

## Respond to editor comments

### (1) Comment from Editor

I have gone through your responses to the referee and the modifications to your manuscript and the issue of how realistic the proposed mechanism would be for excitation of chorus waves in the environment of the inner magnetosphere still remains.

For example:

\* Your response to the referee's comment "The instability requires electrons, or even ideal beam with  $v_z = U_g$ , which is not realistic either" was that "The beam under consideration is effective...". However, I don't think the point of the referee was whether the beam is effective or not but rather, whether such beams can actually be observed in the inner magnetosphere. If they are not observed, then the mechanism is not realistic for the inner magnetosphere environment.

\* Your response to the referee's comment "the favorable wave normal from the proposed mechanism is quite oblique, which is not consistent with often observed quasi-parallel propagation in the literature" was "There are extensive data on the wave normal angle measurements on board THEMIS and Van Allen Probes. There is no theoretical model for oblique electromagnetic chorus." Nonetheless, no references of such observations are provided.

### (2) Author's response

**We would like to thank the Editor for important comments. We have prepared a point-by-point answer to comments below. The responses are marked in bold.**

Editor's Comments:

I have gone through your responses to the referee and the modifications to your manuscript and the issue of how realistic the proposed mechanism would be for excitation of chorus waves in the environment of the inner magnetosphere still remains.

**Response:**

**Accumulated experimental data and theoretical experience showed that progress in the chorus excitation problem solution is due to some modification of the formalism.**

Editor's Comments:

Your response to the referee's comment "The instability requires electrons, or even ideal beam with  $v_z = U_g$ , which is not realistic either" was that "The beam under consideration is effective...". However, I don't think the point of the referee was whether the beam is effective or not but rather, whether such beams can actually be observed in the inner magnetosphere. If they are not observed, then the mechanism is not realistic for the inner magnetosphere environment.

**Response:**

**We explain additionally the expression for the effective distribution function (11). Assume that a short non-spreading pulse propagates at a constant velocity ( $v_{phz} = v_{gz} = u_G$ ) along the magnetic field in a plasma with an arbitrary undisturbed distribution function  $f_0(\vec{v})$ . The wave-particle interaction within a short pulse is determined by the characteristic time of the electron velocity variation  $\tau$ . For correct accounting of the wave-particle interaction, it is necessary to know the distribution function averaged over the time scale  $\tau$ . Let us consider the effective distribution function inside the pulse. Inside the pulse there is a population of electrons which have flown into the interaction region together with the pulse and is fly together with it. The contribution of these particles to the effective distribution function is proportional to  $n_b \delta(v_z - u_G)$ . The suprathermal electrons with other velocities or flight moments do not give an appreciable contribution to the effective distribution function within the pulse since they cross the pulse in a narrow corridor in Fig. 1 too quickly on a time scale  $\tau$  or do not have contact with the pulse. Thus, we take the effective distribution function (11) to analyze the evolution of a short electromagnetic pulse. We added this text to the manuscript.**

Editor's Comments:

Your response to the referee's comment "the favorable wave normal from the proposed mechanism is quite oblique, which is not consistent with often observed quasi-parallel propagation in the literature" was "There are extensive data on the wave normal angle

measurements on board THEMIS and Van Allen Probes. There is no theoretical model for oblique electromagnetic chorus.” Nonetheless, no references of such observations are provided.

**Response:**

**We added references to the manuscript and extended the bibliography.**

(3) Author's changes in manuscript

Page 1 line 21

The text

"There are extensive data on the wave normal angle measurements on board THEMIS (Li et al., 2013) and Van Allen Probe (Li et al., 2016b)."

is added

Page 5 line 15

The text

"We explain additionally the expression for the effective distribution function (11). Assume that a short non-spreading pulse propagates at a constant velocity ( $v_{phz} = v_{gz} = u_G$ ) along the magnetic field in a plasma with an arbitrary undisturbed distribution function  $f_o(\vec{v})$ . The wave-particle interaction within a short pulse is determined by the characteristic time of the electron velocity variation  $\tau$ . For correct accounting of the wave-particle interaction, it is necessary to know the distribution function averaged over the time scale  $\tau$ . Let us consider the effective distribution function inside the pulse. Inside the pulse there is a population of electrons which have flown into the interaction region together with the pulse and is fly together with it. The contribution of these particles to the effective distribution function is proportional to  $n_b \delta(v_z - u_G)$ . The suprathermal electrons with other velocities or flight moments do not give an appreciable contribution to the effective distribution function within the pulse since they cross the pulse in a narrow corridor in Fig. 1. too quickly on a time scale  $\tau$  or do not have contact with the pulse. Thus, we take the effective distribution function (11) to analyze the evolution of a short electromagnetic pulse."

is added

Page 8 line 20

The text

"Li, W., Bortnik, J., Thorne, R.M., Cully, C.M., Chen, L., Angelopoulos, V., Nishimura, Y., Tao, J.B., Bonnell, J.W. and LeContel, O.: Characteristics of the Poynting flux and wave normal vectors of whistler-mode waves observed on THEMIS, J. Geophys. Res., 118, 1461, doi:10.1002/jgra.50176, 2013."

is added

Page 8 line 24

The text

"Li, W., Santolik, O., Bortnik, J., Thorne, R.M., Kletzing, C.A., Kurth, W.S., and Hospodarsky, G.B.: New chorus wave properties near the equator from Van Allen Probes wave observations. Geophys. Res. Lett., 43, 4725, doi: 10.1002/2016GL0687, 2016."

is added

Some minor corrections were made to edit the manuscript.

## Respond to referee#2 comments

### (1) Comment from Referee#2

A mechanism responsible for amplification of short VLF electromagnetic pulses based on the cold beam is proposed. A conditional growth rate of short electromagnetic pulses of whistler wave propagating with the wave normal angle equal to the Gendrin angle (presenting the less dispersive wave packet) is calculated. This condition is often discussed as the mechanism of the creating the wave power gap around 0.5 fce. Using the assumption of the cold beam (actually delta function) and making this the key point in the approximation makes the results almost inapplicable to the inner magnetosphere processes. A couple of questions should be addressed before the manuscript might be considered for publication (please, see detailed comments below). Some inaccuracy in the terminology used in the text makes hard to read the manuscript.

The introduction is pure. I think some description of recent results presenting the chorus wave properties, the chorus generation mechanisms (especially beam driven) have to be presented and discussed. Some discussion of the previous results based on the similar initial conditions (for example, Starodubtsev, et al. (1999) Resonant Cherenkov emission of whistlers by a modulated electron beam. *Physics of Plasmas*, 6(7), 2862–2869. <https://doi.org/10.1063/1.873244> and Pivovarov et al., (2012). Excitation of VLF waves by an electron beam injected into the ionosphere. *Journal of Geophysical Research: Space Physics*, 100(A9), 17515–7526. <https://doi.org/10.1029/95JA01156> and some others from the references therein) have to be included and the new effects obtained in the manuscript need to be emphasized.

Condition (5) in the context looks too artificial and needs some more discussion because so cold electron beams are not observed in the magnetosphere that makes all results to be related more to laboratory plasma.

The condition of 'a short electromagnetic pulse' is not well established and explained

The term 'the Doppler resonances' is not often used for  $\omega - k_z v_z = \omega_B$ . I suggest to change that to the more conventional ones.

L45: "Cos Theta \_BPA=0.6" – please, specify is that definition for the angle of the maximal growth rate? This is not clear here and after. Actually, the authors discuss a very specific case of propagation at the Gendrin angle. This condition gives only one solution (if any) for the particular beam velocity, i.e. properties of the amplified wave are determined by the beam parameters. Thus, discussion about the maximal growth rate seems to be slightly irrelevant here. In the following the authors discuss only one “maximal” growth rate, i.e. only one particular beam velocity.

The statement "The beam pulsed amplifier mechanism is effective for electromagnetic pulses with the duration  $t_p \leq 10/\gamma_{BP} \sim 0.1$  s less than the dispersion and nonlinear time scales" needs to be explained in more details.

(2) Author's response

**We would like to thank the Reviewer#2 for the time he/she spent reading, and commenting our manuscript. We have prepared a point-by-point answer to his/her comments below. The responses are marked in bold.**

Reviewer's Comments:

A mechanism responsible for amplification of short VLF electromagnetic pulses based on the cold beam is proposed. A conditional growth rate of short electromagnetic pulses of whistler wave propagating with the wave normal angle equal to the Gendrin angle (presenting the less dispersive wave packet) is calculated. This condition is often discussed as the mechanism of the creating the wave power gap around  $0.5 f_{ce}$ . Using the assumption of the cold beam (actually delta function) and making this the key point in the approximation makes the results almost inapplicable to the inner magnetosphere processes. A couple of questions should be addressed before the manuscript might be considered for publication (please, see detailed comments below). Some inaccuracy in the terminology used in the text makes hard to read the manuscript.

The introduction is pure. I think some description of recent results presenting the chorus wave properties, the chorus generation mechanisms (especially beam driven) have to be presented and

discussed. Some discussion of the previous results based on the similar initial conditions (for example, Starodubtsev, et al. (1999) Resonant Cherenkov emission of whistlers by a modulated electron beam. *Physics of Plasmas*, 6(7), 2862–2869. <https://doi.org/10.1063/1.873244> and Pivovarov et al., (2012). Excitation of VLF waves by an electron beam injected into the ionosphere. *Journal of Geophysical Research: Space Physics*, 100(A9), 17515–7526. <https://doi.org/10.1029/95JA01156> and some others from the references therein) have to be included and the new effects obtained in the manuscript need to be emphasized.

**Response:**

**This short theoretical manuscript is addressed to professionals who are well aware of the published achievements of experimental and theoretical research on the chorus radiation in the magnetosphere. Therefore, the bibliography contained only references that are important for this original study.**

Reviewer's Comments:

Condition (5) in the context looks too artificial and needs some more discussion because so cold electron beams are not observed in the magnetosphere that makes all results to be related more to laboratory plasma.

**Response:**

**We derived the condition (5) directly from the Vlasov equation with the stable distribution function  $f_{\circ}(\vec{v})$ .**

Reviewer's Comments:

The condition of 'a short electromagnetic pulse' is not well established and explained

**Response:**

**The chosen pulse shape is modeled and corresponds to the shot noise well known in electronics (see, e.g., (Rytov et al., 1989)).**

Reviewer's Comments:

The term 'the Doppler resonances' is not often used for  $\omega - k_z v_z = \omega_B$ . I suggest to change that to the more conventional ones.

**Response:**

**According to reviewer's suggestion the term "the Doppler resonances" is replaced by "the resonance conditions".**

Reviewer's Comments:

L45: "Cos Theta\_BPA=0.6" – please, specify is that definition for the angle of the maximal growth rate? This is not clear here and after. Actually, the authors discuss a very specific case of propagation at the Gendrin angle. This condition gives only one solution (if any) for the particular beam velocity, i.e. properties of the amplified wave are determined by the beam parameters. Thus, discussion about the maximal growth rate seems to be slightly irrelevant here. In the following the authors discuss only one “maximal” growth rate, i.e. only one particular beam velocity.

**Response:**

**It is easy to verify that the function  $\gamma(\theta)$  in the equation (15) has maximum at**

**$\cos^2 \theta_{BPA} = 0.6$ . We note that, according to the dispersion equation (14), the conditional instability takes place over a wide range of angles with a smaller growth rate.**

Reviewer's Comments:

The statement "The beam pulsed amplifier mechanism is effective for electromagnetic pulses with the duration  $t_p \leq 10/\gamma_{BP}$   $\sim 0.1$  s less than the dispersion and nonlinear time scales" needs to be explained in more details.

**Response:**

**The text "less than the dispersion and nonlinear time scales" is replaced by "at the linear stage of the pulse evolution".**

(3) Author's changes in manuscript

Page 4 line 3

The text

"the Doppler resonances"

is replaced by

"the resonance conditions"

Page 7 line 4

The text

"less than the dispersion and nonlinear time scales"

is replaced by

"at the linear stage of the pulse evolution"

# An excitation mechanism for discrete chorus elements in the magnetosphere

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**Abstract.** A beam pulsed amplifier mechanism responsible for effective amplification of short VLF electromagnetic pulses is proposed. Effective amplification near the magnetic equator outside the plasmasphere is considered. A conditional growth rate of short electromagnetic pulses is calculated. Obtained results can explain some important features of the oblique electromagnetic chorus emissions without hiss-like radiation background.

## 5 1 Introduction

VLF chorus emissions are very intense electromagnetic plasma waves that are naturally excited near the magnetic equatorial plane outside the plasmasphere (Burtis and Helliwell, 1969; Burton and Holzer, 1974; Tsurutani and Smith, 1974). Impressive experimental results of the chorus emission study were obtained within the framework of the CLUSTER project. These results have been presented in detail in many papers (e.g., Santolik, 2009). It is very important that chorus are a succession of discrete  
10 emissions.

For electromagnetic chorus with the wave vectors predominantly along the magnetic field significant theoretical results were obtained. The usual kinetic cyclotron instability (Besselov and Trakhtengerts, 1986) is sometimes possible for plasma parameters in the excitation region, but this instability typically has not a sufficient growth rate to explain the rate of the chorus emission modification. The theory of chorus excitation based on the so-called backward wave oscillator (BWO) mechanism  
15 is now well known (see e.g. Trakhtengerts, 1995; Trakhtengerts et al., 2007; Nunn et al., 2009). The implementation of this mechanism is closely related to the hiss-like radiation background and its dynamics. However, the chorus emissions are often recorded without the hiss-like radiation background. In (Omura et al., 2008; Fu et al., 2014) the generation process of the chorus emissions is analyzed by both in theory and simulation assuming that the initial cyclotron wave growth is driven by the strong temperature anisotropy of energetic electrons. To explain the chorus spectrogram, the authors take into account  
20 inhomogeneity of the magnetic field and nonlinear wave decay, or the non monotonic energy spectrum of particles. At present, there are significant achievements in the theoretical study of generation electrostatic chorus with the wave vectors near the resonance cone (see, e.g., (Li et al., 2016a)).

There are extensive data on the wave normal angle measurements on board THEMIS (Li et al., 2013) and Van Allen Probe (Li et al., 2016b).

Some problems connected with the theoretical analysis of chorus formation remain unsolved, for example, the excitation mechanism of oblique electromagnetic chorus has not been studied. In this paper, we introduce a possible mechanism of oblique electromagnetic chorus excitation without hiss-like radiation background. This mechanism is related to the effective amplification of short electromagnetic pulses from the noise level even in a stable plasma. The amplification takes place in a suitable frequency band near the magnetic equator.

## 2 Wave-particle interaction under special conditions

We consider small-scale wave processes near the local minimum of the magnetic field, which typically is close to the magnetic equator where the plasma is almost homogeneous. We use a linearized Vlasov equation for the disturbed distribution function of energetic non-relativistic electrons  $f_{\sim}$  in a quasineutral cool background plasma

$$\begin{aligned} & \frac{\partial f_{\sim}}{\partial t} + v_z \frac{\partial f_{\sim}}{\partial z} + \mathbf{v}_{\perp} \frac{\partial f_{\sim}}{\partial \mathbf{r}_{\perp}} - \\ & - \frac{e}{mc} \mathbf{v} \times \mathbf{B} \frac{\partial f_{\sim}}{\partial \mathbf{v}} - \frac{e}{m} (\mathbf{E}_{\sim} + \frac{1}{c} \mathbf{v} \times \mathbf{B}_{\sim}) \frac{\partial f_{\circ}}{\partial \mathbf{v}} = 0, \end{aligned} \quad (1)$$

where  $e$  is the absolute value of the electron charge,  $m$  is the electron mass,  $c$  is the speed of light in free space,  $v_z = dz/dt$  and  $\mathbf{v}_{\perp} = d\mathbf{r}_{\perp}/dt$  are the components of electron velocity  $\mathbf{v}$  along and across to the ambient magnetic field  $\mathbf{B}$ ,  $f_{\circ}(\mathbf{v})$  is the undisturbed distribution function,  $\mathbf{E}_{\sim}$  and  $\mathbf{B}_{\sim}$  are the electric and magnetic field disturbances.

Let a short VLF electromagnetic pulse propagate in a homogeneous plasma along the  $z$  axe directed along the magnetic field. We assume that the short electromagnetic pulse has an envelope determined by the function  $A(\xi)$  with a nearly step-like form, finite duration and a unit value in the main body of the pulse

$$\mathbf{E}_{\sim} = A(\xi) \mathbf{E}_{[\sim]}(t, \mathbf{r}), \quad \mathbf{B}_{\sim} = A(\xi) \mathbf{B}_{[\sim]}(t, \mathbf{r}), \quad (2)$$

where  $\xi = z - v_{gz}t$ ,  $v_{gz}$  is the component of the group velocity along the magnetic field. The chosen pulse shape is modeled and corresponds to the shot noise well known in electronics (see, e.g., (Rytov et al., 1989)). It is convenient to replace  $f_{\sim}$  in Eq. (1) by an expression similar to Eq. (2):

$$f_{\sim} = A(\xi) f_{[\sim]}(t, \mathbf{r}, \mathbf{v}). \quad (3)$$

Upon substitution of Eqs. (2) and (3) in Eq. (1) we obtain

$$\begin{aligned} & \frac{\partial A}{\partial \xi} (-v_{gz} + v_z) f_{[\sim]} + A \left\{ \frac{\partial f_{[\sim]}}{\partial t} + v_z \frac{\partial f_{[\sim]}}{\partial z} + \mathbf{v}_{\perp} \frac{\partial f_{[\sim]}}{\partial \mathbf{r}_{\perp}} - \right. \\ & \left. - \frac{e}{mc} \mathbf{v} \times \mathbf{B} \frac{\partial f_{[\sim]}}{\partial \mathbf{v}} - \frac{e}{m} (\mathbf{E}_{[\sim]} + \frac{1}{c} \mathbf{v} \times \mathbf{B}_{[\sim]}) \frac{\partial f_{\circ}}{\partial \mathbf{v}} \right\} = 0, \end{aligned} \quad (4)$$

We restrict ourselves to the considering the energetic electrons for which

$$v_z \simeq v_{gz} . \quad (5)$$

For these particles, the first term in Eq. (4) is absent because the disturbed distribution function is nonzero only in a small vicinity of the velocity which satisfies to the condition (5). So, under condition (5) the interaction of a short electromagnetic pulse with energetic electrons is described by Vlasov equation (4) without the first term. It is very important that the reduced equation (4) (without the first term) is identical to Vlasov equation for wave-particle interaction in a homogeneous plasma.

5 Note that the particles which do not satisfy to Eq. (5) exist in the plasma, but their interaction with a short electromagnetic pulse is often not important, and we do not take this interaction into account.

Now we point the condition favorable for the existence of a short electromagnetic pulse that propagates along the magnetic field at a constant velocity without the additional phase modulation and smallest dispersion distortion of the pulse front. The solution of this problem is known. According to electrodynamics (see, e.g., (Jacson, 1962; Sommerfeld, 1914)) this takes place  
10 under condition  $v_{ph_z} = v_{gz}$ . This relationship agrees with Eq. (5), if the following two equalities are valid simultaneously:

$$v_{ph_z} = v_{gz} = v_z , \quad (6)$$

where  $v_{ph_z}$  is the component of the phase velocity along the magnetic field. The particles interact with short electromagnetic pulse under condition (6) as in a homogeneous plasma.

If the ion motion in a relatively dense background plasma is not important, then the dispersion equation of electromagnetic waves for the frequency band  $\omega_{LHF} < \omega < \omega_B$  (where  $\omega_{LHF}$  is the lower-hybrid frequency and  $\omega_B$  is the absolute value of the  
15 electron cyclotron frequency) in the quasi-longitudinal approximation (Ginzburg, 1970; Helliwell, 1965) takes the well-known form

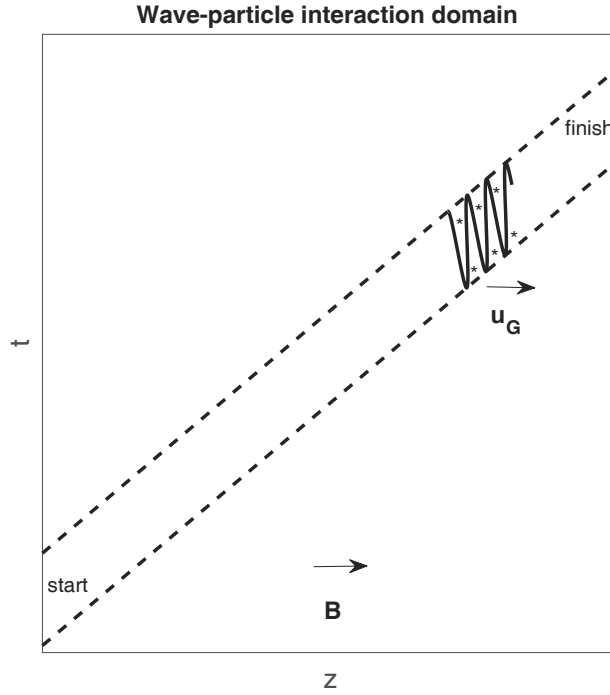
$$\omega = \omega_w(k_z, k_\perp) \equiv \frac{\omega_B |k_z| (k_z^2 + k_\perp^2)^{1/2}}{k_z^2 + k_\perp^2 + (\omega_p/c)^2} . \quad (7)$$

Here,  $k_z$  and  $k_\perp$  are the whistler wave vector components along and across the magnetic field, respectively, and  $\omega_p$  is the electron plasma frequency at the magnetic equator.

Note the first key point of our analysis. It is known (Helliwell, 1995) that according to the dispersion equation (7) the  
20 conditions (6) are fulfilled independent of  $k_\perp$  for two selected velocities along the magnetic field

$$\frac{\omega}{k_z} = \frac{\partial \omega}{\partial k_z} = v_z = \pm u_G , \quad u_G = \frac{c \omega_B}{2 \omega_p} , \quad (8)$$

where  $u_G$  is the Gendrin velocity. We will take into account only one positive velocity in the intermediate calculations.



**Figure 1.** The resonance beam of energetic electrons (stars) and the short electromagnetic pulse (sine) move together in the domain along the corridor between the start and finish lines.

### 3 The beam pulsed amplifier mechanism of the oblique electromagnetic chorus excitation due to the conditional instability of a short electromagnetic pulse

On the plane  $z, t$ , the specific features of particles and wave, which satisfy Eqs. (8), are explained in Fig. 1. Here  $z$  is the axis along the magnetic field, the start and finish lines correspond to the wave-particle interaction region boundaries,  $t$  is the time counted from the pulse crossing the start line. The resonance particles (3) and the short electromagnetic pulse (2) are situated in the domain which moves along a narrow corridor between two parallel dashed lines. The corridor width is determined by the pulse duration. These particles and waves in the domain form a separated plasma subsystem.

Effective wave-particle interaction in a homogeneous plasma in the magnetic field takes place at the resonance conditions  $\omega - k_z v_z = s\omega_B$ , where  $s$  is the integer. Such a resonance condition is part of the mentioned equalities (8) for the Čerenkov

resonance  $s = 0$ . We limit ourselves to this possibility only:

$$\omega - k_z v_z = 0. \quad (9)$$

We expect that the following two simplifications are fulfilled: the radiation power of an individual energetic electron corresponds to the so-called dipole approximation; the specific kinetic effects like the Landau damping are not important. Both of the mentioned simplifications take place for energetic electrons with a sufficiently small dispersion over the transverse and longitudinal velocities (Ginzburg, 1970):

$$k_\perp^2 \overline{v_\perp^2} < \omega_B^2, \quad k_z^2 \overline{(v_z - u_G)^2} < \gamma^2, \quad (10)$$

5 where the overbar means averaging and  $\gamma$  is the instability growth rate. The first inequality makes it possible to expand the Bessel functions for small arguments (this so-called dipole approximation is used for obtaining the expressions of the permittivity tensor). The second inequality makes it possible not to take into account the kinetic effects (like the Landau damping) for instability with a large hydrodynamic growth rate.

Note the second key point for a qualitative calculation of short electromagnetic pulse amplification. According to the previous  
10 comments, the electromagnetic field with suitable frequency in the short pulse changes its value as the electromagnetic wave field in a homogeneous plasma with electron beam described by the effective distribution function:

$$f_b = \frac{n_b}{2\pi v_\perp} \delta(v_\perp) \delta(v_z - u_G), \quad (11)$$

where  $n_b$  is the density of the resonance electron beam. It can be shown that for the wave-particle interaction at the Čerenkov resonance, the results of calculations do not depend on the actual form of the distribution function over perpendicular velocities if the first inequality (10) takes place. We select the effective distribution function (11) with delta function over perpendicular  
15 velocities to simplify the further algebra. On the other hand, the delta function over velocities along the magnetic field exactly follows from the condition (8) for the selected velocity.

**We explain additionally the expression for the effective distribution function (11). Assume that a short non-spreading pulse propagates at a constant velocity ( $v_{phz} = v_{gz} = u_G$ ) along the magnetic field in a plasma with an arbitrary undisturbed distribution function  $f_0(\mathbf{v})$ . The wave-particle interaction within a short pulse is determined by the characteristic time of the electron velocity variation  $\tau$ . For correct accounting of the wave-particle interaction, it is necessary to know the distribution function averaged over the time scale  $\tau$ . Let us consider the effective distribution function inside the pulse. Inside the pulse there is a population of electrons which have flown into the interaction region together with the pulse and is fly together with it. The contribution of these particles to the effective distribution function is proportional to  $n_b \delta(v_z - u_G)$ . The suprathermal electrons with other velocities or flight moments do not give an appreciable  
20 contribution to the effective distribution function within the pulse since they cross the pulse in a narrow corridor in Fig. 1 too quickly on a time scale  $\tau$  or do not have contact with the pulse. Thus, we take the effective distribution function (11) to analyze the evolution of a short electromagnetic pulse.**

We expect that the quasineutral plasma consists of three fractions: unmoved protons; cool electrons; weak electron beam along the magnetic field without thermal dispersion, described by the effective distribution function (11). So, the complicated geophysical problem is reduced to a typical problem of plasma physics (e.g., Akhiezer et al., 1975). We now consider the permittivity tensor  $\varepsilon_{\alpha\beta}$  of such a medium taking into account that the resonance beam density is relatively small. The beam-related terms in the permittivity tensor produce any effect only if they have a resonance pole at  $\omega - k_z u_G = 0$  (see Eq. (9)). There is one such resonance term only in the component  $\varepsilon_{33}$ . The permittivity tensor with mentioned resonance term has the following non-zero components:

$$\begin{aligned}\varepsilon_{11} = \varepsilon_{22} &= 1 - \frac{\omega_p^2}{\omega^2 - \omega_B^2} , \\ \varepsilon_{12} = -\varepsilon_{21} &= i \frac{\omega_B \omega_p^2}{\omega(\omega^2 - \omega_B^2)} , \\ \varepsilon_{33} &= 1 - \frac{\omega_p^2}{\omega^2} - \frac{n_b \omega_p^2}{n_p (\omega - k_z u_G)^2} ,\end{aligned}\tag{12}$$

where  $n_p$  is the background plasma density. The plasma dispersion equation (Ginzburg, 1970) for a plane electromagnetic wave in which all the variables are proportional to  $\exp\{i(-\omega t + k_\perp x + k_z z)\}$ , takes the well-known form:

$$\det \left\| k^2 \delta_{\alpha\beta} - k_\alpha k_\beta - (\omega/c)^2 \varepsilon_{\alpha\beta} \right\| = 0 .\tag{13}$$

- 10 This determinant in the quasi-longitudinal approximation is reduced to the following dispersion equation  $(\omega - \omega_w(k_z, \theta))(\omega - k_z u_G)^2 = (n_b/4n_p)\omega^3 \sin^2 \theta$  and finally, in accordions with Eqs. (7) and (8), we have

$$(\omega - \omega_w(k_z, \theta))(\omega - |k_z| u_G)^2 = \frac{n_b \omega_B^3}{32 n_p} \sin^2 \theta |\cos \theta|^3 ,\tag{14}$$

where the absolute value symbols take into account two selected velocities (8) along the magnetic field. Recall that the dispersion equation (7) was found in the same quasi-longitudinal approximation. Exactly in accordions with Eq. (7) we have

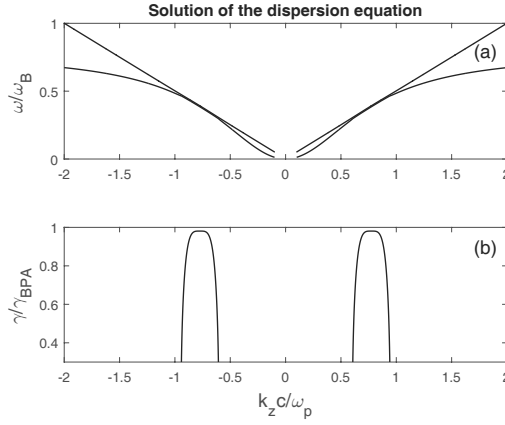
$$\omega_w(k_z, \theta) = \frac{\omega_B (k_z c / \omega_p)^2 |\cos \theta|}{(k_z c / \omega_p)^2 + \cos^2 \theta} ,\tag{7a}$$

where  $\cos \theta = k_z / (k_z^2 + k_\perp^2)^{1/2}$ ,  $\theta$  is the angle between the wave vector and the magnetic field.

- 15 The dispersion equation (14) is a cubic equation for the frequency  $\omega$ . This equation has three roots, one of which (see Fig. 2) for conditions close to optimal (8) corresponds to the short electromagnetic pulse instability with the conditional growth rate

$$\gamma = \frac{\sqrt{3}}{4} \left( \frac{n_b}{4n_p} \sin^2 \theta |\cos \theta|^3 \right)^{1/3} \omega_B .\tag{15}$$

According to Eq. (15), the conditional growth rate of the short electromagnetic pulse instability is maximum for  $\cos^2 \theta_{BPA} = 0.6$ . So, according to Eqs. (8) and (15), it is easy to find the following optimal wave characteristics favorable to the maximum conditional growth rate of a short electromagnetic pulse



**Figure 2.** Numerical solution of the dispersion equation (14) for  $n_b/n_p = 10^{-5}$  and  $\theta = \theta_{BPA}$ : (a) The frequencies (real part of frequencies) dispersion behavior of the longitudinal wave number; (b) Conditional growth rate of a short electrostatic pulse as a function of the longitudinal wave number.

$$\theta_{BPA} \simeq \begin{cases} 39^\circ; & k_{zBPA} \simeq \pm 0.8\omega_p/c; \\ 141^\circ; & k_{\perp BPA} \simeq 0.6\omega_p/c; \end{cases}$$

$$\omega_{BPA} \simeq 0.39\omega_B, \quad \gamma_{BPA} \simeq 0.15\omega_B(n_b/n_p)^{1/3}. \quad (16)$$

Note, that the resonance beam density  $n_b$  is significantly less than the density of energetic electrons but according to the additional calculations, it is enough for explanation of the experimentally recored values of  $\gamma \sim 10^2 \text{ s}^{-1}$  in the chorus excitation region. Now it is possible to estimate an electromagnetic pulse duration  $t_p$ . The beam pulsed amplifier mechanism is effective for electromagnetic pulses with the duration  $t_p \leq 10/\gamma_{BPA} \sim 0.1 \text{ s}$  **at the linear stage of the pulse evolution.**

## 5 4 Conclusions and discussion

The proposed beam pulsed amplifier (BPA) mechanism produces effective amplification of short electromagnetic pulses with frequency close to  $\omega_{BPA}$  in the VLF frequency band near the magnetic equator. The key conditions (8), (16) take place outside the plasmasphere, where  $u_G$  is close to the typical velocity of energetic non-relativistic electrons. In this region the ratio  $(n_b/n_p)^{1/3}$  is not so small. This leads to the amplification of short electromagnetic pulses from the noise level and the formation of discrete VLF emissions.

The threshold of BPA mechanism is mainly determined by the kinetic Čerenkov damping ( $s = 0$ ) of electromagnetic waves in the epithermal background plasma. The magnitude of the decay rate is considerably less than the growth rate  $\gamma_{BPA}$  in the chorus excitation region.

By the further analysis it is possible to explain the gap between the lower and upper chorus frequency bands which are well known experimentally (e.g., (Bell et al., 2009)). Actually, the short electromagnetic pulse excitation takes place for frequency bands both below and above  $\omega_{BPA}$  because of the spectral pulse distortion and the resonance beam velocity decrease as the pulse moves through the wave-particle interaction region.

- 5      Note that electromagnetic signals with a smooth envelope are not amplified due to BPA mechanism during their propagation through the region near the magnetic equator. Therefore, it is possible to explain the excitation of chorus emissions without a hiss-like background.

- 10      BPA mechanism is not connected directly with the energetic electron precipitation into the ionosphere because this mechanism is responsible for the distribution function modification in the velocity space only far enough from the loss cone. On the other hand, exactly after the amplification, a strong electromagnetic pulse during its propagation to the ionosphere can interact with more energetic electrons due to the cyclotron resonance ( $s = 1$ ). Such an interaction produces precipitation of energetic electrons into the ionosphere that is conjugate to the electromagnetic pulse.

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