Response to reviewer

We thank the reviewer for the helpful comments. We have revised the manuscript accordingly. Our point by point response to the review is given below.

Major Questions.

1) In the overall paper (Introduction, Linear vs Nonlinear Dependency, etc.) the Authors miss to cite several previous topical works dealing with the nonlinear and complex dynamics of the Earth's magnetosphere (e.g. Tsurutani, B., et al., GRL, 1990; Vassiliadis, et al., GRL, 1990. Klimas, et al., JGR, 1996; etc.). The same is for what regards previous application of information theory methods to space plasma physics and the Earth's magnetosphere. I strongly invite the Authors to revise their introduction and manuscript considering more extensively the previous literature.

We have added the references in Sections 2, 2.1, 2.2 (Linear vs nonlinear, mutual information and cumulant based cost, transfer entropy). We intend this paper to be a short paper that fits Annales Geophysica Communicate format (a few pages long). This constrains the number of citations not just in the introduction, but throughout the entire paper. Nonetheless, we added the references that the reviewer suggested as well as some additional references to provide additional background.

2) I would like to understand why the Authors in making their analysis do not consider instead of Dst its high-resolution version, Sym-H. Indeed, in disentangling the internal magnetospheric dynamics with respect to the external driven one the use of Dst index could be not sufficient, because all the fast internal processes are not contained in this index. I would like to stress that the internal magnetospheric dynamics is generally related to processes taking place in the tail regions which are characterised by timescales shorter than 60-90 minutes. Thus, Dst cannot be able to provide a reasonable information on it. Please, comment your choice and justify it.

We agree with the referee that there are fast internal processes in the tail that are not contained in the Dst index. This limitation would be particularly relevant if we were studying onset phemonena or the initiation of fast flows. For such phenomena it would be much better to use AE or PC or other indices that have high time resolution. Although fast processes in the tail may be relevant to the storm or substorm initiations, which would start the process of particle injections into the magnetosphere, the processes that govern the ring current dynamics are completely different than those that initiate the storm or substorm onsets in the tail. We refer the referee to our previous studies that addressed onset phenomena with these methods on much shorter timescales (1 minute resolution) [Johnson and Wing, External vs Internal Triggering of Substorms: An Information-Theoretical Approach, 2014]. These type of initiation studies are not within the scope of the present study.

For the work described in the paper, we are interested in the dynamics of the Dst and the symmetric ring current (Dst has long been used as a proxy for the symmetric ring current [Rostoker, 2000]). It is well known that the ring current takes a long time build up and decay. For example, Weygand and McPherron [2006] found that the ring current growth time is about 6 hours and decay time > 72 hours. The same study attributed the growth phase to the driving of the solar wind electric field. Several processes have been identified as the causal agents for the ring current decays such as convections of ions out of the front of the magnetosphere, scattering into the loss cone due to wave-particle interactions, ENA charge exchange etc. All these processes reduce the ring current slowly, in the order of ten hours to a few days [e.g., Feldstein et al., 1990; MacMahon and Llop-Romero, 2008]. Hence, the Dst index is adequate for the study that we presently pursue. We have expanded a discussion on the long time scale of Dst dynamics in Section 3.1

3) In section 3.1, Cumulant based analysis, the Authors state that each of the considered variables is Gaussianized. I do not understand this statement. The PDFs of Dst and also external drivers is generally not Gaussian. What do they mean with this statement ? I guess that probably they refer to the fact that time series are normalized to unit variance. Please explain better this statement.

The distribution of Dst and VBs are indeed generally nongaussian. As such, the raw distributions (e.g. distribution of values of Dst) have nonzero higher-order cumulants (e.g., they can have a skew and kurtosis). This property makes it more difficult to interpret whether higher order cumulants in the time evolution arise from the overall shape of the distribution of datapoints or from the time-ordering of the data. To eliminate the inherent nonzero cumulants in the overall distribution of data we construct a rank-ordered map from the original dataset to a proxy dataset of the same length drawn from a Gaussian distribution [*Kennel and Isabelle*, 1992; *Schreiber and Schmitz*, 1996; *Deco and Schürmann*, 2000]. The distribution of the proxy dataset ensures that all cumulants of the distribution beyond second order should in principle vanish. However, the time-ordering of the data can still lead to nonzero cumulants, because the joint probability distribution of Dst($t+\tau$) and Dst(t) may be non-Gaussian even if the distribution of Dst is Gaussian. Moreover, it is simple to construct surrogate data from the Gaussianized data that shares the same autocorrelation by using the same power spectrum, but randomly shifting the phases of the Fourier coefficients. The surrogate data therefore has the same autocorrelation as the original data. Any deviation from the linear statistic is apparent from comparison with the surrogate data, and we interpret these deviations as evidence of nonlinear dependence because we have falsified the hypothesis that the data can be adequately described by linear statistics. That this method works is evident in Johnson and Wing [2005] where we compare analysis using mutual information (using the actual data) and higher order cumulants (using Gaussianized data) and find a very similar result when analyzing Kp data. We have added a paragraph in Section 3.1 to explain why we need to gaussianize the data.

4) If I have correctly understood the cumulant based method, the nonlinear cross-correlation quantity should provide an information of the overall (linear and nonlinear) correlation between VBs and Dst. Thus, how can the Authors state that peaks at 25, 50 and 90 hours are of an internal origin on the basis that they are not present in the auto- correlation of external drivers ? Furthermore, in doing their analysis the Authors have considered Dst records covering 27 years (1974-2001) without discriminating between single geomagnetic storms and multiple geomagnetic storms. So how they can assert that these secondary peaks (which is less

prominent) do not come from such multiple geomagnetic storms but reflects internal processes ? This conclusion seems to me not convincing. To convince the reader that there are secondary peaks in the nonlinear cross-correlation that are of an internal origin, the Authors should make the analysis on a subset of geomagnetic storms which are characterised by only a single negative-peak in Dst.

We establish that there is a clear nonlinear response of Dst to VBs at lags = 3-10, 25, 50, and 90 hours. However, in the self-significance of VBs, there are linear and nonlinear peaks at lags = 3-12 hours. We conclude that the peaks at lags = 25, 50, and 90 must be due to internal processes.

The argument is as follows:
Suppose that Dst is **completely** driven externally by VBs and VBs time series has multiple peaks with 3 hours periodicity, then we would expect Dst to also have multiple peaks with 3 hours periodicity and (VBs,Dst) significance to also have peaks with 3 hours periodicity. However, if (VBs, Dst) significance has peaks with 25 hours periodicity, then we can say that the origin of this peak is not due to inherent nonlinearity in VBs.

If all the peaks in the Dst were externally driven, then in the case multiple storms, it would be expected that the VBs would also have multiple peaks. A peak or peaks in the self-significance of VBs would also show up in the (VBs, Dst) significance. On the other hand, if some of the peaks in the Dst are not externally driven (internally driven), then there would be peaks in the (VBs, Dst) significance that would not be present in the self significance of VBs. The present study uses 27 years of data and should be seen as a statistical study. Any rare or unusual features would appear as small or insignificant peaks in the (VBs, Dst) significance because they have been “averaged” out, but if the features are not rare, then the peaks would be significant.

On the other hand, we cannot entirely rule out other external drivers being responsible for the evolution of Dst, but it is generally accepted that VBs is likely the most important driver for the ring current decays in the recovery phase [e.g., Burton et al., 1975; O’Brien and McPherron, 2000; McPherron and P’Brien, 2001; Weygand and McPherron, 2006], so at least we can conclude that the nonlinearity seen in the response of Dst does not reflect the inherent nonlinearity of the variable considered to be the most or one the most important driver, which is suggestive that the nonlinear dependence identified in Dst is likely the result of magnetospheric processes. We have added a few sentences to clarify this point in Section 4 (summary).

5) Page 11. To my knowledge there should be also other processes/ mechanisms than ion cyclotron waves -particle scattering that could be responsible for ring-current decay. For instance, I remember that also ENA loss mechanisms could contribute to the decay of the ring current. Perhaps, this could be considered in discussing this point.

Several studies found that the ring current decay has two stages due to different processes such as convection of ions out of the front of the magnetopause, ENA charge exchange, and or coulomb scattering [Hamilton et al., 1988, Ebihara et al., 1998, Kozyra et al., 2002, Macmahon and Llop-Romero, 2011]. The ENA charge exchange can contribute to the ring current decay, mainly in the late recovery phase or the second stage, which may begin about 1 day after the storm commencements [Kozyra et al., 2002]. The charge exchange can take several days to deplete the ring current to the baseline level [e.g., Smith et al., 1976]. The interplay of the multiple loss mechanisms may contribute to the multiple peaks in the (VBs,Dst) significance. We have added discussion on charge exchange and other loss mechanisms at the end of Section 3.1.

6) In the Transfer Entropy analysis section few details are given about the way Transfer Entropy and Mutual Information are computed. To my knowledge binning procedure and PDF computing method are critical issues in evaluating these quantities. I believe that more information should be provided to make the reader able to reproduce the results.

We used the same procedure as described in our previous work [Wing et al., 2016]. The number of bins (n_b) needs to be chosen properly, but fortunately, there are some guidelines that can be followed and usually there is a range of n_b that would work. In general, we would like to maximize the amount of information. Having too few bins would lump too many points into the same bin, leading to loss of information. Conversely, having too many bins would leave many bins with 0 or a few number of points, which also leads to loss of information. *Sturges* [1926] proposes that for a normal distribution, optimal $n_b = \log_2(n) + 1$ and bin width (w) = range/n_b, where n = number of points in the dataset, range = maximum value – minimum value of the points. In practice, there is usually a range of n_b that would work. We have added a discussion on binning at the end of Section 2.2.

7) The result on the time delay (8-11 hr) between the information transfer from Vsw and Dst looks very long. The Earth's magnetosphere is expected to respond to solar wind changes on shorter timescale and this is also the case of ring-current. This is also corroborated by the capability of several Artificial Neural Network models of the Earth's magnetospheric response that consider a time delay of 1-2 hours as input variables for predicting Dst (see e.g. Wu and Lundstedt, JGR, 1997; Lundstedt et al., GRL, 2002; Pallocchia et al., Ann. Geophys., 2006). The Authors should motivate this result with more physical considerations.

Our result on the transfer of information from the VBs to the Dst with lag times of 8–11 is consistent with previous studies. For example, Borovsky et a. [1998] found that the solar wind takes 4 hr to reach the midnight region of the geosynchronous orbit and 15 hr to reach the noon region of the geosynchronous orbit. We have modified Section 3.2 to include this discussion.

Our data analysis aims to discover the dynamics of the magnetosphere, which may differ from that of a neural network model or any other models. We cannot say that we are familiar with the neural network models used for predicting Dst that are referenced by the reviewer. Hence, we would restrain from commenting on them. For example, we do not know if the modelers have considered inputting solar wind parameters in the last 11 hours and how their neural networks would respond to such a large input parameters. However, in a recent model for forecasting Dst, it has become evident that a long time history of solar wind parameters is necessary. For instance, in [Lazzús, et al. (2017), Forecasting the Dst index using a swarm-optimized neural network, Space Weather] Dst forecast model, input parameters of the last 6 hours or more are used. It is not clear how the performance of this model compares with those of other models.

There are many paradigms of neural networks and each paradigm behaves differently. Our own experience working with neural networks is that the larger the number of input parameters, the larger the networks become and the harder the networks can generalize. On the other hand, there may be benefits from having large number of input parameters (or longer time history) as they may be needed to capture more fully the dynamics of the magnetosphere. So, we see that there is an inherent competition between having smaller input parameters (time history) vs. having larger input parameters (time history) in the neural networks that we work with.

8) Figure 1 is hardly readable. I suggest to expand the X-axis or to include a inset where the first part of X-axis is expanded.
We improved the readability of Figure 1. We think that it is not necessary to have an inset.

Minor points. Some references are missing (there are some question marks at page 8.
The missing references have been furbished. Thank you.

Transfer entropy and cumulant based cost as measures of nonlinear causal relationships in space plasmas: applications to D_{st}

Jay R. Johnson

Andrews University, Berrien Springs, MI, USA

Simon Wing

The Johns Hopkins University, Applied Physics Laboratory, Laurel, Maryland, USA

Enrico Camporeale

Center for Mathematics and Computer Science (CWI), Amsterdam, The Netherlands

Jay R. Johnson, Andrews University, Berrien Springs, MI, 49104, USA. (jrj@andrews.edu)

Simon Wing, The Johns Hopkins University, Applied Physics Laboratory, Laurel, Maryland, 20723, USA (simon.wing@jhuapl.edu)

Enrico Camporeale, Center for Mathematics and Computer Science (CWI), Amsterdam, The Netherlands (e.camporeale@cwi.nl)

Abstract. It is well known that the magnetospheric response to the solar wind is nonlinear. Information theoretical tools such as mutual information, transfer entropy, and cumulant based analysis are able to characterize the nonlinearities in the system. Using cumulant based cost, we show that nonlinear significance of D_{st} peaks at $3-12$ hours lags that can be attributed to VBs, which also exhibit similar behavior. However, the nonlinear significance that peaks at lags 25, 50, and 90 hours can be attributed to internal dynamics, which may be related to the relaxation of the ring current. These peaks are absent in the linear and nonlinear self-significance of VBs. Our analysis with mutual information and transfer entropy show that both methods can establish that there are a strong correlation and transfer of information from V_{sw} to D_{st} at a time scale that is consistent with that obtained from the cumulant based analysis. However, mutual information also shows that there is a strong correlation in the backward direction, from D_{st} to V_{sw}, which is counterintuitive. In contrast, transfer entropy shows that there is no or little transfer of information from D_{st} to V_{sw}, as expected because it is the solar wind that drives the magnetosphere, not the other way around. Our case study demonstrates that these information theoretical tools are quite useful for space physics studies because these tools can uncover nonlinear dynamics that cannot be seen with the traditional analyses and models that assume linear relationships.

1. Introduction

One of the most practically important concepts in dynamical systems is the notion of causality. It is particularly useful to organize observational datasets according to causal relationships in order to identify variables that drive the dynamics. Understanding causal dependencies can also help to simplify descriptions of highly complex physical processes because it constrains the coupling functions between the dynamical variables. Analysis of those coupling functions can lead to simplification of the underlying physical processes that are most important for driving the system. It is particularly useful from a practical standpoint to understand causal dependencies in systems involving natural hazards because monitoring of causal variables is closely linked with warning.

A common method to establish causal dependencies in a data stream of two variables, e.g., $[a(t)]$ and $[b(t)]$, is to apply linear correlation studies such as *Strangeway et al.* [2005], which showed the relationship between downward Poynting flux and ion ouflows. Causal relationships are typically identified by considering a time-shifted correlation function

$$\lambda_{ab}(\tau) \triangleq \frac{\langle a(t)b(t+\tau)\rangle - \langle a\rangle\langle b\rangle}{\sqrt{\langle a^2\rangle - \langle a\rangle^2}\sqrt{\langle b^2\rangle - \langle b\rangle^2}} \tag{1}$$

where $\langle ...\rangle$ is an ensemble average obtained by drawing samples at a set of measurement times, $\{t_0, t_1, ..., t_N\}$. For example, [*Borovsky et al.*, 1998] used such a method to identify relationships between solar wind variables and plasma sheet variables. The causal dependency that the plasma sheet responds to changes in the solar wind can be identified from the time-shift of the peak of the cross correlation indicating a response time. From this type of analysis it can be found that the plasma sheet generally responds from the tail to the inner magnetosphere consistent with the notion of earthward convection. Such analysis has been particularly useful to help understand plasma sheet transport.

However, the procedure of detecting causal relationships based on linear cross-correlation suffers from a number of limitations. First it should be noted that the statistical accuracy of the correlation function is limited by the resolution and length of the data stream. Second, the linear time series analysis ignores nonlinear correlations, which may be important for energy transfer in the magnetospheric system. For example, substorms are believed to involve storage and release of energy in the magnetotail, which is a highly nonlinear response. Similarly, magnetosphere-ionosphere coupling may also be highly nonlinear involving the nonlinear development of accelerating potentials along auroral field lines and nonlinear current-voltage relationships. Third, the cross-correlation may not be a particularly clear measure when there are multiple peaks or if there is little or no asymmetry in the forward [i.e., $\lambda_{ab}(\tau)$] and backward directions [i.e., $\lambda_{ba}(\tau) = \lambda_{ab}(-\tau)$]. Finally, the cross-correlation does not provide any way to clearly distinguish between two variables that are passively correlated because of a common driver rather than causally related.

In the remainder of this paper, we will discuss other methods to identify causal relationships based on entropy based discriminating statistics such as mutual information and transfer entropy. We will also discuss the cumulant-based method. We will illustrate the shortcomings and strengths of the various methods for studying causality with examples from nonlinear dynamics and space physics.

2. Linear vs Nonlinear Dependency

It is well known that the magnetosphere responds to variation in the solar wind parameters [*Clauer et al.*, 1981; *Baker et al.*, 1983; *Crooker and Gringauz*, 1993; *Papitashvili et al.*, 2000; *Wing and Johnson*, 2015; *Johnson and Wing*, 2015; *Wing et al.*, 2016], and it has been established that the magnetosphere has a significant linear response to the solar wind. However, it is also expected that the magnetosphere has a nonlinear response [*Tsurutani et al.*, 1990; *Vassiliadis et al.*, 1990; *Klimas et al.*, 1998; *Valdivia et al.*, 2013; *Balikhin et al.*, 2011]. The nonlinear response may driven by internal dynamics rather than driven externally [*Wing et al.*, 2005; *Johnson and Wing*, 2005]. For example, the internal dynamics associated with loading and unloading of magnetic energy associated with storms and substorms is nonlinear [e.g., *Johnson and Wing*, 2014, and references therein]. Indeed, the data analysis of *Bargatze et al.* [1985] indicated that the dynamical response of the magnetosphere to solar wind input could not be entirely understood using linear prediction filters.

Suppose that we consider a set of variables $\mathbf{a}$ and $\mathbf{b}$ which could be vectors of variables measured in time and we would like to measure their dependency. Instead of considering the covariance matrix/correlation function, we consider a more general measure of dependency between an input and output is obtained by considering whether

$$P(\mathbf{a},\mathbf{b}) \stackrel{?}{=} P(\mathbf{a})\mathbf{P}(\mathbf{b}). \tag{2}$$

where $P(\mathbf{a},\mathbf{b})$ is the joint probability of input $\mathbf{a}$ and output $\mathbf{b}$ while $P(\mathbf{a})$ and $P(\mathbf{b})$ are the probability of $\mathbf{a}$ and $\mathbf{b}$ respectively. If the relationship holds, then the variables $\mathbf{a}$ and $\mathbf{b}$ are independent. For all other cases, there is some measure of dependency. In the case where the system output is completely known given the input, $P(\mathbf{a},\mathbf{b}) = \mathbf{P}(\mathbf{a})$. The advantage of considering Equation 2 is that it is possible to detect the presence of higher order nonlinear dependencies between the input and output even in the absence of linear dependencies [*Gershenfeld*, 1998].

2.1. Mutual Information and Cumulant based cost

Mutual information and cumulant-based cost are two useful measures that quantify Eq. 2. Mutual information has the advantage that in the limit of Gaussian joint probability distributions, it may be simply related to the correlation coefficient $C_{ab}(\tau)$ defined in equation 1 [*Li*, 1990]. Cumulants have the advantage of good statistics for limited datasets and noisy systems [*Deco and Schürmann*, 2000]. Moreover, for high-dimensional systems it is more efficient to compute moments of the data rather than try to construct the probability density function.

Correlation studies also only detect linear correlations, so if the feedback involves nonlinear processes (highly likely in this case) then their usefulness may be seriously limited. Alternatively, entropy-based measures such as mutual information [*Prichard and Theiler*, 1995; *Materassi et al.*, 2011] and cumulants [*Johnson and Wing*, 2005] are useful for detecting linear as well as nonlinear correlations. The mutual information is constructed from the probability distribution function of the variables and may be computed using an quantization procedure where data is binned such that the samples $[a(t)]$ are assigned discrete values $\hat{a} \in \{a_1, a_2, ..., a_n\}$ of an alphabet $\aleph_1$ and $[b(t)]$ is assigned discrete values $\hat{b} \in \{b_1, b_2, ..., b_m\}$ of an alphabet $\aleph_2$. The *ad hoc* time-shifted mutual entropy

$$\mathcal{M}_{ab}(\tau) \triangleq \sum_{\hat{a}\in\aleph_1, \hat{b}\in\aleph_2} p(\hat{a}(t+\tau), \hat{b}(t)) \log\left(\frac{p(\hat{a}(t+\tau), \hat{b}(t))}{p(\hat{a})p(\hat{b})}\right) \tag{3}$$

has been used as an indicator of causality, but suffers from the same problems as time-shifted cross correlation when it has multiple peaks and long range correlations.

Similarly, examination of time-shifted cumulants could be used as an indicator of causality in a nonlinear system. In this case, we can define a discriminating statistic

$$D^C = \sum_{q=1}^{\infty} \sum_{i_1,...,i_q \in \Pi_q} K^2_{1 i_2 ... i_q} \tag{4}$$

where

$$\begin{aligned}
K_i &= C_i = \langle z_i \rangle \qquad (4)\\
K_{ij} &= C_{ij} - C_i C_j = \langle z_i z_j \rangle - \langle z_i \rangle \langle z_j \rangle \\
K_{ijk} &= C_{ijk} - C_{ij} C_k - C_{jk} C_i - C_{ik} C_j + 2 C_i C_j C_k \\
K_{ijkl} &= C_{ijkl} - C_{ijk} C_l - C_{ijl} C_k - C_{ilk} C_j - C_{ljk} C_i \\
&\quad - C_{ij} C_{kl} - C_{il} C_{kj} - C_{ik} C_{jl} + 2(C_{ij} C_k C_l \\
&\quad + C_{ik} C_j C_l + C_{il} C_j C_k + C_{jk} C_i C_l + C_{jl} C_i C_k \\
&\quad + C_{kl} C_i C_j) - 6 C_i C_j C_k C_l
\end{aligned}$$

are the cumulants

$$C_{i...j} = \int d\mathbf{z} P(\mathbf{z}) z_i ... z_j \equiv \langle z_i ... z_j \rangle \tag{5}$$

of the joint probability distribution for variables $z_1, ..., z_j$.

With only two variables, a and b, defined above, we can consider the cost function

$$D^C_{a,b}(\tau) = D_C(a(t), b(t+\tau)) \tag{6}$$

The presence of nonlinear dependence has been identified by comparing the cumulant cost for a time series with the cumulant based cost of surrogate time series, which are constructed to have the same linear correlations as in [*Johnson and Wing*, 2005]). Significance measures the difference in the discriminating statistic from the mean of the discriminating statistic of the surrogates in terms of the spread of the surrogates, σ.

In Section 3, we will show an application of cumulant based analysis to the disturbance storm-time index (D_{st}). In principle, the cross-correlation, mutual information, and cumulant-based cost should be independent of the selection of measurement points if the system is stationary; therefore, time stationarity can be examined by comparing these discriminating statistics for groups of measurements drawn from different windows of time as in [*Johnson and Wing*, 2005; *Wing et al.*, 2016].

2.2. Transfer entropy

Another method for determining causality is the one-sided transfer entropy [*Schreiber*, 2000; *De Michelis et al.*, 2011; *Materassi et al.*, 2014; *Wing et al.*, 2016, 2018], which is based upon the conditional mutual information

$$\mathcal{M}_C(x, y|z) \triangleq \sum_{x\in\aleph_1} \sum_{y\in\aleph_2} \sum_{z\in\aleph_3} p(x, y, z) \log\left(\frac{p(x, y, z)p(z)}{p(x, z)p(y, z)}\right) \tag{7}$$

The conditional mutual information measures the dependence of two variables, x and y, given a conditioner variable, z. If either x or y are dependent on z the mutual information between x and y is reduced, and this reduction of information provides a method to eliminate coincidental dependence, or conversely to identify causal dependence.

Transfer entropy considers the conditional mutual information between two variables using the past history of one of the variables as the conditioner.

$$\mathcal{T}_{a\to b}(\tau) = \sum_{\hat{a}\in\aleph_1} \sum_{\hat{a}^{(k)}\in\aleph_1^{(k)}} \sum_{\hat{b}\in\aleph_2} p(\hat{a}(t+\tau), \hat{a}^{(k)}(t), \hat{b}(t)) \log\left(\frac{p(\hat{a}(t+\tau)|\hat{a}^{(k)}(t), \hat{b}(t))}{p(\hat{a}(t+\tau)|\hat{a}^{(k)}(t))}\right) \tag{8}$$

where $\hat{a}^{(k)}(t) = [\hat{a}(t), \hat{a}(t-\Delta), ..., \hat{a}(t-(k-1)\Delta)]$. The standard definition of transfer entropy takes $k = 1$ (no lag), but keeping a higher embedding dimension could in principle provide a more precise measure (for example, if a has periodicity a dimension of 2 may provide better prediction of future values of a from its past time series and therefore lower the transfer entropy. Transfer entropy as a discriminating statistic has the following advantages. First in the absence of information flow from a to b (i.e., $a(t+\tau)$ has no additional dependence from $b(t)$ beyond what is known from the past history of $a^{(k)}(t)$)

$p(\hat{a}(t+\tau)|\hat{a}^{(k)}(t), \hat{b}(t)) = p(\hat{a}(t+\tau|\hat{a}_{(k)}(t))$ and the transfer entropy vanishes. The transfer entropy is also highly directional so that $\mathcal{T}_{a\to b} \neq \mathcal{T}_{b\to a}$. The advantage can be clearly seen for dynamical systems where variables are forward differenced and the transfer entropy is clearly one-sided while mutual information and correlation functions can even be symmetric [*Schreiber*, 2000]. This measure also accounts for static internal correlations, which can be used to determine whether two variables are driven by a common driver or whether the variable b is causally driving the variable a.

Both mutual information and transfer entropy require binning of data. As mentioned in *Wing et al.* [2016], the number of bins (n_b) needs to be chosen properly and there are some guidelines that can be followed. In general, we would like to maximize the amount of information. Having too few bins would lump too many points into the same bin, leading to loss of information. Conversely, having too many bins would leave many bins with 0 or a few number of points, which also would lead to loss of information. *Sturges* [1926] proposed that for a normal distribution, optimal $n_b = log_2(n) + 1$ and bin width $(w) = range/n_b$, where n = number of points in the dataset, $range$ = maximum value $-$ minimum value of the points. In practice, there is usually a range of n_b that would work.

3. Application to space weather: D_{st} analysis

D_{st} (disturbance storm time index) is an hourly index that gives a measure of the strength of the symmetric ring current that, in turn, provides a measure of the dynamics of geomagnetic storms [*Dessler and Parker*, 1959]. Because of its global nature, D_{st} is often used as one of the several indices that represent the state of the magnetosphere. For example, *Balasis et al.* [2011] used the cumulative square amplitude of D_{st} time series as a proxy for energy dissipation rate in the magnetosphere and found that it fits well a power law with log-periodic oscillations, which was interpreted as evidence for discrete scale invariance in the D_{st} dynamics.

When plasma sheet ions are injected into the Earth inner magnetosphere, they drift westward around the Earth, forming the ring current. Studies have shown that the substorm occurrence rate increases with solar wind velocity (high speed streams) [e.g., *Kissinger et al.*, 2011; *Newell et al.*, 2016]. An increase in the solar wind electric field, VB_z, can increase the dawn-dusk electric field in the magnetotail, which in turn determines the amount of plasma sheet particles that move to the inner magnetosphere [e.g., *Friedel et al.*, 2001]. Studies have shown that the electric field, VBs (V_{sw} $\times$ southward IMF B_z) or VB_z, has a strong effect on the ring current dynamics [*Burton et al.*, 1975; *O'Brien and McPherron*, 2000; *McPherron and O'Brien*, 2001; *Weygand and McPherron*, 2006].

For the present study, we examine the relationships between solar wind velocity (V_{sw}) and VBs with D_{st}. We use D_{st} records in the period 1974 $-$ 2001 obtained from Kyoto University World Data Center for Geomagnetism (http://swdcwww.kugi.kyoto-u.ac.jp/index.html). The corresponding solar wind data are obtained from IMP-8, ACE, WIND, ISEE1, and ISEE3 observations. The ACE SWEPAM and MAG data; and the WIND MAG data are obtained from CDAWeb (http://cdaweb.gsfc.nasa.gov/). The WIND 3DP data are obtained from the 3DP team directly. The ISEE1 and ISEE3 data are obtained from UCLA (these datasets are also available at NASA NSSDC [http://nssdc.gsfc.nasa.gov/space/]). The IMP8 data come directly from the IMP teams. The solar wind is propagated with minimum variance technique [*Weimer et al.*, 2003] to

GSM (X, Y, Z) = (17, 0, 0) R_E to produce 1-min files, from which hourly averaged solar wind parameters are constructed.

3.1. Cumulant based analysis

Section 2.1 presents the method of cumulant based cost. Here, we show an application of cumulant based cost to detect nonlinear dynamics in D_{st}. We consider the forward coupling between a solar wind variable such as VBs and D_{st}, which characterizes the ring current response to the solar wind driver. We therefore consider the nonlinear cross-correlations of the vector

$$\mathbf{c}(t, \tau) = \{VBs(t), D_{st}(t + \tau)\} = \{z_1, z_2\} \tag{9}$$

The generalization of cost is based on realizations of $\{z_1, z_2\}$. In this case, each variable is Gaussianized with unit variance to eliminate static nonlinearities (i.e. higher order self-correlations in VB_s and D_{st} are eliminated so that the cost measures only cross-dependence between VBs and D_{st}). This procedure is explained in the next paragraph.

The distribution of D_{st} and VBs are generally non-Gaussian. As such, the raw distributions (e.g., distribution of values of D_{st}) may have nonzero higher-order cumulants (e.g., they can have a skew and kurtosis). This property makes it more difficult to interpret whether the higher order cumulants in the time evolution arise from the overall shape of the distribution of data points or from the time-ordering of the data. To eliminate the inherent nonzero cumulants in the overall distribution of data, we construct a rank-ordered map from the original dataset to a proxy dataset of the same length drawn from a Gaussian distribution [*Kennel and Isabelle*, 1992; *Schreiber and Schmitz*, 1996; *Deco and Schürmann*, 2000]. The distribution of the proxy dataset ensures that all cumulants of the distribution beyond second order should in principle vanish. However, the time-ordering of the data can still lead to nonzero cumulants, because the joint probability distribution of $D_{st}(t+\tau)$ and $D_{st}(t)$ may be non-Gaussian even if the distribution of D_{st} is Gaussian. Moreover, it is simple to construct surrogate data from the Gaussianized data that shares the same autocorrelation by using the same power spectrum, but randomly shifting the phases of the Fourier coefficients. The surrogate data therefore has the same autocorrelation as the original data. Any deviation from the linear statistic is apparent from comparison with the surrogate data, and we interpret these deviations as evidence of nonlinear dependence because we have falsified the hypothesis that the data can be adequately described by linear statistics. This method has been successfully employed in *Johnson and Wing* [2005] where K_p record was analyzed with mutual information and cumulants.

In Figure 1 we plot the significance obtained from the year 1999 as a function of time delay, τ. Significance extracted from $\{VBs(t), D_{st}(t+\tau)\}$ and $\{VBs(t), VBs(t+\tau)\}$ for 1999 are plotted in panels (a) and (b), respectively. It should be noted that there is a strong linear response at around 3 hour time delay. As shown in Figure 1a, there is a clear nonlinear response with peaking around 3−10, 25, 50 and 90 hours lasting for approximately 1 week. In contrast, in Figure 1b, the nonlinearity only has one broad peak around $3-12$ hours in the self-significance for VBs, suggesting that the nonlinear and linear peaks at $\tau = 3-12$ hours in in Figure 1a i may be associated with VBs. We will revisit the solar wind causal relationship with D_{st} using transfer entropy in Section 3.2.

The absence of the nonlinear peaks at $\tau = 25$, 50, and 90 hours in the self-significance for VBs (Figure 1b) suggest that these nonlinearities in $\{VBs(t), D_{st}(t+\tau)\}$ are related to internal magnetospheric dynamics. As the D_{st} index is thought to reflect storm activity, it is reasonable that nonlinear significance would decay on the order of 1 week as storms commonly last around that time. The strong nonlinear responses at $\tau = 25$, 50, and 90 hours are likely related to multiple modes of relaxation of the ring current following the commencement of storms. It should also be noted that other nonlinearities detected by even higher order cumulants may also be present; however, the calculation demonstrates the nonlinear nature of the underlying dynamics.

A common scenario for storm-ring current interaction is the following. A storm compresses the magnetosphere and intensifies the magnetic field in the magnetosphere and energetic particles are injected into the ring current region. The ring current intensifies as more particles are injected during the main phase of the storm, which can last ~ 6 hours [*Weygand and McPherron*, 2006]. Once the injection stops, the ring current begins to decay and the storm enters the recovery phase. Conservation of magnetic moment implies that anisotropies develop in the ring current and plasma sheet. Anisotropy drives the ring current plasma unstable to ion cyclotron waves. The ion cyclotron waves scatter energetic ions into the loss cone so that they are lost from the ring current. Nonlinear interaction between waves and particles keeps the plasma near marginal instability with a steady loss of energetic particles due to wave-particle scattering. Other loss mechanisms include charge exchange, coulomb scattering, and convective of ions to the front of the magnetopause. The ring current decay can have two stages [*Kozyra et al.*, 2002]. In the first stage, the ring current decays rapidly and the loss mechanisms can be attributed to convective out flow, pitch-angle scattering in the ring current, and O^+ charge exchange [e.g., *Weygand and McPherron*, 2006; *Hamilton et al.*, 1988]. The second stage may typically begin about one day from the commencement of the storm (see, for example, Figure 7 of *Kozyra et al.* [2002]). In the second stage, the decay rate is slower and is attributed mainly to H^+ charge exchange [*Hamilton et al.*, 1988] and can take several days to deplete the ring current to the baseline level [*Smith et al.*, 1976]. We can speculate that the multiple nonlinear response lag times that are detected with the cumulant-based approach are likely the relaxation of the ring current due to complex interplay of multiple loss processes.

3.2. Transfer entropy

As mentioned in Section 2.2, transfer entropy gives a measure of how much information is transfered from one variable to another. We have applied transfer entropy and mutual information to the relationship between the V_{sw} and D_{st} for the period $1974 - 2001$. The result is shown in Figure 2. Note that the mutual information measure suggests strong correlations between prior values of D_{st} and V_{sw}. This finding suggests that D_{st} could be a driver of V_{sw}, which is counterintuitive. On the other hand, the transfer entropy clearly shows that this information transfer in the backward direction ($D_{st} \rightarrow V_{sw}$) does not rise above the noise level (the horizontal blue lines indicate mean and standard deviation of 100 surrogate data sets where the data was randomly reordered.) This result is expected because it is the solar wind that drives the magnetosphere, not the other way around. The transfer of information from V_{sw} to D_{st} peaks at $\tau = 8 - 11$ hours. The cumulant based analysis in Section 3.1 shows that the response of D_{st} to VBs has similar time scale. This time scale is consistent with the 4 to 15 hours transport time for the solar wind to reach the midnight and noon regions of the geosynchronous orbit, respectively, from the dayside magnetopause [*Borovsky et al.*, 1998]. The analysis presented here illustrates the power of the transfer entropy for accessing causality.

4. Summary

We recently used mutual information, transfer entropy, and conditional mutual information to discover the solar wind drivers of the outer radiation belt electrons [*Wing et al.*, 2016]. Because V_{sw} anticorrelates with solar wind density (n_{sw}), it is hard to isolate the effects of V_{sw} on radiation belt electrons, given n_{sw} and vice versa. However, using conditional mutual information, we were able to determine the information transfer from n_{sw} or any other solar wind parameters to radiation belt electrons, given V_{sw} (or any other solar wind parameters). We also showed that the triangle distribution in the radiation belt electron vs. solar wind velocity plot [*Reeves et al.*, 2011] can be understood better when we consider that V_{sw} and n_{sw} transfer information to radiation belt electrons with 2 days and 0 day ($<$ 24 hr) lags, respectively. Also recently, we used transfer entropy to better understand the causal parameters in the solar cycle and their response lag times [*Wing et al.*, 2018].

As a follow up to *Wing et al.* [2016, 2018], the present study demonstrates further how information theoretical tools can be useful for space physics and space weather studies. Cumulant based analysis can be used to distinguish internal vs. external driving of the system. Both mutual information and transfer entropy give a measure of shared information between two variables (or vectors). However, unlike mutual information, transfer entropy is highly directional. To illustrate, we apply mutual information, transfer entropy, and cumulant based analysis to investigate the dynamics of D_{st} index.

Our analysis with mutual information and transfer entropy indicates that there are strong linear and nonlinear correlations and transfer of information, respectively, in the forward direction between V_{sw} and D_{st} ($V_{sw} \rightarrow D_{st}$). However, mutual information indicates that there is also a strong correlation in the backward direction ($D_{st} \rightarrow V_{sw}$), which is puzzling and counterintuitive. In contrast, the transfer entropy indicates that there is no information transfer in the backward direction ($D_{st} \rightarrow V_{sw}$), as expected because it is the solar wind that drives the magnetosphere, not the other way around. The transfer of information from V_{sw} to D_{st} peaks at $\tau = 8 - 11$ hours.

Using the cumulant-based significance, we have established that the underlying dynamics of D_{st} is in general nonlinear exhibiting a quasiperiodicity which is detectable only if nonlinear correlations are taken into account. The strong nonlinear responses of D_{st} to VBs at $\tau = 25$, 50, and 90 hours are likely related to multiple modes of relaxation of the ring current from multiple loss mechanisms following the commencement of storms. It is, of course, possible that these nonlinearities are caused by solar wind drivers other than VBs. However, the timing of these nonlinearities would put them well in the recovery phase of a storm and previous studies suggested that the ring current decays in the recovery phase are strongly influenced by VBs [*Burton et al.*, 1975; *O'Brien and McPherron*, 2000; *McPherron and O'Brien*, 2001]. The nonlinearities at $\tau = 3 - 12$ hours are not caused by internal dynamics but rather by the solar wind driver, which is similar with the time scale for the solar wind transport time from the dayside magnetopause to the inner magnetosphere. This time scale is consistent with the time scale for the information transfer from the solar wind to D_{st} obtained from transfer entropy analysis.

Although linear models are useful, our results indicate that these models have to be used with cautions because solar wind − magnetosphere system is inherently nonlinear. Hence, nonlinearities generally need to be taken into account in order to describe the system accurately. Local-linear models (which include slow evolution of parameters) may be able to handle some nonlinearities, but it is expected that these local-linear models would have difficulties if the dynamics suddenly and rapidly change.

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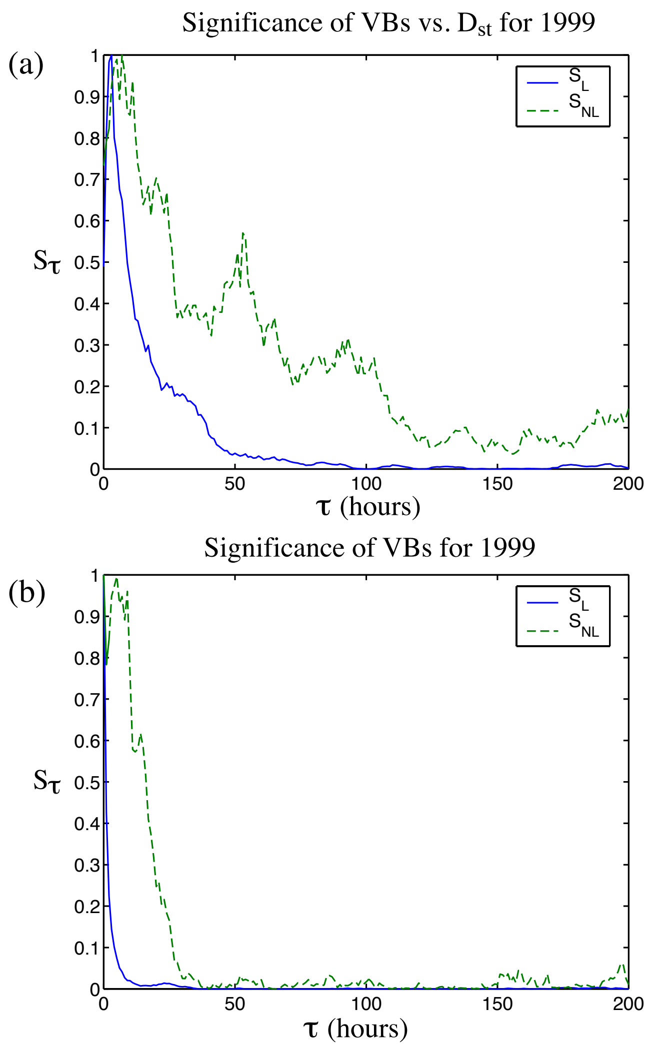

Figure 1. Significance extracted from (a) $\{VBs(t), D_{st}(t-\tau)\}$ and (b) $\{VBs(t), VBs(t-\tau)\}$ for 1999. It should be noted that there is a strong linear response at around 3 hour time delay. There is a clear nonlinear response with a strong peak around 50 hours lasting for approximately 1 week. The longterm nonlinear response is absent in the solar wind data indicating that the longterm nonlinear correlations between VBs and $\mathrm{D_{st}}$ are the result of internal magnetospheric dynamics.

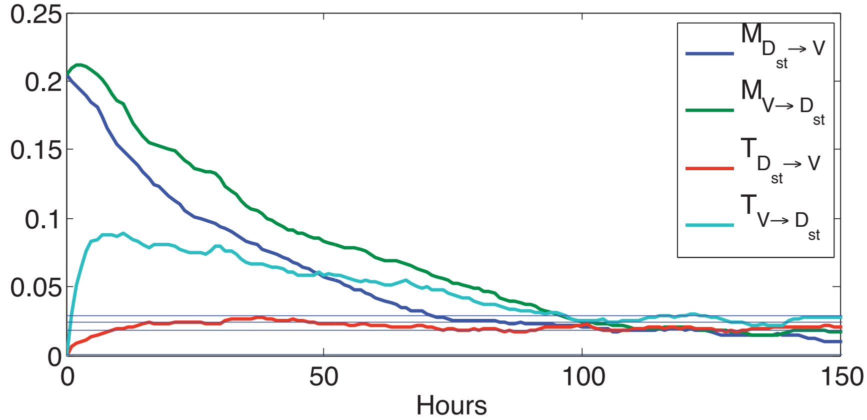

Figure 2. Comparison of mutual information and transfer entropy measures to determine causal driving of the magnetosphere as characterized by D_{st}. Note that causal driving appears to peak somewhat later (11 hours) than indicated by mutual information (2 hours) indicating that internal dynamics likely are very important initially. The backward transfer entropy is below the noise level for all values indicating that D_{st} in no way influences the upstream solar wind velocity. Such a conclusion could not be inferred from the mutual information measure.