

## Reply to referee comments – angeo-2018-69 (revision 2)

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– *In general, I am happy with the adjustments made to the paper. The authors may still want to consider a number of issues, listed below, that require further clarification.*

1. *When I first read the manuscript I was quite surprised by the difference between the simulation and analytic estimates of the diffusion time, of about two orders of magnitude. From the revised manuscript, I understand that the major reason behind is the neglect of the electron-neutral collisions in the hybrid simulations (and related contribution to the conductance), with additional impact of numerical diffusion (p.2, L4-9). According to the Response to my comments (L79), including the ion term in the conductance estimate results in a diffusion time longer by 10–20%. Conversely, the electron contribution to the conductance (and diffusion time) is some 80–90%, i.e. by neglecting this contribution the diffusion time is 5 to 10 times shorter than its actual value. Therefore, considering the electron contribution would increase the simulation proxy of 1000 s (p.2, L2) to some 500010000 s, i.e., 1.5-3 h. This reduces the difference between the simulation and analytic result by one order of magnitude. While my expertise on simulations is limited and I cannot fully judge the importance of numerical diffusion, I find this smaller difference more reasonable. A more detailed discussion (e.g., in the Intro and/or in Section 2.2), perhaps including the quantitative aspect above, may help the reader to get the message better.*

– Reply: Good idea. We added an explanation as to why our estimate of the diffusion time is so different from the hybrid simulation by two orders of magnitude. Absence of electron-neutral collisions is certainly one of the reasonable explanations. Follow-

ing the suggestion by the referee, we performed an order-of-magnitude estimate for the artificial or numerical diffusion. We found out that the numerical diffusion originates in the spatial smoothing procedure to stabilize numerical instabilities (oscillations and divergence). The smoothing procedure in the simulation can explain the difference in the conductivity estimate by about two orders of magnitude. So, both the absence of electron-neutral collisions and the spatial smoothing in the simulation can reasonably explain the difference in the conductivity estimate.

- We added the following text in section 2.3.

Page 7 line 2–6.

“For reference, we repeat the calculation of the diffusion time using only the electron term in the conductivity. The diffusion time from the electron term is in the range from about 40,000 s (11 hours) to about 146,000 s (40 hours). The ion contribution makes a difference in the diffusion time by about 10–20 %. Note that the peak Pedersen conductivity is about  $14 \text{ S m}^{-1}$  from the electron term and about  $0.035 \text{ S m}^{-1}$  (Tab. 2 right column), i.e., the ion contribution is only about 0.2 % at the peak of the Pedersen conductivity height-profile.”

- We also added a new section (Sec. 3 “Discussion”) and discuss the difference from the simulation results in new section 3.1. Accordingly, the conclusion section was moved to new section 4 “Concluding remark”.

Page 7, line 8 to page 8, line 17.

“It is interesting to observe the difference between the simulation and analytic estimates of the diffusion time by about two orders of magnitude. The major reason behind this difference most likely lies in the neglect of the electron-neutral collisions and the numerical diffusion in the hybrid simulations.

- Electron contribution. The ion term in the conductance estimate contributes to a longer diffusion time by 10–20 %. In other words, the electron contribution to the conductance (and diffusion time) is somewhat 80–90 %, i.e., by neglecting this contribution the diffusion time is 5 to 10 times shorter than its actual value. Therefore, considering the electron contribution would increase the simulation proxy of 1000 s to 5000–10000 s, i.e., 1.5–3 hours. This reduces the difference between the simulation and analytic result by one order of magnitude.
- Numerical diffusion. Problem of numerical oscillation, rounding or cutoff error, and numerical instabilities in the computation are suppressed either by adding an artificial diffusion or imposing a spatial smoothing (Winske and Omidi, 1993; Bagdonat, 2002). Here we estimate the diffusion coefficient or the conductivity for the

smoothing procedure. First, we rewrite the magnetic diffusion equation

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} \quad (5)$$

into the time advancing formula as

$$55 \quad \mathbf{B}(t + \Delta t) = \eta \nabla^2 \mathbf{B}(t) \Delta t \quad (6)$$

$$\simeq \frac{\eta}{\ell^2} \mathbf{B}(t) \Delta t, \quad (7)$$

where  $\eta$  denotes the diffusion coefficient of the magnetic field,  $\mathbf{B}$  is the magnetic field,  $\Delta t$  is the time step in the simulation, and  $\ell$  is the length scale of the gradient. Second, in the smoothing method, the magnetic field is smoothed by the following procedure (Winske and Omidi, 1993; Bagdonat, 2002; Müller et al., 2011),

$$\mathbf{B}(t) \rightarrow \mathbf{B}(t) - \alpha_{\text{sm}} (\mathbf{B}(t) - \langle \mathbf{B} \rangle), \quad (8)$$

where  $\alpha_{\text{sm}}$  is a free parameter called the the smoothing factor (its value of  $\alpha_{\text{s}}$  is typically 0.01 to 0.1) and  $\langle \mathbf{B} \rangle$  is the locally-smoothed magnetic field. Now, by comparing the right-hand side of Eq. (7) with the smoothing term  $\alpha_{\text{sm}} \mathbf{B}$  in Eq. (8), we obtain a relation between the smoothing factor  $\alpha_{\text{sm}}$  and the diffusion coefficient  $\eta$  as follows,

$$\eta = \frac{\alpha_{\text{sm}} \ell^2}{\Delta t}. \quad (9)$$

Using the relation to the conductivity  $\eta = (\mu_0 \sigma)^{-1}$ , Eq. (9) is expressed as

$$\sigma = \frac{\Delta t}{\mu_0 \alpha_{\text{sm}} \ell^2}. \quad (10)$$

We evaluate Eq. (10) using the following values: the smoothing parameter  $\alpha_{\text{sm}} = 0.01$  (i.e., 1 % spatial smoothing) from Müller et al. (2011),  $\ell = 100$  km (grid size in the simulation) from Bößwetter et al. (2004) and Martinecz et al. (2009),  $\Delta t = 1$  s (typical ion gyroperiod in the solar wind), and  $\mu_0 = 4\pi \times 10^{-7}$  H m<sup>-1</sup> (permeability of free space), and obtain the numerical conductivity (or the smoothing conductivity)  $\sigma_{\text{sm}} = 7.96 \times 10^{-3}$  S m<sup>-1</sup>. Hence, the numerical conductivity equivalent to the spatial smoothing procedure for the purpose of numerical damping in the hybrid simulation is of the order of  $10^{-2}$  S m<sup>-1</sup>. The physical conductivity from our study is 1–10 S m<sup>-1</sup> at the peak.

The magnetic diffusion time is  $L^2 \mu_0 \sigma$  and the diffusion length scale  $L$  is the same as the grid scale  $l$ . Hence, the diffusion time is proportional to the conductivity in our problem, and the difference in the diffusion time by two orders of magnitude between the hybrid simulations (about 1000 s) and our semi-analytic estimate (about 100,000 s) can reasonably be explained by the spatial smoothing procedure in the simulation.”

– 2. I would appreciate as well a more comprehensive discussion of the conductivity estimate (which controls, essentially, the diffusion time), including a comparison with the Earth case. This would cast the matter into a broader perspective and help pointing out the (different) parameter regime at Venus. In particular, the actual value of the conductivity is much higher than at the Earth, by about four orders of magnitude, which has to do, I guess, with the weaker magnetic field and higher neutral density. At the same time, the Pedersen conductance appears to be dominated by electrons, unlike at the Earth, where is dominated by ions, with important consequences, e.g. on the simulation results (as commented above).

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– Reply: The most essential difference is the lack of strong magnetic field at the Venus. This leads to a lower gyro-frequency at higher altitudes and the electron-neutral collision frequency becomes dominant at altitudes of 150 km. The electron density is nearly the same ( $10^5 \text{ cm}^{-3}$ ) between the two planets at the relevant altitudes (150 km at the Venus, 130 km at the Earth). So, the essential difference in the conductivity estimate between the Venus and the Earth comes from the ion-to-electron mass ratio (about 20,000) and this factor is slightly compensated by the ratio of the electron-neutral collision frequency at the Earth to the ion-neutral collision frequency at the Venus (a factor of about 0.2). In summary, the ratio of the peak conductivity at the Venus to that at the Earth is in the range between 6,000 and 10,000.

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– We added a new subsection 3.2 “Comparison with the Earth ionosphere”.

Page 8, line 18 to page 9, line 17.

“It is also interesting to observe that the Pedersen conductivity at the Venus is much higher than at the Earth by about four orders of magnitude. This difference can be explained as follows. We write formulas separately for the electron Pedersen conductivity  $\sigma_{p,e} \text{ S m}^{-1}$  as

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$$\sigma_{p,e}[\text{Sm}^{-1}] = \frac{n_e e^2}{m_e} \frac{\nu_{en}}{\nu_{en}^2 + f_{ge}^2} \quad (11)$$

$$\simeq 2.8 \times 10^{-2} \times n_e[\text{cm}^{-3}] \times \frac{\nu_{en}[\text{Hz}]}{(\nu_{en}[\text{Hz}])^2 + (f_{ge}[\text{Hz}])^2} \quad (12)$$

and the ion Pedersen conductivity  $\sigma_{p,i} \text{ S m}^{-1}$  as

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$$\sigma_{p,i}[\text{Sm}^{-1}] = \frac{n_e e^2}{m_i} \frac{\nu_{in}}{\nu_{in}^2 + f_{gi}^2} \quad (13)$$

$$\simeq 1.3 \times 10^{-6} \times n_e[\text{cm}^{-3}] \times \frac{\nu_{in}[\text{Hz}]}{(\nu_{in}[\text{Hz}])^2 + (f_{gi}[\text{Hz}])^2}, \quad (14)$$

where a mass ratio of  $m_i/m_p = 11.6$  or  $m_i/m_e = 2.1 \times 10^4$  is used ( $m_i$  is the ion mass,  $m_p$  is the proton mass, and  $m_e$  is the electron mass).

The Pedersen conductivity at the Venus  $\sigma_p^{(V)}$  is primarily contributed by the electron-neutral collisions. By ignoring the gyro-frequency, the Venus Pedersen conductivity is

approximately as follows.

$$\sigma_p^{(V)} \sim \frac{n_e e^2}{m_e \nu_{en}} \quad (15)$$

The Pedersen conductivity at the Earth  $\sigma_p^{(E)}$  is, in contrast to the Venus case, contributed by the ion-neutral collisions.

$$\sigma_p^{(E)} \sim \frac{n_e e^2}{m_i \nu_{in}} \quad (16)$$

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The reason for this is that the gyro-frequency exceeds the electron-neutral collision frequency at an altitude of 60–70 km and above due to a stronger magnetic field (than that of the Venus). Now we compare Eq. (15) with Eq. (16).

Using the facts that (1) the electron density is almost the same ( $n_e \sim 10^5 \text{ cm}^{-3}$ ) between the Venus (peak altitude  $z = 150 \text{ km}$ ) and the Earth (peak altitude  $z = 130 \text{ km}$ ), (2) the typical mass ratio from the ions to the electrons is about 20,000 (about 11 proton mass), and (3) the collision frequency is roughly of the same order between the Venus and the Earth,  $\nu_{en}^{(V)} = 250 \text{ Hz}$  and  $\nu_{in}^{(E)} = 50 \text{ Hz}$  at the Earth, respectively, we obtain the ratio of the peak Pedersen conductivity from the Venus to the Earth as follows:

$$\frac{\sigma_{p,e}^{(V)}}{\sigma_{p,i}^{(E)}} \sim \frac{m_i \nu_{in}^{(E)}}{m_e \nu_{en}^{(V)}} \sim 10^4. \quad (17)$$

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The difference in the peak Pedersen conductivity by nearly four orders of magnitude can essentially represent the difference in the Pedersen current carrier in the different magnetic field environments: the Pedersen current is carried by the electrons at the Venus and by the ions at the Earth. More detailed calculations of the peak Pedersen conductivity at the Earth and the Venus are shown in Tab. 2.”

– Table 2 was added, too.

**Table 2.** Comparison of Pedersen conductivity estimate at the peak altitudes of the conductivity,  $z = 130$  km at the Earth and  $z = 150$  km at the Venus. Electron density value at the Earth is taken from Kelly (1989) at  $z = 130$  km. Electron density value at the Venus is from Kilore and Luhmann (1991) during the solar maximum at an altitude of  $z = 150$  km (at the peak of Pedersen conductivity). Electron gyro-frequency is calculated from the nominal magnetic field magnitude. Collision frequency values at the Earth are from Kertz (1989) and that at the Venus are from Dubinin et al. (2014). A mean mass ratio of 11.6 is used between the ions and the protons.

	Earth	Venus
magnetic field $B$	$3.0 \times 10^5$ nT	$1.0 \times 10$ nT
electron density $n_e$	$1.2 \times 10^5$ cm $^{-3}$	$2.9 \times 10^5$ cm $^{-3}$
electron gyro-frequency $f_{ge}$	$8.4 \times 10^4$ Hz	$2.8 \times 10^2$ Hz
electron-neutral collision frequency $\nu_{en}$	$1.3 \times 10^3$ Hz	$2.5 \times 10^2$ Hz
electron Pedersen conductivity $\sigma_{p,e}$	$6.2 \times 10^{-4}$ S m $^{-1}$	$1.4 \times 10^1$ S m $^{-1}$
ion gyro-frequency $f_{gi}$	$3.9 \times 10^1$ Hz	$1.3 \times 10^{-2}$ Hz
ion-neutral collision frequency $\nu_{in}$	$5.0 \times 10^1$ Hz	$1.1 \times 10$ Hz
ion Pedersen conductivity $\sigma_{p,i}$	$2.3 \times 10^{-3}$ S m $^{-1}$	$3.5 \times 10^{-2}$ S m $^{-1}$

115 – 3. *The newly introduced para at p. 3, L9–18, is somewhat confuse:*  
3.1. *First, I am not sure if the geometric aspect, emphasized there, is the key one for the diffu-*  
*sive process. I think the energetic aspect is at least as important, since diffusion takes time be-*  
*cause energy is dissipated along the way (magnetic energy is converted to heat). In the extreme*  
120 *case where conductivity is (quasi)zero, magnetic field penetration is (quasi)instantaneous. I*  
*think the energetic aspect, not the geometric one, is the main reason behind using the Ped-*  
*ersen conductance to estimate the diffusion time. Energy dissipation is achieved by Pedersen*  
*current, while Hall current has no energetic effect.*

125 3.2. *Second, in order to clarify better the geometric aspect, the average orientation of the*  
*magnetic field in the (current carrying layer of the) ionosphere should be indicated. From*  
*the text, I infer that magnetic field is essentially included in the ionospheric plane (like at the*  
*Earth equator). Correct?*

– Reply:

3.1 Agreed. We added the energy dissipation aspect of the conductivity before the ge-  
ometrial aspect.

130 3.2 Yes.

– Changes in the manuscript:

Page 3, line 10–13.

135 “Above all, the Pedersen conductivity is relevant to the problem of diffusion time esti-  
mate. The reason for this is that magnetic diffusion takes time because energy is dissi-  
pated along the way (magnetic energy is converted to heat). It is the Pedersen current by  
which the energy dissipation is achieved. The Hall current, in contrast, has no energetic  
effect. From a geometrical point of view,”

140 Page 3, line 14–15

“or in the current-carrying layer of the ionosphere”

– 4. *Height integration (point 5 of Section 2.2) and conductivity panel of Fig. 1: My guess is*  
*that most of the conductance (height integrated conductivity) is confined around the maximum*  
*at 140 km, perhaps within 20 to 40 km. Can you indicate what is the (percentagewise) contri-*  
145 *bution of this (current carrying) layer, and what of the rest? (including the altitude regions,*  
*low and high, subject to extrapolation)*

– Reply:

150 Yes, the most of the conductivity is confied to a layer between an altitude of 140 km and  
170 km. Maximum of the conductivity is around an altitude of 150 km. We calculated  
the contribution of this layer to the condutivity, and obtain a percentage of 78–85 %

during the solar maximum and 88–91 % during the solar minimum.

– Changes in the manuscript:

Page 5, line 27–30.

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“Most of the conductance (height-integrated conductivity) is confined around the maximum at 150 km. The contribution of this current-carrying layer from 140 km to 170 km in altitude is around 78–85 % during the solar maximum (the variation comes from the choice of the magnetic field mode and the ion profile) and 88–91 % during the solar minimum.”

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– 5. *Minors:*

- p.2, L1: *simulation \*time\* as*

- L3: *numerical diffusion superimposes the physical diffusion ⇒ numerical resistivity significantly exceeds physical resistivity (?)*

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- L9: *the ICME ⇒ an ICME*

- L16: *linear to the ⇒ linear in the*

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- L17: *the magnetic energy stored in ⇒ magnetic energy supplied to (?)*

- L23: *for the reason of ⇒ to infer*

- L26: *Delete even and during the solar minimum.*

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- L30: *Correct down to 90 km (1000 km, according to p7, L7); by the Bepi ⇒ during the Bepi*

- L33: *problem into ⇒ problem to*

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- p.3, L9: *There are three different kinds of conductivity in the plasma ⇒ In general, conductivity in a magnetized plasma is a tensor, whose components are*

- p.4, L16–18 and caption of Fig. 1: *Please refer also to ion-neutral collisions, as included in the revised manuscript.*



- 185 - p.7, L5: *in a variety* ⇒ *for a variety*; *exampl,e* ⇒ *example*;
- L5–6: *BepiColombo plans two Venus flyby maneuvers* ⇒ *two Venus flyby manoeuvres are planned for BepiColombo*
- 190 - L6: *Delete to an altitude.*
- L8–9: *How long does it take the flyby?*
- Ack: *Venur*
- 195
- Reply:
    - Done.
    - Changes in the manuscript:
      - Page 1, line 23, “simulation time”
      - 200 Page 2, line 2, “numerical resistivity...”
      - Page 2, line 7, “an ICME”
      - Page 2, line 15, “linear in the”
      - Page 2, line 16, “magnetic energy supplied to the ionosphere”
      - Page 2, line 22, “to infer”
      - 205 Page 2, line 24–25, “which shortens...”
      - Page 2, line 29–30, “down to 1000 km...”
      - Page 2, line 33, “problem to”
      - Page 3, line 9, “In general...”
      - Page 6, figure 1, line 2, “and that...”
      - 210 Page 9, line 25, “example”
      - Page 9, line 25, “two Venus flyby manoeuvres are...”
      - Page 9, line 26, “down to”
      - Page 9, line 29, “for several hours to a day”
      - Page 10, Acknowledgments, line 3, “Venus”
  - The following references were added.
    - Bagdonat, T. B.: Hybrid simulations of weak comets, PhD thesis, Tech. Univ. Braunschweig, Braunschweig, 2002.
    - Kelly, M. C.: The Earths Ionosphere, Academic Press, Inc., San Diego, 1989.
    - Kertz, W.: Einführung in die Geophysik II, BI Hochschultaschenbücher, 1989.
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- Winske, D., and Omidi, N.: *Hybrid codes: Methods and applications*, Terra Publication, Tokyo, 1993.