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The mirror mode: A superconducting space plasma analogue

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Abstract.– We re-examine the physics of the magnetic mirror mode in its final state of saturation, the thermodynamic equilibrium, to demonstrate that the mirror mode is the analogue of a superconducting effect in a classical anisotropic-pressure space plasma. Two different spatial scales are identified which control the behaviour of its evolution. These are the ion inertial scale $\lambda_{im}(\tau)$ based on the excess density $N_m(\tau)$ generated in the mirror mode, and the Debye scale $\lambda_D(\tau)$. The Debye length plays the role of the correlation length in superconductivity. Their dependence on the temperature ratio $\tau = T_\parallel/T_\perp < 1$ is given, with $T_\perp$ the critical temperature. The mirror mode equilibrium structure under saturation is determined by the Landau-Ginzburg ratio $\kappa_D = \lambda_{im}/\lambda_D$, or $\kappa_\rho = \lambda_{im}/\rho$, depending on whether the Debye length or the thermal-ion gyroradius ρ serve as correlation lengths. Since in all space plasmas $\kappa_D \gg 1$, plasmas with λ_D as relevant correlation length always behave like type II superconductors, naturally giving rise to chains of local depletions of the magnetic field of the kind observed in the mirror mode. In this way they provide the plasma with a magnetic bubble texture. The problem becomes more subtle when ρ is taken as correlation length. In this case the evolution of mirror modes is more restricted. Their existence as chains or trains of mirror bubbles implies that another threshold, $V_A > v_{\perp th}$, is exceeded.

1 Introduction

Under special conditions high-temperature collisionless plasmas may develop properties which resemble those of superconductors. This is the case with the mirror mode when the anisotropic pressure gives rise to local depletions of the magnetic field similar to the Meissner effect in metals where it signals the onset of superconductivity (Kittel, 1963; Fetter and Walecka, 1971; Huang, 1987; Lifshitz and Pitaevskii, 1998), i.e. the exclusion of friction between the current and the lattice. In collisionless plasmas there is no lattice, the plasma is frictionless, thus it already is ideally conducting which, however, does not mean that it is superconducting! For being superconducting, additional properties are required. These, as we show below, are given in the saturation state of the mirror mode.

The mirror mode is a non-oscillatory plasma instability (Chandrasekhar, 1961; Hasegawa, 1969; Gary, 1993; Southwood and Kivelson, 1993; Kivelson and Southwood, 1996) which evolves in anisotropic plasmas (for a recent review see Sulem, 2011, and references therein). It has been argued that it should readily saturate by quasilinear depletion of the temperature anisotropy (cf., e.g. Noreen et al., 2017, and references therein). Observations do not support this conclusion. In fact, we recently argued (Treumann and Baumjohann, 2018) that the large amplitudes of mirror-mode oscillations observed in the Earth's magnetosheath, magnetotail and elsewhere, like other planetary magnetosheaths, in the solar wind and generally in the

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heliosphere, (see., e.g. Tsurutani et al., 1982, 2011; Zhang et al., 1998, 2008, 2009; Lucek et al., 1999a, b; Volwerk et al., 2008, and others) are a sign of the impotence of quasilinear theory of limiting growth of the mirror instability. Instead, mirror modes should be subject to weak kinetic turbulence theory (Sagdeev and Galeev, 1969; Davidson, 1972; Tsytovich, 1977; Yoon, 2007, 2018; Yoon and Fang, 2007) which allows them to evolve until becoming comparable in amplitude to the ambient magnetic field long before any dissipation can set on.

This is not unreasonable, because all those plasmas where the mirror instability evolves are ideal conductors on the scales of the plasma flow. On the other hand, no weak turbulence theory of the mirror mode is available yet as it is difficult to identify the various modes which interact to destroy quasilinear quenching. The frequent claim that whistlers (lion roars) excited in the trapped electron component would destroy the temperature anisotropy is erroneous, because whistlers (Thorne and Tsurutani, 1981; Baumjohann et al., 1999; Maksimovic et al., 2001; Zhang et al., 1998) grow on the expense of a small component of anisotropic resonant particles only (Kennel and Petschek, 1966). Depletion of the resonant anisotropy does not affect the bulk temperature anisotropy responsible for the mirror instability. On the other hand, construction of a weak turbulence theory of the mirror mode poses serious problems. One therefore needs to refer to other means of description of its final saturation state.

Since measurements suggest that the observed mirror modes are about stationary phenomena it seems reasonable to tackle them within a thermodynamic approach, i.e. assuming that they can be described as the stationary state of interaction between the ideally conducting plasma and the magnetic field. This can be most efficiently done when the free energy of the plasma is known which, unfortunately, is not the case. Magnetohydrodynamics does not apply, and the formulation of a free energy in the kinetic state is not available. For this reason we refer to some phenomenological approach which is guided by the phenomenological theory of superconductivity. There we have the similar phenomenon that the magnetic field is expelled from the medium due to internal quantum interactions, known as the Meissner effect. This resembles the evolution of the mirror mode though in our case the interactions are not in the quantum domain. This is easily understood if considering the thermal length $\lambda_h = \sqrt{2\pi\hbar^2/m_eT}$ and comparing it to the shortest plasma scale, viz. the inter-particle distance $d_N \sim 1/\sqrt[3]{N}$. The former is, for all plasma temperatures T, in the atomic range while the latter in space plasmas for all densities N is many orders of magnitude larger. Plasmas are classical. In their equilibrium state classical thermodynamics applies to them. In the following we boldly ask for the thermodynamic equilibrium state of a mirror unstable plasma.

2 Properties of the mirror instability

The mirror instability evolves whence the magnetic field B in a collisionless magnetised plasma with an internal pressure/temperature anisotropy $T_\perp > T_\parallel$, where the subscripts refer to the directions perpendicular and parallel to the ambient magnetic field, drops below a critical value

$$B < B_{crit} \approx \sqrt{2\mu_0 N T_{i\perp}}\left(\Theta_i + \sqrt{\frac{T_{e\perp}}{T_{i\perp}}}\Theta_e\right)^{\frac{1}{2}} \left|\sin\theta\right| \tag{1}$$

where $\Theta_i = \left(T_\perp/T_\parallel - 1\right)_j > 0$ is the temperature anisotropy of species $j = e, i$ (for ions and electrons) and θ is the angle of propagation of the wave with respect to the ambient magnetic field (cf., e.g., Treumann and Baumjohann, 2018). Here any possible temperature anisotropy in the electron population has been included but will be dropped below as it seems (Noreen et al., 2017) that it does not provide any further insight into the physics of the final state of the mirror mode.

The important observation is that the existence of the mirror mode depends on the temperature difference $T_\perp - T_\|$ and the critical magnetic field. To some degree this resembles the behaviour of magnetic fields under superconducting conditions. To demonstrate this, we take $T_\perp$ as reference – or critical – temperature. The critical magnetic field becomes a function of the temperature ratio $\tau = T_\|/T_\perp$. Once $\tau < 1$ and $B < B_{crit}$ the magnetic field will be pushed out of the plasma to give space to an accumulated plasma density and surface currents on the boundaries of the (partially) field-evacuated domain.

The τ-dependence of the critical magnetic field can be cast into the form

$$\frac{B_{crit}(T_\|)}{B^0_{crit}} = \left[\tau^{-1}(1-\tau)\right]^{\frac{1}{2}} = \left(\frac{T_\perp}{T_\|}\right)^{\frac{1}{2}}\left(1-\frac{T_\|}{T_\perp}\right)^{\frac{1}{2}} \tag{2}$$

which indeed resembles that in the phenomenological theory of superconductivity. Here

$$B^0_{crit} = \sqrt{2\mu_0 N T_{i\perp}}\,\big|\sin\theta\big| \tag{3}$$

and the critical threshold vanishes for $\tau = 1$ where the range of possible unstable magnetic field values shrinks to zero; the limits $T_\| = 0$ or $T_\perp = \infty$ make no physical sense.

Though the effects are similar to superconductivity, the temperature dependence is different from that of the Meissner effect in metals in their isotropic low-temperature super-conducting phase. In contrast, in an anisotropic plasma the effect occurs in the high-temperature phase only while being absent at low temperatures. Nevertheless, the condition $\tau < 1$ indicates that the mirror mode, concerning the ratio of parallel to perpendicular temperatures, is a *low-temperature* effect in the high-temperature plasma phase. This may suggest that even in metals high-temperature superconductivity might be achieved more easily for anisotropic temperatures, a point we will follow elsewhere.

Since the plasma is ideally conducting, any quasi-stationary magnetic field is subject to the penetration depth, which is the inertial scale $\lambda_{im} = c/\omega_{im}$, with $\omega^2_{im} = e^2 N_m/\epsilon_0 m_i$ based on the density N_m of the plasma component involved into the mirror effect. The mirror instability is a slow purely growing instability with real frequency $\omega \approx 0$. On these low frequencies the plasma is quasi-neutral. In metallic superconductivity this length is the London penetration depth which refers to electrons as the ions are fixed to the lattice. Here, in the space plasma, it is rather the ion scale because the dominant mirror effect is caused by mobile ions in the absence of any crystal lattice. Such a “magnetic lattice” structure is ultimately provided under conditions investigated below by the saturated state of the mirror mode, where it collectively affects the trapped ion component on scales of an internal correlation length.

3 Free energy

In the thermodynamic equilibrium state the quantity which describes the matter in the presence of a magnetic field $\boldsymbol{B}$ is the Gibbs free energy density

$$G = F - \frac{1}{2\mu_0}\delta\boldsymbol{B}\cdot\boldsymbol{B} \tag{4}$$

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where F is the Helmholtz free energy density which, unfortunately, is not known. In magnetohydrodynamics it can be formulated but becomes a messy expression which contains all contributions of magnetohydrodynamic waves. The total Gibbs free energy is the volume integral of this quantity over all space. Since this is stationary, one has

$$\frac{d}{dt}\int d^3x\, G = 0 \tag{5}$$

In order to restrict to our case we assume that F in the above expression, which contains the full dynamics of the plasma matter, can be expanded with respect to the normalised density N_m of the plasma component which participates in the mirror instability:

$$F = F_0 + aN_m + \frac{1}{2}bN_m^2 + \cdots \tag{6}$$

Normalisation is to the ambient density N_0, thus attributing the dimension of energy density to the expansion coefficients a, b. An expansion like this one is always possible in the spirit of a perturbation approach as long as the total density $N/N_0 = 1+N_m$ with $N_m < 1$. It is thus clear that N_m is not the total ambient plasma density N_0 which is itself in pressure equilibrium with the ambient field B_0 under static conditions expressed by $N_0T = B_0^2/2\mu_0$ under the assumption that no static current $\boldsymbol{J}_0$ flows in the medium. Otherwise its Lorentz force $\boldsymbol{J}_0 \times \boldsymbol{B}_0 = -T\nabla N_0$ is compensated by the pressure gradient force already in the absence of the mirror mode and includes the magnetic stresses generated by the current. This case includes a stationary contribution of the free energy F_0 around which the mirror state has evolved.

What concerns the presence of the mirror mode, we know that it must as well be in balance between the local plasma gradient ∇N_m of the fluctuating pressure and the induced magnetic pressure $(\delta\boldsymbol{B})^2/2\mu_0$. Note that all quantities are stationary; the prefix δ refers to deviations from "normal" thermodynamic equilibrium, not to variations. Moreover, we have Maxwell's equations which in the stationary state reduce to

$$\nabla\times\delta\boldsymbol{B} = \mu_0\delta\boldsymbol{J}, \quad \text{and} \quad \delta\boldsymbol{B} = \nabla\times\boldsymbol{A} \tag{7}$$

accounting for the vanishing divergence by introducing the fluctuating vector potential $\boldsymbol{A}$ (where we drop the δ-prefix on the vector potential). This enables to write the kinetic part of the free energy of the particles involved in the canonical operator form

$$\frac{\boldsymbol{p}^2}{2m} = \frac{1}{2m}\Big| -i\alpha\nabla - q\boldsymbol{A}\Big|^2 \tag{8}$$

referring to ions of positive charge $q > 0$, and the constant α naturally has the dimension of a classical action. In this form the momentum acts on a complex dimensionless "wave function" $\psi(\boldsymbol{x})$ whose square

$$\big|\psi(\boldsymbol{x})\big|^2 = N_m \tag{9}$$

we identify with the above used normalised density. Unlike quantum theory, $\psi(\boldsymbol{x})$ is not a single particle wave function, it rather applies to a large compound of trapped particles (ions) in the mirror mode which behave similarly and are bound together by some correlation length which is to be discussed later. It enters the expression for the free energy density thus providing the

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units of energy density to the expansion coefficients a, b. In the quantum case (as for instance in the theory of superconductivity) we would have $\alpha = \hbar$; in the classical case considered here, α remains undetermined until a connection to the mirror mode is obtained. Clearly, $\alpha \gg \hbar$ cannot be very small because the gradient and the corresponding wave vector $\boldsymbol{k}$ involved in the operation ∇ are of the scale of the inverse ion gyro-radius in the mirror mode. Hence, we suspect that $\alpha \propto T/\omega_p$ where T is a typical plasma temperature in energy units, and ω_p is a typical frequency of collective ion oscillations in the plasma. Any such oscillations naturally imply the existence of some correlation length which binds the particles to exert a collective motion of the ions which give rise to field and density fluctuations. Such frequencies can be either plasma $\omega_p = \omega_i = e\sqrt{N/\epsilon_0 m}$ or cyclotron $\omega_c = eB/m$ frequencies. For the ion mirror mode the choice is that $q = +e$, and $m = m_i$.

Retaining the quantum action and dividing by the charge q, the factor of the Nabla operator becomes $\hbar/q = \Phi_0 e/2\pi q$. Hence, α is proportional to the number $\nu = \Phi/\Phi_0$ of elementary flux elements in the ion-gyro cross section, which in a plasma is a large number due to the high temperature $T_\perp$. Let us now define the fluctuating normalised mirror density excess $N_m > 1$ through the "wave function" square $|\psi(\boldsymbol{x})|^2$

$$N_m(\boldsymbol{x}) = \left|\psi(\boldsymbol{x})\right|^2 = \psi^*(\boldsymbol{x})\psi(\boldsymbol{x}) \tag{10}$$

With these assumptions in mind we can write for the free energy density up to second order in N_m

$$F = F_0 + a\left|\psi\right|^2 + \frac{1}{2}b\left|\psi\right|^4 + \frac{1}{2m}\Big|\big(-i\alpha\nabla - q\boldsymbol{A}\big)\,\psi\Big|^2 + \frac{\delta\boldsymbol{B}\cdot\boldsymbol{B}_0}{2\mu_0} \tag{11}$$

This is to be inserted into the Gibbs free energy density and used in the Gibbs free energy. Then the last term is absorbed by the Gibbs potential. As conventionally done, varying the Gibbs free energy with respect to $\boldsymbol{A}$ and ψ yields an equation for the "wave function" $\psi(\boldsymbol{x})$

$$\left[\frac{1}{2m}\big(-i\alpha\nabla - q\boldsymbol{A}\big)^2 + \,a\, + \,b\left|\psi\right|^2\right]\psi = 0 \tag{12}$$

which is recognised as a nonlinear Schrödinger equation. Such equations appear in plasma physics whence waves undergo modulation instability and evolve towards solitons.

It is known that the nonlinear Schrödinger equation can be solved by inverse scattering methods and, under certain conditions, yields either single solitons or trains of solitary solutions. To our knowledge, the nonlinear Schrödinger equation has not yet been derived for the mirror instability because no slow wave is known which would modulate its amplitude. Whether this is possible is an open question which we will not follow up here. Hence the quantity α remains undetermined for the mirror mode. Instead, we chose a phenomenological approach which is suggested by the similarity of both, the mirror mode effect in ideally conducting plasma and the above obtained nonlinear Schrödinger equation to the phenomenological Landau-Ginzburg theory of metallic superconductivity.

In the thermodynamic equilibrium state the above equation does not describe the mirror mode amplitude itself. Rather it describes the evolution of the "wave function" of the compound of particles trapped in the mirror mode magnetic potential $\boldsymbol{A}$ which it modulates. This differs from superconductivity where we have pairing of particles, escape from collisions with the lattice and superfluidity of the paired particle population at low temperatures. In the ideally conducting plasma we have no

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collisions but, under normal conditions, also no pairing and no superconductivity though, in the presence of some particular plasma waves, attractive forces between neighbouring electrons can sometimes evolve (Treumann and Baumjohann, 2014). In superconductivity the pairing implies that the particles become correlated, an effect which in plasma must also happen whence the superconducting mirror mode Meissner effect occurs, but it happens in a completely different way via correlating large numbers of particles, as we will exemplify farther below.

The wave function $\psi(\boldsymbol{x})$ describes only the trapped particle component which is responsible for the maintenance of the pressure equilibrium between the magnetic field and plasma. In a bounded region one must add boundary conditions to the above equation which imply that the tangential component of the magnetic field is continuous at the boundary and the normal components of the electric currents vanish at the boundary because the current has no divergence. The current, normalised to N_0, is then given by

$$\delta \boldsymbol{J} = \frac{iq\alpha}{2m}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) - \frac{q^2}{m}\left|\psi\right|^2\boldsymbol{A} \tag{13}$$

which shows that the main modulated contribution to the current is provided by the last term, the product of the mirror particle density $\left|\psi\right|^2 = N_m$ times the vector potential fluctuation $\boldsymbol{A}$, which is the mutual interaction term between the density and magnetic fields. One may note that the vector potential from Maxwell's equations is directly related to the magnetic flux Φ in the wave flux tube of radius R through its circumference $A = \Phi/2\pi R$.

One also observes that under certain conditions in the last expression for the current density the two gradient terms of the ψ function partially cancel. Assuming $\psi = \left|\psi(\boldsymbol{x})\right|\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$ the current term becomes

$$\delta \boldsymbol{J} = \frac{q\alpha}{m}\boldsymbol{k}\left|\psi\right|^2 - \frac{q^2}{m}\left|\psi\right|^2\boldsymbol{A} \tag{14}$$

The first term is small in the long wavelength domain $k\alpha \ll 1$. Assuming that this is the case for the mirror mode, which implies that the first term is important only at the boundaries of the mirror bubbles whre it comes up for the diamagnetic effect of the surface currents, the current is determined mainly by the last term which can be written

$$\delta \boldsymbol{J} \approx -\frac{q^2 N_m}{m}\boldsymbol{A} = -\epsilon_0\omega_{im}^2\boldsymbol{A} \tag{15}$$

This is to be compared to $\mu_0\delta\boldsymbol{J} = -\nabla^2\boldsymbol{A}$ thus yielding the penetration depth

$$\lambda_{im}(\tau) = c/\omega_{im}(\tau) \tag{16}$$

which is the ion inertial length based on the relevant temperature dependence of the particle density $N_m(\tau)$ for the mirror mode, where we should keep in mind that the latter is normalised to N_0. Thus, identifying the reference temperature as $T_\perp$, we recover the connection between the mirror mode penetration depth and its dependence on temperature ratio τ from thermodynamic equilibrium theory in the long wavelength limit with main density N_0 constant on scales larger than the mirror mode wavelength.

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4 Magnetic penetration scale

So far we considered only the current. Now we have to relate the above penetration depth to the plasma, the mirror mode. What we need, is the connection of the mirror mode to the nonlinear Schrödinger equation. Because treating the nonlinear Schrödinger equation is very difficult even in two dimensions, this is done in one dimension, assuming for instance that the cross section of the mirror structures is circular with relevant dimension the radius. In the presence of a magnetic wave field $\boldsymbol{A}\neq 0$ Eq. (12) under homogeneous or nearly homogeneous conditions, with the canonical gradient term neglected, has the thermodynamic equilibrium solution

$$N_m = |\psi|^2 = -\frac{a}{b} - \frac{q^2 N_0}{2mb} A^2 > 0 \tag{17}$$

which implies that either a or b is negative. In addition there is the trivial solution $\psi = 0$ which describes the initial stable state when no instability evolves. The Helmholtz free energy density in this state is $F = F_0$. Equation (11) tells that the thermodynamic equilibrium has free energy density

$$F = F_0 - \frac{q^2 a N_0}{2mb} A^2 - \frac{a^2}{2b} = F_0 - \frac{q^2 a N_0}{2mb} A^2 - \frac{B_{crit}^2}{2\mu_0} \tag{18}$$

where the last term is provided by the critical magnetic field which is the external magnetic field. Thus $b > 0$ and $a < 0$, and the dependence on temperature τ can be freely attributed to a. Comparison with Eq. (2) then yields that

$$a(\tau) = -B^0_{crit}\sqrt{\frac{b}{\mu_0}}\,\tau^{-\frac{1}{2}}(1-\tau)^{\frac{1}{2}} \tag{19}$$

At critical field one still has $\boldsymbol{A} = 0$. Hence the density at critical field is

$$N_m(\tau) = \frac{|a(\tau)|}{b} = \frac{B^0_{crit}}{\sqrt{b\mu_0}}\tau^{-\frac{1}{2}}(1-\tau)^{\frac{1}{2}} \tag{20}$$

which shows that the distortion of the density vanishes for $\tau = 1$ as it should be. This expression can be used in the magnetic penetration depth to obtain its critical temperature dependence

$$\lambda_{im}(\tau) = \left[\frac{m^2 b}{\mu_0 q^4 \left(N_0 B^0_{crit}\right)^2}\frac{\tau}{(1-\tau)}\right]^{\frac{1}{4}} \quad \mathrm{m} \tag{21}$$

which suggests that the critical penetration depth vanishes for $\tau = 0$. However, $\tau = 0$ is excluded by the argumentation following (2) and Eq. (20), because it would imply infinite trapped densities. In principle, $\tau \geq \tau_{min}$ cannot become smaller than a minimum value which must be determined by other methods referring to measurements of the maximum density in thermodynamic equilibrium. One should, however, keep in mind that $B^0_{crit}(\theta) \propto |\sin\theta|$ still depends on the angle θ which enters the above expressions.

The last two expressions still contain the undetermined coefficient b. This can be expressed through the minimum value of the anisotropy τ_{min} at maximum critical density $N_m \lesssim 1$ as

$$b = \frac{\left(B^0_{crit}\right)^2}{\mu_0}\tau^{-1}_{min}(1-\tau_{min}) \tag{22}$$

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(Note that for $N_m > 1$ the above expansion of the free energy F becomes invalid. It is not expected, however, that the mirror mode allows the evolution of sharp density peaks which locally double the density.) With this expression the inertial length becomes

$$\frac{\lambda_{im}(\tau)}{\lambda_{i0}} = \left[\frac{\tau}{\tau_{min}}\left(\frac{1-\tau_{min}}{1-\tau}\right)\right]^{\frac{1}{4}} \tag{23}$$

When the mirror mode saturates away from the critical field, the magnetic fluctuation grows until it saturates as well, and one has $\boldsymbol{A} \neq 0$. The increased fractional density N_m is in perpendicular pressure equilibrium with the magnetic field distortion $\delta\boldsymbol{B}$ through

$$\begin{aligned} N_{sat}T_\perp &= \frac{1}{2\mu_0 N_0}\left(\boldsymbol{B}_0 - \delta\boldsymbol{B}_{sat}\right)^2 - \frac{B_0^2}{2\mu_0 N_0} \\ &= \frac{1}{2\mu_0 N_0}\left[\nabla^2(A^2) - (\nabla\boldsymbol{A})^2\right]_{sat} - \frac{\boldsymbol{B}_0\cdot\nabla\times\boldsymbol{A}}{\mu_0 N_0} \\ &\approx -\frac{q^2 a(\tau_{sat})}{2mb}A_{sat}^2 \end{aligned} \tag{24}$$

There is also a small local contribution from the magnetic stresses which results from the surface currents at the mirror boundaries in which only a minor part of the trapped particles is involved. This is indicated by the approximate sign.

The last two lines yield for the macroscopic penetration depth the expression (21). We thus conclude that Eq. (21) is also valid at saturation with $\tau = \tau_{sat}$. Measuring the saturation wavelength λ_{sat} and saturation temperature anisotropy τ_{sat} determines the unknown constant b through (22) with τ_{min} replaced with τ_{sat}. Clearly

$$\tau_{min} \leq \tau_{sat} < 1 \tag{25}$$

as the mirror mode might saturate at temperature anisotropies larger than the permitted lowest anisotropy. Moreover, measurement of τ_{sat} at saturation, the state in which the mirror mode is actually observed, immediately yields the normalised saturation density excess $N_m(\tau_{sat})$ from Eq. (20) which then from pressure balance yields the magnetic decrease, i.e. the mirror amplitude. To some extent this completes the theory of the mirror mode in as far as is relates the density at saturation to the saturated normalised temperature anisotropy at given $T_\perp$ and determines the scale λ_{im} and $\delta B(\tau_{sat})$.

5 Determination of α

Since observations always refer to the final thermodynamic state, when the mirror mode is saturated, the anisotropy at saturation can be measured, and the value of the unknown constant α in the Schrödinger equation can also be determined. Expressed through b and λ_{im} at τ_{sat} it becomes

$$\alpha = \sqrt{2m}\lambda_{sat} = \frac{m}{q}\sqrt{\frac{b}{\mu_0 N_0|a(\tau_{sat})|}} \tag{26}$$

What is interesting about this number is that it is much larger than the quantum of action $\hbar$ but at the same time is sufficiently small, which in retrospect justifies the neglect of the gradient term in the former section. It represents the elementary action in

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a mirror unstable plasma, where the characteristic length is given by the inertial scale $\alpha/\sqrt{2m} = \lambda_{sat}$ respectively maximum of the normalised density N_m. One may note that α is not an elementary constant like $\hbar$. It depends on the critical reference temperature $T_\perp$, and it depends on τ. Its constancy is understood in a thermodynamic sense.

Our argument applies when $\boldsymbol{A} \neq 0$. In this case Eq. (12) reads

$$-\frac{\alpha^2}{2ma}\frac{d^2 f(x)}{dx^2} - f(1-f^2) = 0, \quad \text{where} \quad f(x) = \frac{\psi(x)}{|\psi_\infty|} < 1 \tag{27}$$

and $|\psi|_\infty = \sqrt{N_{max}(x_\infty)}$ is given by the maximum density excess in the centre x_∞ of the magnetic field decrease. Clearly this equation defines a natural scale length which is given by

$$\lambda_\alpha = \alpha/\sqrt{2m|a(T_\perp,\tau)|} \tag{28}$$

which, identifying it with λ_{sat}, yields the above expression for α. For x_∞ large the equation for f can be solved asymptotically when $df/dx = 0$ for $f^2 = 1$ corresponding to a maximum in N_m. It is then easy to show by multiplication with df/dx that

$$\frac{df(x)}{\sqrt{1-f^2}} = \sqrt{2}\lambda_\alpha dx \tag{29}$$

which has the Landau-Ginzburg solution

$$f(x) = \tanh\left[\frac{x-x_\infty}{\sqrt{2}\lambda_\alpha}\right] \tag{30}$$

This implies that the excess density assumes the shape

$$N_m = N_{sat}\tanh^2\left[\frac{x-x_\infty}{\sqrt{2}\lambda_\alpha}\right] \tag{31}$$

The above condition on the vanishing gradient of f at x_∞ warrants the flat shape of the excess density at maximum and the equally flat shape of the magnetic field in its minimum. Of course, these considerations apply strictly only to the one-dimensional case. It is, however, not difficult to generalise them to the cylindrical problem with radius r in place of x. The main qualitative properties are thereby retained. In the next section we will turn to the question of generation of chains of mirror mode bubbles, as this is the case which is usually observed in space plasmas.

Since the quantum of action enters the magnetic quantum flux element $\Phi_0 = 2\pi\hbar/e$ we may also conclude that in a mirror unstable plasma the relevant magnetic flux element is given by $\Phi_m = \alpha/q$.

Indentification of α is an important step. With its knowledge in mind the nonlinear Schrödinger equation for the hypothetical saturation state of the mirror mode is (up to the coefficient b which, however, is defined in (22) and can be obtained from measurement) completely determined and thus ready for application of the inverse scattering procedure which solves it under any given initial conditions for the mirror mode. It thus opens up the possibility to further investigate the final evolution of the mirror mode. Executing this programme should, under various conditions, provide the different forms of the mirror mode in its final thermodynamic equilibrium state. This is left as a formally sufficiently complicated exercise which will not be treated in the present communication. Instead, we ask for the conditions under which the mirror mode evolves into a chain of separated mirror bubbles, which requires the existence of a microscopic though classical correlation length.

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6 The problem of the correlation length

The present phenomenological theory of the final thermodynamic equilibrium state of the mirror mode is modelled after the phenomenological Landau-Ginzburg theory of superconductivity as presented in the cited textbook literature. From the existence of λ_{im} we would conclude that, under mirror instability, the magnetic field inside the plasma volume should decay to a minimum value determined by the achievable minimum τ_{sat} of the temperature ratio. This conclusion would, however, be premature and contradicts observation where chains or trains (cf., e.g., Zhang et al., 2009, for examples) of mirror mode fluctuations are usually observed, which presumably are in their saturated state having had sufficient time to evolve. In fact, observations of mirror modes in their growth phase have to our knowledge never yet been reported. On the other hand, in no case known to us a global reduction of the gross magnetic field in an anisotropic plasma has been identified yet.

It is clear that in any real collisionless high temperature plasma neither N_m can become infinite, nor can τ drop to zero. Since it is not known how and on which way the mirror mode saturates, its growth must ultimately be stopped when the particle correlation length comes into play. In a plasma the shortest natural correlation length is the Debye length λ_D which under all conditions is much shorter then the above estimated penetration length. Referring to the Debye length, the Landau-Ginzburg parameter, i.e. the ratio of penetration to correlation lengths, in a plasma as function of τ becomes

$$\kappa_D \equiv \frac{\lambda_{im}(\tau)}{\lambda_D(\tau)} \gg 1 \qquad (32)$$

a quantity that is large. Writing for the Debye length

$$\lambda_D^2(\tau) = \lambda_{D\perp}^2 \frac{1+\tau/2}{N_m(\tau)}, \qquad \lambda_{D\perp}^2 = \frac{4}{3}\frac{T_\perp/m_i}{\omega_{i0}^2} \qquad (33)$$

the Landau-Ginzburg parameter can be expressed in terms of τ, exhibiting only a weak dependence on the temperature ratio $\tau < 1$:

$$\kappa_D(\tau) = \frac{\lambda_{i0}}{\lambda_{D\perp}}\sqrt{\frac{1+\tau/2}{2}} \gg 1 \qquad (34)$$

Thus, κ_D is practically constant and about independent on the temperature anisotropy. Its value $\kappa_{D0} = \lambda_{i0}/\lambda_D$ at $\tau = 1, T_\| = T_\perp$ refers to the isotropic case when no mirror instability evolves.

This is an important finding because it implies that in a plasma the case that the magnetic field would be completely expelled from the volume of the plasma cannot be realised. Different regions of extension longer than the correlation length λ_D are uncorrelated. They therefore behave separately lacking knowledge about their uncorrelated neighbours. Each of them experiences the penetration scale and adjust itself to it. This is in complete analogy to Landau-Ginzburg theory. Thus, once the main magnetic field in an anisotropic plasma drops below threshold, the plasma will necessarily evolve into a chain of nearly unrelated mirror bubbles which interact with each other because each occupies space. In superconductivity this corresponds to a type II superconductor. Mirror unstable plasmas in this sense behave like type II superconductors. They decay into regions of normal magnetic field strength and embedded domains of spatial scale $\lambda_m(\tau)$ with reduced magnetic field. These regions contain an excess plasma population which is in pressure and stress balance with the magnetic field. Its diamagnetism (perpendicular

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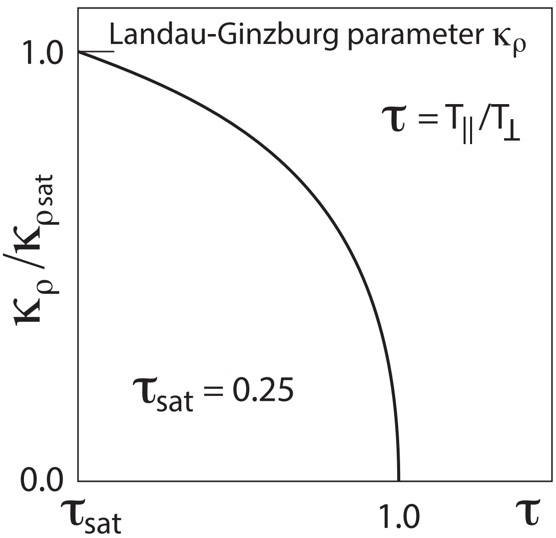

Figure 1. The Landau-Ginzburg parameter $\kappa_\rho/\kappa_{\rho,sat}$ as function of anisotropy ratio $\tau = T_\|/T_\perp < 1$ for the particular choice $\tau_{sat} = \frac{1}{4}$. The parameter κ_ρ refers to the thermal gyroradius as the short-scale correlation length, as explained in the text. It maximises at saturation anisotropy $\tau = \tau_{sat}$ and vanishes for $\tau = 1$ when no instability sets on. For any given ratio τ the value of κ_ρ lies on a curve like the one shown. There is a threshold for the mirror mode to evolve into bubbles which it must overcome. It is given by the ratio $\kappa_{\rho,sat} > 1$ of the critical Alfvén speed to perpendicular thermal velocity.

pressure) keeps the magnetic field partially out and causes weak diamagnetic currents to flow along the boundaries of each of the partially field-evacuated domains. This trapped plasma behaves analogously to the pair plasma in metallic superconductivity, this time however at the high plasma temperature being bound together not by pairing potentials but by the Debye potential over the Debye correlation length.

The Debye length is a natural scale in a plasma. Within a mutual distance of one Debye length, particles are not completely independent. They are weakly bound together by their incompletely compensated electric charge fields. However, the Debye length is a very short scale, in fact the shortest collective scale in the plasma, and though it must have an effect on the collective evolution of particles in plasmas, there could also be longer scales on which the particles are correlated.

Such a scale would, for instance, be the ion gyroradius. For the low frequencies of the mirror mode, the magnetic moment $\mu(\tau) = T_\perp/B(\tau) = \mathrm{const}$ of the particles is conserved in their dynamics, which implies that all particles with same magnetic moment $\mu(\tau)$ behave about collectively, at least in the sense of a gyro-kinetic theory.

However, though $\mu(\tau)$ is a constant of motion, it still is a function of the anisotropy through the dependence of the magnetic field on τ. Expressing the gyroradius through the magnetic moment

$$\rho(\tau) = \sqrt{\frac{2\mu(\tau)}{e\omega_{ci}(\tau)}} = \rho_0\sqrt{\frac{\tau}{1-\tau}}, \qquad \rho_0 = \sqrt{\frac{2mT_\perp}{e^2\left(B^0_{crit}\right)^2}} \tag{35}$$

it can be taken as another kind of collective correlation scale as on scales longer than ρ it collectively binds particles of same magnetic moment which, in particular, are magnetically trapped like those which are active in the mirror instability. Below the gyroradius charged particles are magnetically free. ρ is the scale where the particles magnetise, start feeling the magnetic field effect and collectively enter another phase in their dynamics. This scale is much longer than the Debye length and may be more appropriate for describing the saturated behaviour of the mirror mode. Thus one may argue that, as long as the penetration depth (inertial sale) exceeds ρ, the gyroradius is the relevant correlation length. Only when it drops below the gyroradius, the Debye length takes over. The Landau-Ginzburg parameter then becomes

$$\kappa_\rho(\tau) = \frac{\lambda_{im}(\tau)}{\rho(\tau)} = \frac{\lambda_{i0}}{\rho_0}\left[\frac{\tau_{sat}}{\tau}\frac{1-\tau}{1-\tau_{sat}}\right]^{\frac{1}{4}} \tag{36}$$

This ratio depends on the temperature anisotropy $\tau = T_\parallel/T_\perp$, which is a measurable quantity and the important parameter, while it saturates at $\kappa_{\rho,sat} = \lambda_{i0}/\rho_0$, the ratio of inertial length to gyroradius at critical field. This ratio is not necessarily large. It can be expressed by the ratio of Alfvén velocity V_A to perpendicular ion thermal velocity $v_{\perp th}$

$$\kappa_{\rho,sat} = \frac{\lambda_{i0}}{\rho_0} = \frac{V_A\big(B^0_{crit}\big)}{v_{\perp th}} > 1 \tag{37}$$

Hence, when referring to the ion gyroradius as correlation length, the mirror mode would evolve and saturate into a chain of mirror bubbles only, when the Alfvén speed $V_A > v_{\perp th}$ exceeds the perpendicular thermal velocity of the ions. [Since $B^0_{crit} \propto \big|\sin\theta\big|$, highly oblique angles are favoured. The range of optimum angles has recently been estimated (Treumann and Baumjohann, 2018).] This is to be multiplied with the τ-dependence, of which Figure 1 gives an example. The value of this function is always smaller than one. For a chain of mirror bubbles to evolve in a plasma, the requirement $\kappa_\rho > 1$ can then be written as

$$1 \le \frac{\tau}{\tau_{sat}} < \frac{\kappa^4_{\rho,sat}}{1+\big(\kappa^4_{\rho,sat}-1\big)\tau_{sat}} \tag{38}$$

which is always satisfied for $\tau_{sat} < 1$ and $\kappa_{\rho,sat} > 1$, i.e. the Alfvén speed exceeding the perpendicular thermal speed, which indeed is the crucial condition for mirror modes to evolve into chains and become observable, with the gyroradius playing the role of a correlation length. Mirror mode chains in the present case are restricted to comparably cool anisotropic plasma conditions, a prediction which can be checked experimentally to decide whether or not the gyroradius serves as correlation length.

Otherwise, when the above condition is not satisfied and $\tau < 1$ is below threshold, a very small and thus probably not susceptible reduction in the overall magnetic field is produced in the anisotropic pressure region over distances $L \gg \rho$, much larger than the ion gyroradius. Observation of such domains of reduced magnetic field strengths under anisotropic pressure/temperature conditions would indicate the presence of a large-scale typ I classical Meissner effect in the plasma. Such a reduction of the magnetic field would be difficult to explain otherwise and could only be understood as confinement of plasma by discontinuous boundaries of the kind of tangential discontinuities.

The relative rarity of observations of mirror-mode chains or trains seems to support the case that the gyroradius, not the Debye length, plays the role of the correlation length in a magnetised plasma under conservation of the magnetic moments

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of the particles. From basic theory it cannot be decided which of the two correlation lengths, the Debye length λ_D or the ion gyroradius ρ, dominates the dynamics and saturation of the mirror mode. A decision can only be established by observations.

7 Conclusions

Mirror modes in the anisotropic collisionless space plasma apparently represent a classical thermodynamic analogue to a superconducting equilibrium state. In contrast to metallic superconductivity which is described by the Landau-Ginzburg theory to which we refer here or, on the microscopic quantum level, by BCS-pairing theory, the problem of circumventing friction and resistance is of no interest in space plasmas which evolve towards mirror modes. High temperature plasmas are classical systems in which no pairing occurs and BCS theory is not applicable. Those plasmas are already ideally conducting with the possible exception that some anomalous resistance may develop from high-frequency kinetic instabilities or turbulence but is of little importance in the zero frequency mirror mode even asymptotically, in the long term thermodynamic limit, where anomalous resistance may contribute to decay of the mirror-surface currents which develop and flow along the boundaries of the mirror bubbles. The times when this happens are very long compared with the saturation time of the mirror instability and transition into the thermodynamic quasi-equilibrium which has been considered here.

The more interesting finding concerns the explanation why at all, in an ideally conducting plasma, mirror bubbles can evolve. Fluid and simple kinetic theory demonstrate that mirror modes occur in the presence of temperature anisotropies thereby identifying the linear growth rate of the instability. The present theory contributes to clarification of the mechanism and its final thermodynamic equilibrium as a nonlinear effect driven by the available free energy. The perpendicular temperature in this theory plays the role of a critical temperature. When the parallel temperature drops below it, which means that $1 > \tau > \tau_{min}$, mirror modes evolve. Interestingly the anisotropy is restricted from below. The parallel temperature cannot drop below a minimum value. This value is open to determination by observations.

The observation of chains of mirror bubbles, which provide the mirror-unstable plasma a particular magnetic texture, suggests that the plasma, in addition to being mirror unstable, is subject to another correlation length which determines the spatial structure of the mirror texture in the saturated thermodynamic quasi-equilibrium state. This correlation length can be either taken as the Debye scale λ_D, which then naturally makes it plausible that many such mirror bubbles evolve, because in all magnetised plasmas the magnetic penetration depth by far exceeds the Debye length and makes the Landau-Ginzburg parameter based on the Debye length $\kappa_D \gg 1$. Otherwise the role of a correlation length could also be played by the thermal ion gyroradius ρ. In this case the conditions for the evolution of the mirror mode with the many observed bubbles become more subtle, because then $\kappa_\rho \gtrsim 1$ occurs under additional restrictions implying that the Alfvén speed exceeds the perpendicular thermal speed. This prediction has to be checked and possibly verified experimentally. A particular case of the dependence of the gyroradius based Landau Ginzburg parameter κ_ρ is shown graphically in Fig. 1.

To the space plasma physicist the present investigation may look a bit academic. However, it provides a physical understanding of how mirror modes saturate, why they evolve into chains of many bubbles or magnetic holes, what the conditions are that this may happen.

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It also relates the measurable saturated magnetic amplitudes of mirror modes to the saturated anisotropy τ_{sat} and the Landau-Ginzburg parameter κ, transforming both into experimentally accessible quantities. These should be of use in the development of a weak-kinetic turbulence theory of magnetic mirror modes as the result of which mirror modes can grow to the observed large amplitudes which are known to by far exceed the simple quasilinear saturation limits.

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