Interactive comment on *Multiscale estimation of the field-aligned current density* by Costel Bunescu et al.

Anonymous Referee #1

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1 Summary

This manuscript presents a novel technique for assessing field-aligned currents across a range of scales, extending upon previous work present by (Bunescu et al., 2015). That previous work applied minimum variance analysis in sliding windows across a range of scales to determine the planarity and orientation of field-aligned currents. Using this technique, the field-aligned current density for each scale is determined by calculating the current from the gradient of the maximal variance magnetic field perturbation. While there is merit in this idea, the described technique does not sufficiently address the non-orthogonality of the scales used, which limits this techniques usefulness and how the results may be interpreted. As such, it is my recommendation that the authors revise this.

MSMVA technique is a time domain analysis based on the procedure described in (Bunescu et al., 2015). Besides the multiscale information on planarity and orientation, the main point of MSMVA is the determination of characteristic scales of the measured FAC elements and their locations, provided by $\partial_w \lambda_\eta$. In this manuscript, we use the MSMVA analysis to compute the local FAC density and we find consistent results for the synthetic generated FACs and also for the Swarm events analyzed in the manuscript. We agree with the referee that there is room for improvement, but we plan this for a future study. The main criticism from both referees is related to the non-orthogonality of the basis functions. As explained below, we regard this as a feature of the technique and not as a week point. We note that various other spectral methods were already tried in other studies, e.g. Stasiewicz and Potemra (1998), with applications for wave field characterization, e.g. Alfven waves. Stasiewicz and Potemra (1998) analysis is based on the decomposition using orthogonal wavelets. As far as we checked, this paper is cited mainly for the small-scale FACs observations by Freja and not for further application of the orthogonal-wavelet decomposition of the FAC density. Nevertheless, in a future study, we plan to complement the MSMVA analysis with the spectral type of analysis (e.g. (Stasiewicz and Potemra, 1998)) in order to better characterize both the quasi-stationary (spatial FAC structures) and the temporal-structures (e.g. wave phenomena). As of now, we just note that an orthogonal basis function may be intrinsically associated with issues that limit its use to derive the scale and location of FAC elements - as suggested in our study by the results derived with the (almost orthogonal) logarithmic sampling scheme.

I would note that the manuscript does provide a good level of detail in relatively accessible language and notation, which is a credit to the authors.

2 Major adjustments

The work on the revision of the manuscript prompted additional consistency checks, which helped us to understand better the results and limitations of the multiscale analysis, as well as to improve the computation and the interpretation of the MSMVA parameters. Among those, perhaps the main improvement is that $\partial_w \lambda_\eta$ does not simply provide the normal scale of the FAC sheet, but the crossing length along spacecraft track. In order to obtain the FAC thickness, one has to project this length onto the FAC normal. The correction of the scale information is done for all MSMVA quantities and for both scale sampling schemes, by assuming that the spacecraft track is essentially in northward direction, i.e., along the x axis of the MFA system. Basically, this correction is needed because the analysis is done in the MFA (x, y) frame, whereas thickness is defined in the FAC sheet (ξ,η) frame. In order to explain this behavior we added a paragraph at the end of section (2.3). In this paragraph we explain also that the amplitude of $\partial_w \lambda_\eta$ and the local FAC density have to be corrected as well for the dependence on thickness instead of the parameter w.

Following this adjustment, we decided to include also a proper parametrization of the orientation in the synthetic data. The new FAC system from section (3.1) includes now orientations of 0° and 40° for the FD and FU FACs, respectively. The new results show the scale correction in all quantities. The amplitude corrected $j_\|$ and $\partial_w \lambda_\eta$ show now the same amplitudes for the two FACs. The scale and amplitude correction of the multiscale parameters has also impact on the application of MSMVA to Swarm events. While the amplitude of $j_{\|}$ was correctly computed for Swarm events, the scale correction was not applied in the submitted manuscript. Thus, all profiles of MSMVA quantities (Figures 6, 10, and 14) now contain correction of scales (i.e., adjustments in the respective abscissa values) and the various scales and orientations mentioned in the text for all Swarm events (sections (4.2), (4.3), and (4.4)) were changed accordingly. Note that the previous work by Bunescu et al. (2015) did not include tests on inclined FAC structures. Nevertheless, the results of Bunescu et al. (2015) are still fine since the analysis was illustrated with east-west aligned synthetic FACs and also the application to measured data was done on essentially east-west aligned aurora.

Following the referees' comments we looked closer at the scale weightings of the local FAC density to obtain a global FAC density estimate. The conclusion is that the weighting concept is not mature enough to be applied to measured data. One option would have been to still keep the weightings for the synthetic data where we have a good reference FAC density (input FAC density) to compare with global FAC density (output of the weighting). However, such a setup, with weightings only for synthetic data, might still confuse the reader, who could wonder why the wighting is not applied also to measured data. We thus agree with the referees that the weighting is better suited to further work, oriented towards the reconstruction of the total FAC density. A paragraph was added at the end of section (2.4) to explain that we cannot simply integrate over scales to obtain a global FAC density and we also pointed out that proper scale weighting is needed because of the lack of orthogonality of the basis functions. As a consequence of removing the weighting, section (2.5) was removed and the manuscript was cleaned from the references to the weighting of FAC estimates. The associated changes are indicated by the green cuts through text in all sections. The bottom panels in Figures 1 and 2 were removed. Also the two bottom panels in the left and right plots of Figures 5, 9, and 13 were removed.

Following the removal of the global FAC estimates (weighted FAC estimates) we compare the local multiscale FAC density, at the scale given by $\partial_w \lambda_\eta$ (assumed to dominate at a specific location), with single- and dual-spacecraft FAC estimates obtained at the same time/position. We added paragraphs to each section showing Swarm observations (sections (4.2), (4.3), (4.4)), where we compared the FAC estimates and quantified the differences percentage wise. For the synthetic data (sections (3.1), (3.2)) we compared the local multiscale FAC density with the input FAC density. We also updated two paragraphs from the conclusion section to indicate the agreement between the different FAC estimates and also the consistency of the linear and logarithmic scale samplings.

The previous version of the manuscript included two FAC estimates based on the dual-spacecraft methods, FD and LS FAC estimates. A more careful analysis showed that both FD and LS show basically similar FAC density estimates. The differences were caused, essentially, by the time tags assignment. Following discussion with the coauthors we decided to keep only the FD dual-spacecraft estimate in order to reduce the amount of information and panels in Figures 4, 8, and 12. Moreover, at the moment the dual-spacecraft LS estimate is not publicly available and the application of the dual-spacecraft LS technique to Swarm is not published. The associated changes are indicated by the blue cuts through text in all sections.

The revised manuscript includes the mapping of the optical frames to the geographic frame. This change leads to adjustments in the interpretation of the optical information. Thus, the paragraphs describing the optical data were updated accordingly for each Swarm event. For the event on 17 February 2015 the mapped optical data show that actually the southward auroral structure looks curled. For the event on 15 January 2015 we now clearly see a large scale auroral structure inclined by about 20° towards south, and small embedded auroral structures of different inclinations, rather east-west aligned. Also, for the last Swarm event the optical data are now more consistent with the results of the MSMVA analysis.

Section (2.6) was integrated into the discussion section, since we do not actually show comparisons with spectral techniques.

Finally, we found an error in the computation of $\partial_w \lambda_\eta$ for the logarithmic scale sampling scheme. This was caused by using the same code as for the linear scheme, where we have a constant discretization in the scale array, dw=const, whereas for the logarithmic scheme this is not the case. The results of the linear and logarithmic schemes show now a better consistency.

Various other corrections of the text were needed to better explain certain features or correct small errors. We hope that the revised manuscript is clearer, follows better the target of this work and avoids confusion on side subjects.

3 Specific comments

There is much potential merit in the analysis technique described. However, in my opinion there is a potential underlying flaw that drastically limits the usefulness of this technique; the scales examined are not independent. As described, the minimum variance technique is applied to a collection of increasing scales by simply varying the window length over which the analysis is performed. As such, this does not isolate fluctuations on these scales and there is potential for the scales to 'bleed' into one another. The manuscript does discuss this in a limited fashion, noting that the scales are not orthogonal, however this does not go beyond a discussion.

The model functions implicitly employed to represent the magnetic field measurements are piece-wise linear functions of a certain length w, interpreted as the scale of the underlying current structure. The corresponding FAC density profile is a step function of the same width w, and centered at the same reference time $tcen$. This approach is compatible with established FAC estimators based on finite differencing. Actual magnetic profiles in the auroral zone are quite similar to these underlying piece-wise linear model functions, at least closer than perfectly smooth functions such as the ones employed for producing the synthetic data in section (3) (which are preferred there because of analytic tractability). Hence, we assume that our FAC scalogram performs actually better on real data than on the synthetic examples. Nonzero correlations among different piece-wise linear model functions lead to the non-orthogonal behavior criticized by both referees. The overall implications, however, depend on the particular subset of model functions associated with the chosen sampling scheme: (a) If for a given scale w all available center times $tcen$ are used, model functions with neighboring $tcen$ are strongly correlated, resulting in a highly redundant and very non-orthogonal representation. This scale sampling scheme we call "linear". (b) If for a given scale w the chosen center times $tcen$ are separated by the scale w, the model functions are only weakly correlated, resulting in a representation that is much less redundant and closer to orthogonality. This scale sampling scheme we call "logarithmic". The underlying logic is the same as for the Haar wavelet transform. By comparing the results of linear versus logarithmic scale sampling for synthetic data, one finds that localization of center time/location and scale is more accurate with the linear sampling scheme. In logarithmic sampling, the center location of a current structure is heavily constrained by the scale w that thus effectively represents the uncertainty of the $tcen$ (note also the uncertainty relation in wavelet analysis). Here our emphasis is on constraining FAC scales and center locations using a visualization tool, not on a full reconstruction of the FAC profile, thus we prefer to use a highly redundant set of model functions instead of an orthogonal and thus non-redundant one. Since the synthetic data are smooth profiles, and the scales are the widths of Gaussian profiles, we cannot expect that the piece-wise linear model functions identify the parameters perfectly. We updated the discussion section of the manuscript with this paragraph

As a result, the calculated FACs at each scale are comparable to the total FAC, particularly at the smaller scales and the sum of the FACs across all scales is not the total FAC.

At each scale the FAC density, as well as the MVA analysis, is applied on the detrended magnetic field perturbation obtained by removing the average signal on the respective window. Practically, at a fixed window/scale we compute the MVA and FAC density on the residual perturbation obtained by extracting a running window average. This procedure should partially remove the background / the large scale trend. We agree that one cannot completely and uniquely separate between scales. This is more difficult when the FAC elements have comparable intensities and thicknesses which results in comparable perturbations around the neighboring scale. The resolution of the method in terms of scale identification depends on the characteristics of the superposed FAC structures, e.g. the ratio of the intensities, and thicknesses, relative orientations, and localizations. In the auroral region, there are larger differences between the large/mesoscale FACs and the superposed small-scale FACs, both in terms of intensities and scales. Thus, we expect that our analysis is appropriate to separate such scales.

In order to better illustrate the technique with synthetic data, we adjusted equations (9) and (11) in section (3). The previous equations did not include properly the parametrization of the orientation. One cannot find the inclination when working in the (ξ,η) frame of the FAC sheet. We now rotate B_η to compute the components B_x and B_y which are subject to MVA. The FAC structure included in section (3.2) emphasizes the use of the technique for visualization of the local FAC density and of the other characteristics of the FACs. This FAC structure consists of a large-scale double FAC system, with fwhm thickness of about 117 km ($\sigma_\perp$=50 km) for each FAC sheet, with superposed small scale FACs of $\sim$11.7 km ($\sigma_\perp$=5 km) each. The small-scale FACs are organized in two sandwiches of 3 sheets and are centered on the large-scale FACs. The orientation of the large scale FAC system is $\theta_l^{(1)}$=0°/$\theta_l^{(1)}$=40° for FD/FU FACs, similar to the FAC system in section (3.1). For simplicity the orientation of all small-scale FACs is $\theta_l^{(1)}$=0°. To show more quantitative results we included profiles (vertical cuts) trough the spectrograms. Figure 2 (panel (p)) indicates that the local FAC density at the large and small scale is roughly consistent with the input FAC density. The local FAC density for FD/FU is 4μA/m^2/-4.5μA/m^2 and indicate a good agreement with the input of $\pm 5\mu$A/m^2. For the small scale FACs centered on FD/FU we have -12μA/m^2/-16μA/m^2, which is roughly consistent with expected input FAC density of $\sim$-16μA/m^2/-12μA/m^2.

It is not clear to me exactly how to address this. The technique described in (Bunescu et al., 2015) potentially enables different scales to be determined at different times by determining local maxima in $\partial_w \lambda_{\max}$, so some iterative process which identifies the relevant scales, filters the data at those scales, then runs the minimum variance analysis on those may be appropriate. Alternatively, filtering at a select number of scales will remove some of the 'bleed' between scales. These additions will not remove the non-orthogonality problem (discrete wavelet analysis or similar would be needed for that), band-pass filtering to attempt to isolate given scales should improve the results of the current calculation and remove the need to apply the weighting functions.

We also think of an iterative algorithm, but for the purpose to reconstruct the observed FAC density. We plan to iteratively identify the most intense planar FAC structures, that can be large or small scale FACs, as indicated by R_λ and $\partial_w \lambda_{\max}$, and apply MSMVA to the successive residuals obtained by separating the identified FACs. However, the problem might not be uniquely determined, and before further effort to develop the technique and reconstruct the FAC signature, we plan to gain more experience with its use and apply it to more events.

The filtering using zero-phase band-pass filtering comes with other difficulties. We agree that the filtering of the large scale R1/R2 current system would improve the results of the MSMVA analysis at the small- and mesoscale range. But at the same time an unappropriate filtering might introduce additional features. This is likely to be the case even with a good filtering scheme and the reason why we prefer to apply the method on the perturbation obtained only by removing a model magnetic field. Reprocessing/filtering (zero phase) is essentially a projection and distorts the interpretation. It may be preferred on theoretical grounds because of a seemingly more unique interpretation but should depend on the type of filter and then again on the model functions.

I note that in order to attempt to correct for the issue of the total multi-scale FAC, the manuscript describes three ways to weight the data: either taking the mean of the FACs across all scales; or multiplying by either the window width or one over the window width. These are somewhat contradictory to the aims of the paper as they either equally weight all scales or weight the to the larger or smaller scales. However, the principle of this analysis is to determine the most important scales. I believe that by applying the appropriate filtering, the need for these weightings will be removed.

Regarding the pre-processing of the data by band-pass filtering, this would imply an apriori selection of a specific scale range and a bias to the result. Admittedly, the weighting scheme has a similar problem - imperfectly cured for the time being by using a couple of different weights. Initially we thought to change the title of section (2.5) to "Global FAC estimation derived from the multiscale FAC" to partially remove the confusion on the purpose of this global weightings, and to add a paragraph to stress that we do not reconstruct the FAC density but aim for a quantity to qualitatively compare with the other single and dual-spacecraft methods which are not providing deconvoluted information on the FAC density. After more discussions with the coauthor we decided to remove completely the weightings from the paper. As already mentioned above, this change resulted in a few changes of the manuscript, indicated by the green color.

4 Technical comments

Figures should have panels labeled. While the panels are described in the captions, none are actually label-led.

All panels are now labeled.

The figures all appear to be fairly low resolution. For multi-panel figures, this makes them hard to examine in detail. Please provide higher resolution figures.

We now provide high resolution eps figures for the linear sampling scheme for both synthetic and Swarm data, whereas for the logarithmic sampling scheme only for the Swarm events. Saving the plots for the logarithmic scheme as eps for the synthetic data turns out to highly affect the discrete character of the respective results. Therefore, in this case we saved the figures as png that does not alter the results.

U1, 2 etc. are not labeled in Figure 4.

We included the U and D labels in the hodogram representation, right panel of Figures 4, 8 and 12. For the other figures, e.g. multipanel figures, these labels would complicate the layout too much. It is difficult to add them in the multipanel figures because of limited space. We explain in the text that the color of the interval is similar to the color of the left vertical line.

Figures 5, 9 and 13 all have a mis-labelled Y-axis in the top left plot (this should be "Magnetic Field (nT)" or similar)

We corrected this error.

The caption for Figure 6 does not described the coloured traces. Furthermore, the dashed lines only appear to be in two panels.

In the submitted manuscript we included the dash line only in the FAC density (panels d of Figures 6, 10, and 14), whereas the dash line in panels (a) of the same Figures was indicating an arbitrary reference level of planarity, R_λ =100. We now indicate this reference level by a solid blue line. We added also the other dash traces indicating the results for the logarithmic scanning in all panels of Figures 6, 10, and 14. The color of the traces indicates that the respective profile is taken around the center of the FAC indicated by a solid line of the same color in spectrograms (Figures 5, 9, 13). For instance, we used black and magenta for the U1 and D1 FACs in Figures 5 and 6.

Each hodogram is missing the label for the Y-axis

Previously this axis was removed to reduce the width of Figures 4, 8, and 12 , to fit to one column. We now included the Y axis.

In general, the description of the MSMVA panels in the text should be improved – it is somewhat hard to follow e.g. panel 9e4 etc. I would recommend unique letters for each panel.

We followed the suggestion and changed to unique letters. We did not develop much the discussion about MSMVA results because the basics and some applications (mainly $\partial_w \lambda_{\max}$ quantity) were shown in (Bunescu et al., 2015). By including the inclination in the synthetic FACs (section (3.1)) we now extended the description of the results also for the respective numeric experiment, where we can control the input. For synthetic FAC systems one can control the relative orientation between FAC elements.

P1. Line 11 - the abstract notes that the multiscale FAC is compared with input data and Swarm data, but gives no indication of how good or bad the comparison is.

We now make quantitative comparison between the different FAC density estimates for both synthetic and Swarm data. We compare the local multiscale FAC density with the input synthetic data, as well as with the single- and dual-spacecraft FAC density Swarm products, at the same position/time. As already mentioned above, we improved both the synthetic data and Swarm events sections by adding paragraphs where we make these comparisons and derive the differences between the FAC density estimates. We note that there are also differences between the dual- and single-spacecraft Swarm products even for events which presumably fulfill the assumptions, e.g. the event from 2015-02-17 with planar FACs.

P1. Line 17 – while I agree that solar wind-magnetosphere coupling is a key driver, there is an element of ionospheric feedback into the system which should not be ignored.

Adjusted: "subject to ionospheric feedback" added to the sentence and S-M extended to "S-M-I" in the next sentence.

P1. Line 24 – above the ionosphere, one tends to measure magnetic perturbations due to the in-situ field-aligned currents rather than the ionospheric Pedersen currents

Adjusted. Now the sentence reads: "While above the ionospheres one measures the magnetic perturbation of the field-aligned current (closed in the ionosphere mainly by the Pedersen current), the magnetic perturbation observed on ground is related mainly to the Hall component of the ionospheric current

P2. Line11 – I suggest you reword this – it reads as though the maximum width was around 400-500 m but the average was greater than that. I believe you mean the peak of the distribution was 400-500 m

Adjusted: Trondsen and Cogger (1997) addressed the scale distribution of the black aurora, found to peak around 400-500 m, with an average of 615 m (range between 200 m and 1 km).

P2. Lines 21-32 – please be clear as to whether these scales are in-situ, in which case C3 the height they were measured is important, or mapped to some common altitude

All these scales are indeed mapped to the ionosphere - now made explicit in the text.

P.18 Line 8 – You suggest that your technique is useful for comparing SwA and SwC data, but do not then go on to make this comparison. It would be interesting to see that (or remove this comment).

We removed this comment. We show only the single spacecraft results without comparison.

P.19 Line 14 – is this event a unipolar or multi-polar event from (Wu et al., 2017)

Text was adjusted accordingly - “unipolar” added to the sentence.

The authors may also be interested in a study by (Peria et al., 2013) who examined used MVA to statistically examine auroral zone crossings by FAST.

We included this reference in the introduction section and briefly commented the results.

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