

We thank the anonymous reviewer for the valuable suggestions.

Concerning the specific comments:

- *“I find the paper to be technically very good, but the reduction of noise over the continents does not seem so important. CM5 was based upon CHAMP data only, but newer M2 models from Swarm (see Sabaka et al., 2018) show much less noise contamination in general, which will probably also be reduced over continents. ... In fact, the residuals for the trial functions in figure 6 (CM5) are over a smaller range of 0.0-0.3 than those of the figure 7 (synthetic) range 0.0-0.6, which makes me wonder if the appearance of the residuals for CM5 look more exaggerated than they really are.”*

It is true that the scales of Figures 6 and 7 are different. However, this is to illustrate different aspects of the approximations via the different trial functions, as we explain in a bit more detail in our reply to the last question in the technical corrections. It was not meant to exaggerate the residual of one magnetic field model over the other.

- *“However, what I find very important about this work are the physical based trial functions themselves because they allow a direct connection between the data and the ocean velocity and conductivity parameters. I think the most important part of the paper would be an added section that addresses solving the inverse problem of inferring these parameters from the data, that is, CM5, using these trial functions. The section should include discussions on:*
 - 1) How does the \mathbf{u} field corresponding to figure 6 from CM5 compare with the TPX08-ATLAS model?*
 - 2) Can you solve for the 1D conductivity sigma in the ball \mathbb{B}_a like was done in Grayver et al. (2016)? Even if you cannot do this at least discuss the issues like sensitivity, observability, regularization, etc.*
 - 3) By fixing u to TPX08-ATLAS, can you say anything about the sensitivity or observability of sigma in the shell \mathbb{S}_a ?”*

As the reviewer suggests, the crucial aspect is the connection between the oceanic magnetic signal and the underlying quantities. In particular, \mathbf{u} can be obtained directly from the approximation of \mathbf{B}_{oc} via the proposed trial functions, due to the linear connection. A comparison with TPX08-ATLAS has not been included in the paper since we based our trial functions solely on divergence-free (depth-integrated) current systems, and which we assume has lead to reconstructed current systems that are not comparable to TPX08-ATLAS. However, the inclusion of trial functions also based on non-divergence-free current systems and a thorough study and comparison to existing velocity field models will be part of future work. It should be said, though, that one cannot expect the resolution of, e.g., TPX08-ATLAS solely based on magnetic field satellite data.

Concerning questions 2) and 3): due to the nonlinear connection between the magnetic field and the underlying conductivity, it is significantly more difficult to reconstruct σ (assuming a fixed \mathbf{u}) than reconstructing \mathbf{u} (assuming a fixed σ). Of course, one could base our trial functions on different conductivity models and then compare the approximations of \mathbf{B}_{oc} . However, our approach does not provide a nice and simple possibility of reconstructing σ simultaneously together with \mathbf{B}_{oc} . Therefore, at the moment, we focus on the simultaneous reconstruction of \mathbf{u} and \mathbf{B}_{oc} .

These issues are now briefly addressed in the penultimate paragraph of the introduction of the paper.

Concerning the technical corrections:

- “Define \mathbf{E} and μ_0 in Equation (1).”

We added the mentioned definitions.

- “Perhaps you could give some more detail about the Regularized Orthogonal Functional Matching Pursuit.”

The functional matching pursuit is essentially a method that iteratively picks functions from a pre-defined set of trial functions (“dictionary”) for a given minimization problem (in our case, data misfit plus a regularization term). Since the minimization algorithm is not the crucial aspect of this paper (we chose the functional matching pursuit due to its flexibility but it is not necessary to use that algorithm for the proposed trial functions), we desisted from a further explanation of the algorithm. For a more detailed explanation of the algorithm, we supplied some references in the beginning of Section 2.

- “What is the meaning of the norm with respect to \mathcal{H} in Equation (2)?”

We added an according footnote.

- “Should d have a subscripted i or $i - 1$ in Equation (2)?”

No, we minimize over all $d \in \mathcal{D}$ in order to find the next d_i . The function d_{i-1} is implicitly contained in both \mathbf{B}_{i-1} and \mathbf{R}_{i-1} .

- “Define λ Equation (2).”

An explanation has been added.

- “On page 4 you say $y_{0,0}^{(2)} = \mathbf{y}_{0,0}^{(3)} = \mathbf{0}$ for convenience, but the real reason is to ensure uniqueness?”

This is in fact only for convenience in order to be able to let all sums start with $n = 0$ since neither $\mathbf{y}_{n,k}^{(2)}$ nor $\mathbf{y}_{n,k}^{(3)}$ are actually defined for $n = 0$ because the denominator becomes zero in Equations (4) and (5).

- “In the context of eigenvalue equation (not numbered) on page 6, you should state that the denominator of equation 8 is equal to $\hat{g}^T \hat{g}$. If this is not correct, then please explain the eigenvalue equation more thoroughly.”

Indeed, in case of a normalized function the denominator is equal to $\hat{g}^T \hat{g}$. We described this in more detail now.

- “What model does \mathbf{B}_{main} come from?”

We used the main field from CHAOS-5, which we now also cited.

- “What are the units of the map in figure 4a?”

The unit in Figure 4a would be m^2/s^2 and in Figure 4b it would be nT^2 . However, in the revision, to avoid confusion, we deleted the units both for Figure 4a and 4b since the accumulated energy is no actual ‘physical’ energy.

- “On page 9 you define \mathcal{D}_1 from $n = 0, \dots, 20$, but if these functions are the solid harmonics, then should you be starting at $n = 1$ instead? ”

We also incorporate the case $n = 0$ which is $\mathbf{h}_{0,0}(r\xi) = \frac{1}{r^2}\xi Y_{0,0} = \frac{1}{r^2\sqrt{4\pi}}\xi$.

- “Define a Reuter grid or give a reference.”

We added a reference.

- “What altitude are the maps in figure 5?”

We rectified the corresponding caption and added the 300km altitude.

- “In all global map figures you should outline the continents so they can be seen.”

We improved the visibility of the coast lines.

- “Figures 6 and 7 should be combined such that each corresponding map uses the same scale and can be seen simultaneously. As stated earlier, it appears that the residuals for the CM5 fits are over a smaller range, which amplifies the features relative to that of X3DG.”

We chose different ways to display the CM5 and the X3DG residuals in order to emphasize certain aspects of the approximation. In Figure 6, where CM5 data is approximated, we chose not to unify the scales in order to improve the display the different spatial structures of the residuals. That is, spherical harmonic create global errors, while the approximation with kernels results in localized errors, and the proposed new trial functions lead to larger residuals over the continents. A unification of the scales would make those differences less visible. In particular, the global structures of the polynomial spherical harmonic solution would hardly be visible in an 0.0 to 0.3nT scale. The goal of Figures 6 and 7 was not to compare the quality of CM5 and X3DG but to illustrate the effect of the choice of different trial functions (since the numerical computation of the physics-based trial functions is based on the X3DG solver, it is not surprising that the residuals over the continents seem better for the approximation of X3DG than for CM5).

Moreover, we added another (new) example in order to more thoroughly discuss the influence of continental ‘noise’ in the data (Figures 8,9,10). Here, we compare the residuals of approximations from data with and without continental ‘noise’ and we also compare corresponding RMS errors in a separate table.