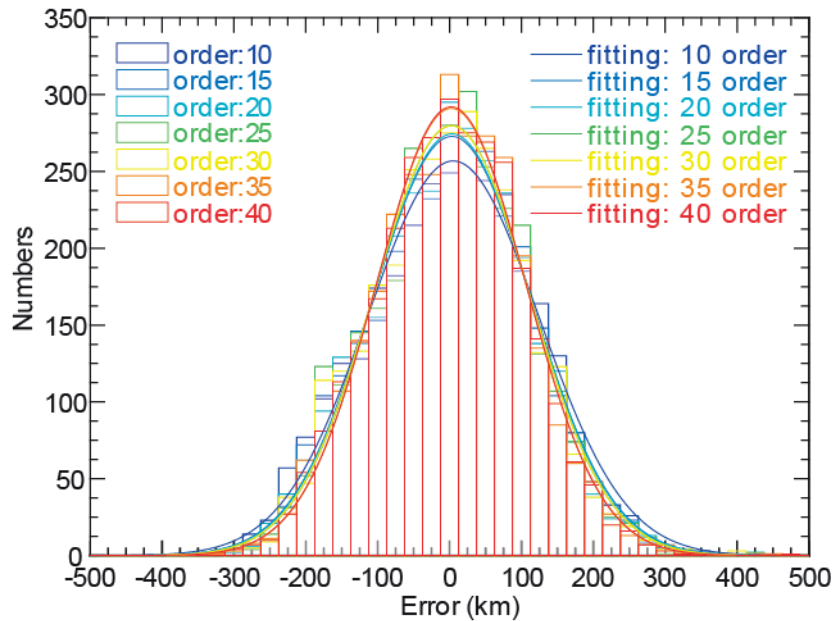


**Fig.2** the optimal ionospheric shell height with different elevation mask angle at PTBB

Figure 3 and Figure 4 show the Fourier models with different order from 10 to 40 for GOLD and their errors. With the increase of order, more details display, the variance of fitting error decreases. The models with 35 order and 40 order are similar, and much different with the other orders. Figure 3 (in the manuscript) shows that the 4-month cycle is also outstanding at GOLD. If we set the order as 30 (the minimum cycle is about 134 days) or smaller, the 4-month cycle will be lost. So we conservatively set the order as 40.



**Fig.3** Fourier fitting results of different order for GOLD



**Fig.4** Errors of the Fourier fittings for GOLD and their Gaussian fitting results

Yes, the missing tails indicate the limits of our approach. The approach attempts to reduce the DCB error by modifying the shell height. While the error is not only caused by the inappropriate shell height, but also caused by mapping function error and VTEC model error.

Figure 4 and 5 top panels show the difference of the DCBs of 2014 in the reference station with the predictions of the Fourier model. However this model has been presented earlier only in term of shell height. It is therefore difficult to understand if it is a good prediction or not. I think a more explicit discussion of the whole validation approach is needed

Reply:

We appreciate the reviewer for raising this comment. We have followed the suggestion. Please see Line 172-176. The difference between DCB released by CODE and DCB calculated by the predicted optimal ionospheric shell heights are plotted in red dots. The difference between DCB released by CODE and DCB calculated by the fixed ionospheric shell height are plotted in black dots. In both Figure 4 and Figure 5, the general distribution and mean value of red dots is smaller than that of black dots, which means the DCB estimation is improved by using the predicted optimal

ionospheric shell heights.

### **Technical corrections**

line 108: I think “it is intuitional and practical” should read “it is intuitive and practical”

#### Reply:

We thank the reviewer for pointing out this mistake. We have accordingly make the revision. Please see Line 112 in the revised manuscript.

line 191: correct “the receiver type of GOLD have been changed” into “the receiver type of GOLD has been changed”.

#### Reply:

We thank the reviewer for pointing out this mistake. We have accordingly make the revision. Please see Line 198 in the revised manuscript.

line 197: I suggest to indicate in the caption that the stations in black are the reference stations for the study. I would also suggest to include in both maps of figure 1 circles centered on the reference station to indicate the distances, e.g. 300, 600, 900, 1200 km, or whichever choice the authors think is significant.

#### Reply:

We thank for your helpful comment. We have modified figure 1 as suggested. Please see Line 203 in the revised manuscript.

line 201: I suggest to indicate more explicitly that in the table the column of “Receiver type” includes the date of change of the receivers in the reference stations.

#### Reply:

We thank for this comment. We have followed the suggestion. Please see Line 208 in the revised manuscript.

Figure 2 vertical axis label contains a typo: Scarge instead of Scargle

Reply:

We thank the reviewer for pointing out this mistake. We have modified figure 3 as suggested. Please see Line 235 in the revised manuscript.

lines 239-241: the description of figure 4 repeats the concept expressed in the previous sentence. To avoid confusion I suggest to simplify the text writing something like: "The results of this comparison are shown in Figure 4".

Reply:

We thank the reviewer for pointing out this mistake. We have accordingly make the revision. Please see Line 249 in the revised manuscript.

line 252: I suggest to rewrite the sentence "Note that some days no result because of missing data", for instance: "Data gap on the figure correspond to days when data from that station are not available".

Reply:

We thank the reviewer for pointing out this mistake. We have accordingly make the revision. Please see Line 259-260 in the revised manuscript.

lines 254-256: I suggest to simplify the sentence to avoid the cumbersome expression "is more concentrated distributed around 0 in a statistical sense".

Reply:

We thank for the reviewer for raising this suggestion. We have followed the reviewer's suggestion and rephrased this sentence. Please see Lines 261-264 in the revised manuscript.

lines 256-258 the wording "can be improved" at that position in this sentence is not grammatically correct.

Reply:

We thank for reviewer for this important comment. We have rephrased this sentence

in the revised manuscript. Please see line 264-266 in the revised manuscript.

line 320: correct "GLOD" into "GOLD".

Reply:

We thank the reviewer for pointing out this mistake. We have accordingly make the revision. Please see Line 329 in the revised manuscript.

Many bibliographic records appear to be incomplete, either the title of the article or the volume number, or doi is missing. Doi should be included without the "https://doi.org/" prefix.

Reply:

We thank the reviewer for the helpful suggestions. We have revised the references accordingly in the revised manuscript.

# **Validation and application of optimal ionospheric shell height model for single-site TEC estimation**

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## **Abstract**

We recently proposed a method to establish optimal ionospheric shell height model based on the international GNSS service (IGS) station data and the differential code bias (DCB) provided by Center for Orbit Determination in Europe (CODE) during the time from 2003 to 2013. This method is very promising for DCB and accurate total electron content (TEC) estimation by comparing to traditional fixed shell height method. However, this method is basically feasible only for IGS stations. In this study, we investigate how to apply the optimal ionospheric shell height derived from IGS station to non-IGS stations or isolated GNSS receivers. The intuitional and practical method to estimate TEC of non-IGS stations is based on optimal ionospheric shell height derived from nearby IGS stations. To validate this method, we selected two dense networks of IGS stations located in US and Europe region. Two optimal ionospheric shell height models are established by two reference stations, namely GOLD and PTBB, which are located at the approximate center of two selected regions. The predicted daily optimal ionospheric shell heights by the two models are

applied to other IGS stations around these two reference stations. Daily DCBs are calculated according to these two optimal shell heights and compared to respective DCBs released by CODE. The validation results of this method present that 1) Optimal ionospheric shell height calculated by IGS stations can be applied to its nearby non-IGS stations or isolated GNSS receivers for accurate TEC estimation. 2) As the distance away from the reference IGS station becomes larger, the DCB estimation error becomes larger. The relation between the DCB estimation error and the distance is generally linear.

### **Keyword**

Ionospheric shell height, Single layer model (SLM), Differential code bias (DCB), Total electron content (TEC)

### **Introduction**

Dual-frequency GPS signals propagation are affected effectively by ionospheric dispersive characteristic. While, by taking advantage of this property, ionospheric TEC along the path of signal can be estimated by using differencing the pseudorange or carrier phase observations from dual-frequency GPS signals. Carrier phase leveling/smoothing of code measurement is widely adopted to improve the precision of absolute TEC observations (Mannucci et al., 1998; Horvath and Crozier, 2007). In general, it is considered that the derived TEC in carrier phase leveling/smoothing

technique consists of slant TEC (STEC), the combination differential code bias (DCB) of satellite and receiver, multipath effects and noise. The DCB is usually considered as the main error source and could be as large as several TECu (Lanyi and Roth, 1988; Warnant 1997).

For TEC and DCB estimations, mapping function with single layer model (SLM) assumption have been intensively studied for many years. Sovers and Fanelow (1987) firstly simplified the ionosphere to a spherical shell. They set the bottom and the top side of the ionospheric shell as  $h-35$  and  $h+75$  km, where  $h$  is taken to be 350 km above the surface of the earth and allowed to be adjusted. In this model, the electron density was evenly distributed in the vertical direction. Based on this model, Sardón et al. (1994) introduced the Kalman filter method for real-time ionospheric VTEC estimation, **which can also be promising prediction of DCBs under adverse conditions (antispoofing, ionospheric disturbances)**. Klobuchar (1987) assumed that STEC equals VTEC multiplied by the approximation of the standard geometric mapping function at the mean vertical height of 350 km along the path of STEC. Lanyi and Roth (1988) further developed this model into a single thin-layer model, and proposed the standard geometric mapping function and the polynomial model. The single thin-layer model assumed that the ionosphere is simplified by a spherical thin shell with infinitesimal thickness. Clyne et al (1989) proposed a mapping function in the form of a polynomial by assuming a homogeneous electron density shell between altitudes of 200 and 600 km. Mannucci et al (1998) presented an elevation scaling mapping function derived from extended slab mode. There are also many modified



mapping function according to the standard geometric mapping function. Schaer (1999) proposed the modified standard mapping function using a reduced zenith angle. Rideout and Coster (2006) presented a new mapping function which replaces the influence of the shell height by an adjustment parameter, and set the shell height as 450 km. Smith et al (2008) modified the standard mapping function by using a complex factor. Based on the electron density field derived from the international reference ionosphere (IRI), Zus et al (2017) recently developed an ionospheric mapping function at fixed height of 450 km with dependence on time, location, azimuth angle, elevation angle, and different frequencies.

Ionospheric shell height is considered to be the most important parameter for mapping function, and the shell height is typically set to a fixed value between 350 and 450 km (Lanyi and Roth, 1988; Mannucci et al., 1998). Birch et al. (2002) proposed an inverse method for estimate the shell height by using simultaneous VTEC and STEC observations, and suggested the shell height is preferred to be a value between 600 and 1200 km. Nava et al. (2007) **utilized multiple stations** to obtain a shell height estimation method by minimizing the mapping function errors, this method is referred as the “coinciding pierce point” technique. Their results indicated that the suitable shell heights for the mid-latitude is 400 km and 500 km during the geomagnetic undisturbed conditions and disturbed conditions, respectively. In the case of the low-latitude, the shell height at about 400 km is suitable for both quiet and disturbed geomagnetic conditions. Jiang et al. (2018) applied this technique to estimate the optimal shell height for different latitude bands. In their case, the optimal

layer height is about 350 km for the entire globe. Brunini et al. (2011) studied the influence of the shell height by using an empirical model of the ionosphere, and pointed out that a unique shell height for whole region does not exist. Li et al. (2018) applied a new determination method of the shell height based on the combined IGS GIMs and the two methods mentioned above to the Chinese region, and indicated that the optimal shell height in China ranges from 450 to 550 km. Wang et al. (2016) studied the shell height for grid-based algorithm by analyzing goodness of fit for STEC. Lu et al. (2017) applied this method to different VTEC models, and investigated the optimal shell heights at solar maximum and at solar minimum.

In the recent study by Zhao and Zhou (2018), a method to establish optimal ionospheric shell height model for single station VTEC estimation has been proposed. This method calculates the optimal ionospheric shell height with regards to minimize  $|\Delta\text{DCB}|$  by comparing to the DCB released by CODE. Five optimal ionospheric shell height models were established by the proposed method based on the data of five IGS stations at different latitudes and the corresponding DCBs provided by CODE during the time 2003 to 2013. For the five selected IGS stations, the results have shown that the optimal ionospheric shell height models improve the accuracies of DCB and TEC estimation comparing to fixed ionospheric shell height of 400 km in a statistical sense. We also found that the optimal ionospheric shell height show 11-year and 1-year periods and is related to the solar activity, which indicated the connection of the optimal shell height with ionospheric physics.

While the proposed optimal ionospheric shell height model is promising for

DCB and TEC estimation, **this method also can be implemented to isolated GNSS receivers not belonging to IGS stations, if we can get the long-term observations and reference values of DCB from the isolated GNSS receivers.** The purpose of this study is to investigate the application of the optimal ionospheric shell height derived from IGS station to non-IGS stations. By considering the spatial correlation of ionospheric electron density, **it is intuitive and practical** to adopt the optimal ionospheric shell height of a nearby IGS station for the non-IGS stations.

The purpose of this study is to investigate the feasibility of applying the optimal ionospheric shell height derived from IGS station to nearby non-IGS GNSS receivers for accurate TEC/DCB estimation. By selecting two different regions in U.S. and Europe with dense IGS stations, we calculate the daily DCBs of 2014 by using the optimal ionospheric shell heights derived from 2003-2013 data of two central stations in two regions. We also try to find the DCB estimation error and its relation to distance away from the central reference station.

## **Method**

In (Zhao and Zhou, 2018), we proposed a concept of optimal ionospheric shell height for accurate TEC and DCB estimation. Based on daily data of single site, this approach searches daily optimal ionospheric shell height, which minimizes the difference between the DCBs calculated by VTEC model for single site and reference values of DCB. For a single site, its long-term daily optimal ionospheric shell heights can be estimated and then modeled. In our case, the polynomial model (Wild, 1994;

Komjathy, 1997) is applied to estimate satellite and receiver DCBs, and the DCBs provided by CODE are used as the reference.

In the polynomial model, the VTEC is considered as a Taylor series expansion in latitude and solar hour angle, which is expressed as follows:

$$T_V(\varphi, S) = \sum_{i=0}^m \sum_{j=0}^n E_{ij} (\varphi - \varphi_0)^i (S - S_0)^j \quad (1)$$

where  $T_V$  denotes VTEC.  $\varphi$  and  $S$  denote the geographic latitude and the solar hour angle of ionospheric pierce point (IPP), respectively;  $\varphi_0$  and  $S_0$  denote  $\varphi$  and  $S$  at regional center.  $E_{ij}$  is the model coefficient.  $m$  and  $n$  denote the orders of the model. A polynomial model fits the VTEC over a period of time. In our case, a VTEC model is generated over 3 hours of time, therefore 8 VTEC models are applied per day. DCB is considered as constant in one day. Since our analysis is based on long-term single site data, we set  $m$  and  $n$  to 4 and 3, respectively. Huang and Yuan (2014) applied the polynomial model with the same orders to TEC estimation.

Based on the thin shell approximation, the observation equation can be written as:

$$T_{os}^{PRN}(\varphi, S) = T_V(\varphi, S) \cdot f(z) + DCB^{PRN} \quad (2)$$

where  $T_{os}^{PRN}$  is slant TEC calculated by carrier phase smoothing, the superscript  $PRN$  denotes GPS satellite.  $DCB^{PRN}$  denotes the combination of GPS satellite and receiver DCB.  $z$  denotes the zenith angle of IPP. According to Lanyi and Roth (1988), the standard geometric mapping function  $f(z)$  is expressed as follows:

$$f(z) = 1/\cos(z) \quad (3)$$

$$z = \arcsin \frac{Re \cdot \cos El}{Re + h} \quad (4)$$

where  $Re$  denotes the earth's radius,  $El$  denotes the elevation angle, and  $h$  denotes the thin ionospheric shell height. Note that  $h$  also affects the location of IPP.

To estimate DCBs, The method above requires a definite thin shell height value. Conversely, if we get the daily solutions of DCBs, the optimal ionospheric shell height can be estimated. The optimal ionospheric shell height is assumed to be between 100 and 1000 km and is defined as the shell height with the minimum difference between  $DCB^{PRN}$  and the reference values. This optimization problem can be written as:

$$\min_{100 < h < 1000} \text{mean}(|\mathbf{DCB}_{\text{ref}} - \mathbf{DCB}|) \text{ s.t. } \mathbf{T} = \mathbf{\Phi} \cdot \mathbf{E} + \mathbf{\theta} \cdot \mathbf{DCB} \quad (5)$$

where  $h$  is the daily optimal ionospheric shell height,  $\mathbf{DCB}_{\text{ref}}$  denotes the vector of the reference values of DCBs, s.t. is the abbreviation for subject to,  $\mathbf{T} = \mathbf{\Phi} \cdot \mathbf{E} + \mathbf{\theta} \cdot \mathbf{DCB}$  is the matrix form of all the observation equations in one day,  $\mathbf{T}$  denotes the vector of  $T_{os}$ ,  $\mathbf{E}$  corresponds to the coefficients of the models,  $\mathbf{DCB}$  is the vector of  $DCB^{PRN}$ ,  $\mathbf{\Phi}$  and  $\mathbf{\theta}$  are the coefficient matrix of  $\mathbf{E}$  and  $\mathbf{DCB}$ , respectively.

After the method above is applied to 11-year data, the estimated optimal ionospheric shell heights can be modeled by a Fourier series, which is expressed as follows:

$$h(x) = a_0 + \sum_{n=1}^k \left( a_n \cos \frac{2n\pi x}{L} + b_n \sin \frac{2n\pi x}{L} \right) \quad (6)$$

where  $k$  is the order of Fourier series and is set to 40,  $a_n$  and  $b_n$  are the model coefficients,  $x$  is the time, and  $L$  is the time span which equals to 4018 days. The maximum frequency of model is  $40/L \approx 0.01$  per day, which corresponds to a period of 100 days. By least square method, the model coefficients can be estimated.

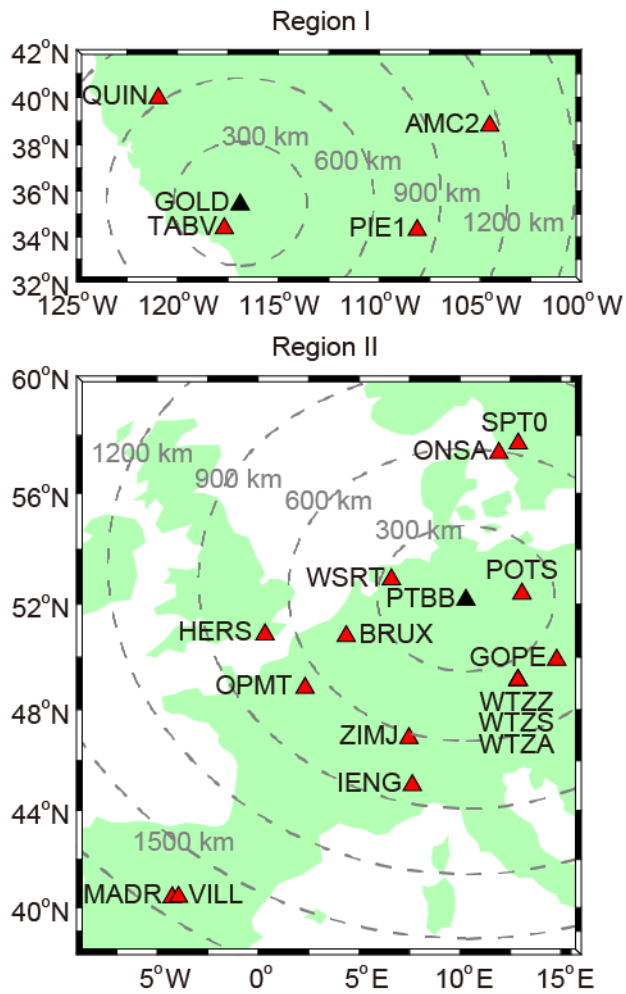
This model can be applied to neighboring stations' DCB estimation. Instead of fixed shell height, this model provide predicted optimal ionospheric shell height. While in the establishment and application of the model, the VTEC model, mapping function and elevation cut-off angle can't change. Both of them affect the optimal ionospheric shell height.

## **Experiment and Results**

The previous section introduced a method to establish daily optimal ionospheric shell height model based on single site with reference values of DCBs. To analyze the improvement of DCB estimation by this model for the reference station and other neighboring stations, we present two experiments to evaluate and validate this method by using IGS stations located in U.S. and Europe region. To ensure the accuracy and consistency of DCB, we only select IGS stations with pseudorange measurements of P1 code, and whose receiver DCBs have been published by CODE.

Figure 1 presents the location and distribution of the selected IGS stations in two regions. Table 1 presents the information of the geographical location, distance to reference station in each region and receiver types of all stations. Based on the

RINEX data of GOLD station in Region I and PTBB station in Region II during the period of 2003-2013, two separate optimal ionospheric shell height models for each region are established by the aforementioned method. Then the model are applied to DCB estimation in 2014 for all the other stations in each region. Note that reference GOLD and PTBB stations are marked with black triangle in the figure. The other neighboring stations are located in different orientations of GOLD and PTBB with different distances, which range from 136 to 1159 km for region I and range from 190.82 to 1712.27 km for region II. In the table, the receiver type is corresponding to 2003~2014 for GOLD and PTBB, and 2014 for the other stations. In region I, the receiver type of GOLD has been changed once in September 2011. The five selected stations used four receiver types in 2014; TABV and PIE1 had the same receiver type. In region II, there are nine receiver types for the sixteen stations. The receiver type of PTBB has changed twice in 2006.



**Fig.1** Geographical location of the selected IGS stations in U.S. region (Region I) and Europe region (Region II). **The black triangle in each plot is the reference station.**

**Table 1** Information for the stations

Name	Latitude (deg)	Longitude (deg)	Distance to GOLD or PTBB (km)	Receiver type and service date
GOLD	35.42	-116.89	0	ASHTECH Z-XII3 ~ 2011-09-14
TABV	34.38	-117.68	136.67	JPS EGGDT 2011-09-19 ~

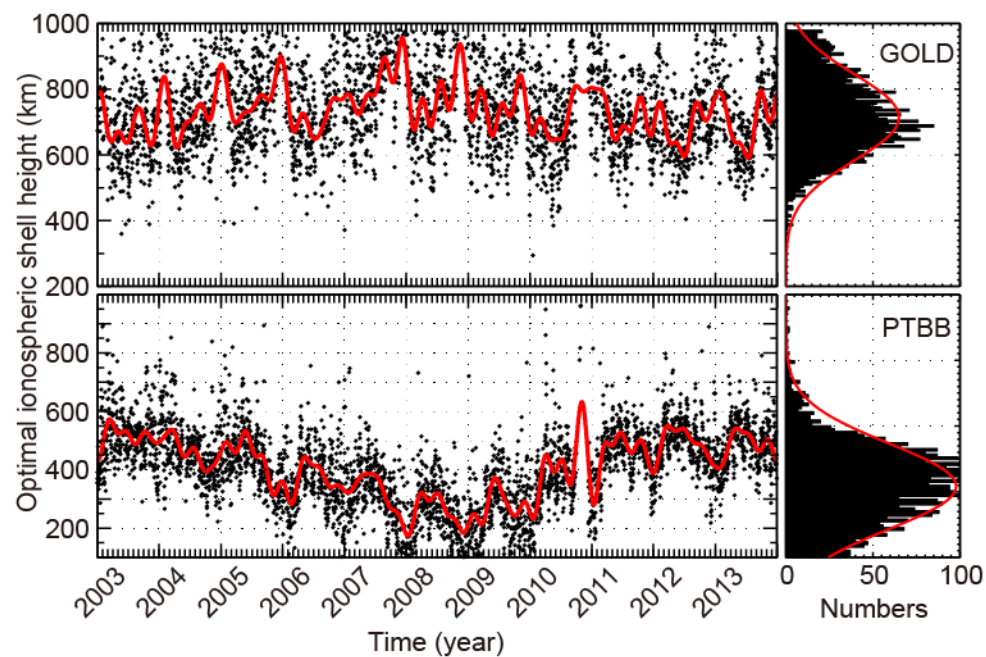


QUIN	39.97	-120.94	619.55	ASHTECH UZ-12
PIE1	34.30	-108.12	810.51	JAVAD TRE_G3TH DELTA
AMC2	38.80	-104.52	1159.09	ASHTECH Z-XII3T
PTBB	52.15	10.30	0	SEPT POLARX2 2006-07-25~ 2006-11-13 ASHTECH Z-XII3T else
POTS	52.38	13.07	190.82	JAVAD TRE_G3TH DELTA
WSRT	52.91	6.60	264.92	AOA SNR-12 ACT
WTZA	49.14	12.88	381.28	ASHTECH Z-XII3T
WTZS	49.14	12.88	381.28	SEPT POLARX2
WTZZ	49.14	12.88	381.28	JAVAD TRE_G3TH DELTA
GOPE	49.91	14.79	401.51	TPS NETG3
BRUX	50.80	4.36	439.03	SEPT POLARX4TR
ONSA	57.40	11.93	593.72	JPS E_GGD
ZIMJ	46.88	7.47	620.79	JAVAD TRE_G3TH DELTA
SPT0	57.72	12.89	641.78	JAVAD TRE_G3TH DELTA
OPMT	48.84	2.33	674.24	ASHTECH Z-XII3T
HERS	50.87	0.34	705.38	SEPT POLARX3ETR
IENG	45.02	7.64	816.64	ASHTECH Z-XII3T
VILL	40.44	-3.95	1696.62	SEPT POLARX4
MADR	40.43	-4.25	1712.27	JAVAD TRE_G3TH DELTA

Figure 2 presents the estimated daily optimal ionospheric shell height of GOLD and PTBB during the period from 2003 to 2013. The left panel shows the variation of the daily optimal ionospheric shell height and the fitting result by (6). From the overall trend, the variations of daily optimal ionospheric shell height for both two stations appear wave-like oscillation during the 11 years period. In the right panel, the

statistical result are fitted by a normal distribution. The mean and the standard deviation (STD) of the normal distribution are 714.3 and 185.4 km for GOLD, respectively. The mean and STD value for PTBB is 416.4 and 184.1 km, respectively.

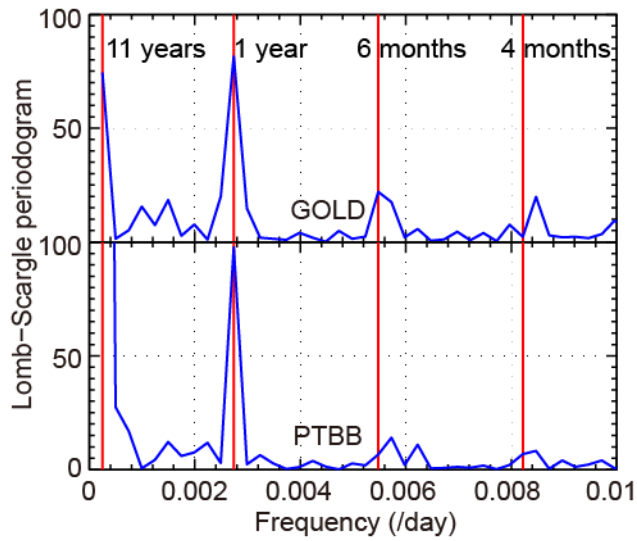
At the end of 2010, a gap appears, for the DCB provided by CODE is simultaneously anomalous (Zhao and Zhou, 2018), and the data during this period are abandoned.



**Fig.2** Variation of the daily optimal ionospheric shell height (black) and the fitting result (red)

Figure 3 presents the amplitude spectra of the daily optimal ionospheric shell height of two reference stations estimated by the Lomb-Scargle analysis (Lomb, 1976; Scargle, 1982). As can be found in Figure 3, the peaks correspond to 11-year, 1-year, 6-month and 4-month cycles. The amplitudes of 11-year and 1-year cycles are more evident than other periods in both two stations. Note that the frequencies above 0.01

per day are discarded because of their small amplitudes. As mentioned earlier, 0.01 per day is about the maximum frequency of (6). This result shows that the optimal ionospheric shell height of GOLD and PTBB is periodic, and the 40th-order of Fourier series is suitable for modelling its variation.

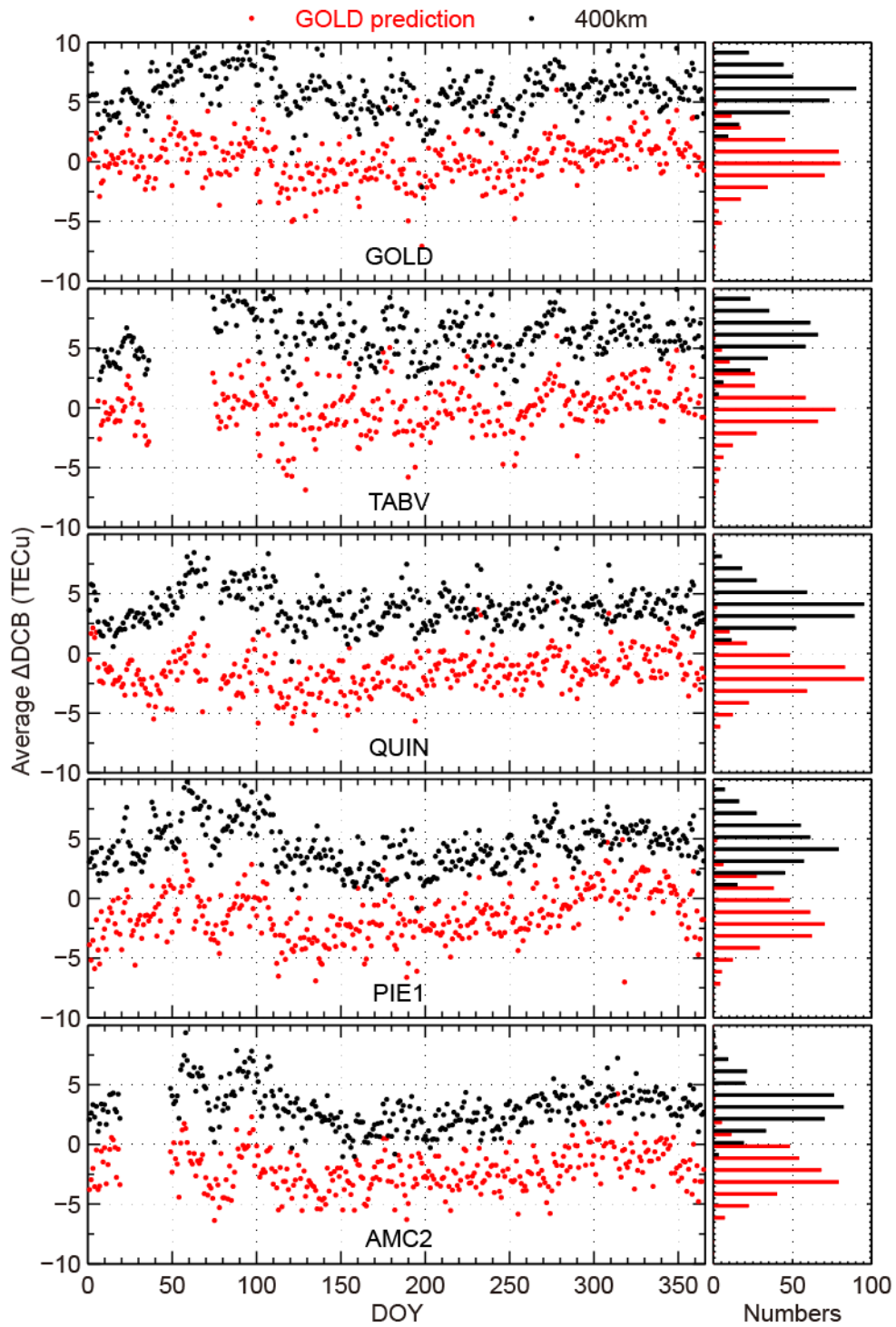


**Fig.3** Lomb-Scargle spectra of the daily optimal ionospheric shell height

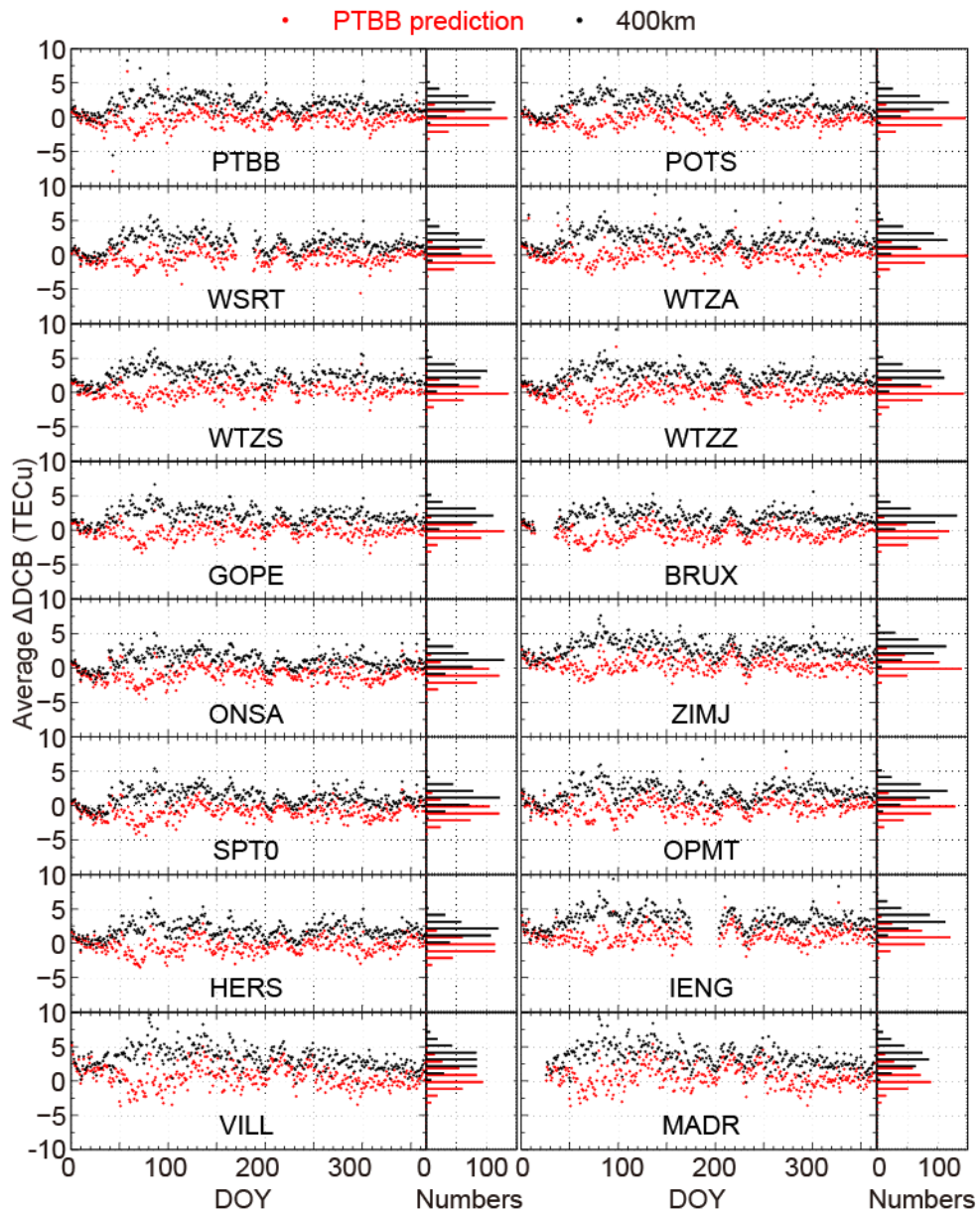
We establish two optimal ionospheric shell height models for each region by the 40th-order of Fourier series based on the 11-year data of GOLD and PTBB. To investigate the availability zone of the optimal ionospheric shell height model, we apply the model to the stations of each region as shown in Figure 1 and Table 1. Based on the predicted daily optimal ionospheric shell heights in 2014 calculated by the model of GOLD and PTBB, the DCBs in all stations of each region are estimated in the form of single station by the polynomial model mentioned earlier. The difference of DCBs in all station in each region calculated by the optimal ionospheric

shell height model from each reference station and DCBs provided by CODE is then compared to the difference of DCBs calculated by fixed ionospheric shell height (400 km) and DCBs released by CODE.

The results of this comparison are shown in Figure 4. The panels for the stations are arranged by their distances to reference station, this is also applied to the following table; from the top panels to the bottom panels, the distance of the corresponding station to the reference station gradually increases. The left and right panels show the daily differences and the histograms of the statistical results in 2014, respectively. For all of the stations, the daily average differences of DCBs calculated by the optimal ionospheric shell height model are reduced compared to the fixed ionospheric shell height. For GOLD and TABV, the reductions are appropriate, the daily average  $\Delta$ DCBs around zero have the most days. For the other stations, the reductions are so much that most of the average  $\Delta$ DCBs are negative. This result shows the improvement of the model seems to be related with the distance to GOLD. Data gap on the figure correspond to days when data from that station are not available. Figure 5 is the same format as Figure 4, which presents the results of Region II. By comparing to the results of fixed ionospheric height, Figure 5 also indicates that the  $\Delta$ DCB calculated by using optimal ionospheric shell heights of PTBB prediction is statistically less than that calculated by using fixed ionospheric shell height. Both Figure 4 and Figure 5 present that the accuracy of DCB estimation can be improved by using optimal ionospheric heights from reference station.



**Fig.4** Comparisons of the average  $\Delta$ DCB calculated by the predicted optimal ionospheric shell heights (red dots) and by the fixed ionospheric shell height (black dots) in 2014 for stations in Region I.



**Fig.5** Comparisons of the average  $\Delta$ DCB calculated by the predicted optimal ionospheric shell heights (red dots) and by the fixed ionospheric shell height (black dots) in 2014 for stations in Region II.

Table 2 presents the quantitative statistical results of average  $\Delta$ DCB in 2014. For

all the stations in each region, the mean values and the root mean squares (RMS) by the optimal ionospheric shell height model are smaller than by the fixed ionospheric height. For Region I, the improvements of TABV are the most significant. Their mean values are reduced to 0.12 and 0.08 TECu, respectively; the root mean squares are reduced by 4.43 and 4.33 TECu, respectively. For Region II, the improvement for DCB estimation are the most obvious for WTZZ, with mean value of  $\Delta$ DCB decreases from 2.34 to 0.02. We could note that TABV and WTZZ station are quite close to the reference stations in each region.

**Table 2** Statistical results of mean ( $\Delta$ DCB) in 2014

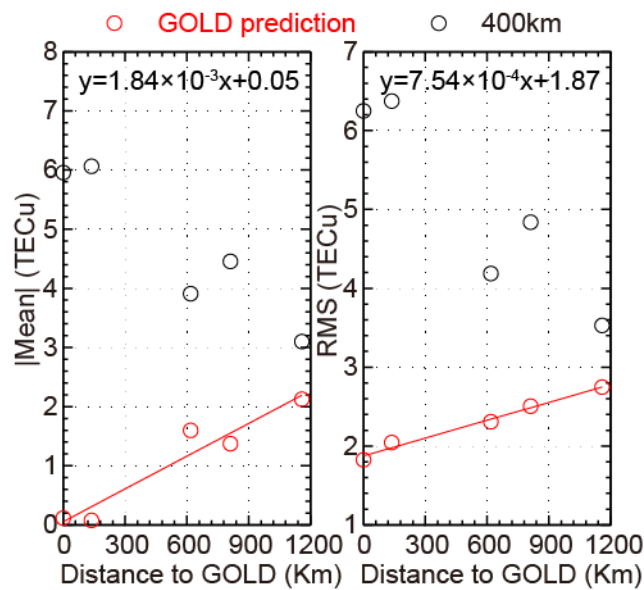
Station	Average $\Delta$ DCB (TECu)		Average $\Delta$ DCB (TECu)	
	Optimal Ionospheric Height		Fixed Ionospheric Height	
	Mean	RMS	Mean	RMS
GOLD	0.12	1.82	5.96	6.25
TABV	0.08	2.04	6.06	6.37
QUIN	-1.60	2.31	3.91	4.19
PIE1	-1.38	2.50	4.46	4.84
AMC2	-2.12	2.75	3.09	3.53
PTBB	-0.28	1.23	1.82	2.26
POTS	-0.27	1.00	1.84	2.18
WSRT	-0.41	1.14	1.65	2.10
WTZA	0.09	1.20	2.38	2.73
WTZS	0.14	0.99	2.48	2.76
WTZZ	0.02	1.14	2.34	2.65
GOPE	-0.17	1.00	2.12	2.41
BRUX	-0.42	1.12	1.86	2.13

ONSA	-0.88	1.40	1.10	1.63
ZIMJ	0.48	1.17	2.87	3.13
SPT0	-0.84	1.40	1.14	1.67
OPMT	-0.29	1.21	1.93	2.35
HERS	-0.37	1.19	1.84	2.19
IENG	1.05	1.57	3.44	3.69
VILL	0.59	1.67	3.30	3.66
MADR	0.66	1.71	3.50	3.86

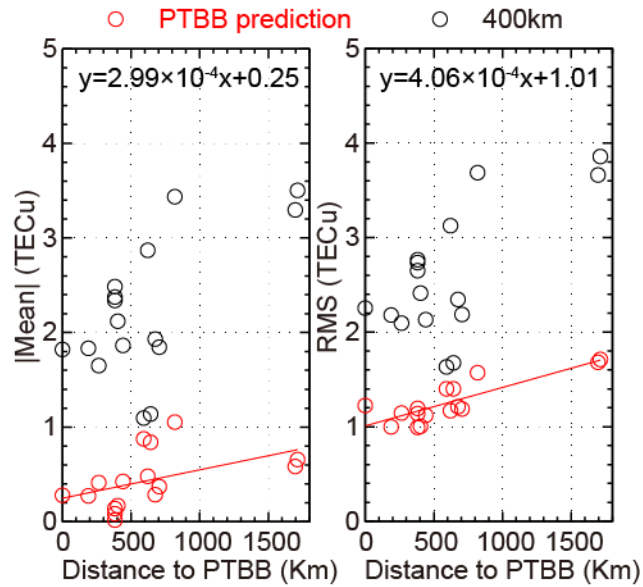
Figure 6 and Figure 7 present the relation between the statistical results of average  $\Delta$ DCB and the distance to reference stations in each region. The left and the right panels in each figure show the relation of the absolute mean value and the root mean square with the distance to GOLD and PTBB, respectively. For all of the stations, the optimal ionospheric shell height model improves the accuracies of DCB estimation compared to the fixed ionospheric shell height in a statistical sense; both of the absolute mean values and the root mean squares become smaller. For the optimal ionospheric shell height model, the absolute mean values present a positive correlation with the distance to reference station GOLD and PTBB in each region, as well as the root mean squares. By using the linear regression, for Region I, the absolute mean value increases at a rate of about 1.84 TECu per 1000 km and start at about 0.05 TECu. The RMS value increases at a rate of about 0.75 TECu per 1000 km and starts at about 1.87 TECu. According to the fitting results, the absolute mean value and the RMS less than 1 TECu and 2.25 TECu in the region around GOLD with a radius of 500 km, and less than 2 TECu and 2.62 TECu for the region with a radius



of 1000 km. For Region II, the absolute mean value increases at a rate of about 0.30 TECu per 1000 km and start at about 0.25 TECu. The RMS value increases at a rate of about 0.41 TECu per 1000 km and starts at about 1.01 TECu. According to the fitting results, the absolute mean value and the RMS less than about 0.40 TECu and 1.21 TECu in the region around PTBB with a radius of 500 km, and less than about 0.55 TECu and 1.42 TECu for the region with a radius of 1000 km. For the two regions, the RMSs presents stronger linear relation with distance comparing to the means.



**Fig.6** Relation of the accuracy for DCB estimation with the distance to GOLD. The red lines are the linear fitting results



**Fig.7** Relation of the accuracy for DCB estimation with the distance to PTBB. The red lines are the linear fitting results

## Summary

In this study, we investigate the implementation and validation of optimal ionospheric shell height derived from IGS station to non-IGS station or isolated GNSS receiver. We establish two optimal ionospheric shell height models by the 40th-order of Fourier series based on the data of IGS station GOLD and PTBB in two separate regions. These two models are applied to the stations in each region, where the distance to **GOLD** ranges from 136.67 to 1159.09 km and the distance to PTBB ranges from 190.82 to 1712.27 km. The main findings are summarized as follows:

- 1) The optimal ionospheric shell height model improves the accuracy of DCB estimation comparing to the fixed shell height for all of the stations in a statistical

sense. This results indicate the feasibility of applying the optimal ionospheric shell height derived from IGS station to other neighboring stations. The IGS station can calculate and predict the daily optimal ionospheric shell height, and then release this value to the nearby non-IGS stations or isolated GNSS receivers.

- 2) For other station in each region, the error of DCB by the optimal ionospheric shell height increases linearly with the distance to the reference GOLD and PTBB station. For the mean and the RMS of the daily average  $\Delta$ DCBs, in region I, the slopes are about 1.84 and 0.75 TECu per 1000 km; in region II, the slopes are about 0.30 and 0.41 TECu per 1000 km. This results indicate the horizontal spatial correlation of regional ionospheric electron density distribution. For different region, the error at 0 km (i.e. the error for the reference station) is different, which should be also considered.

As the requirement of this experiment, we just analyze two regions in mid-latitude due to the insufficiency of long-term P1 data. We also ignore the orientation of isolated GPS receivers to the reference station.

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