Ann. Geophys. Discuss., https://doi.org/10.5194/angeo-2018-129
Manuscript under review for journal Ann. Geophys.
Discussion started: 5 December 2018

Annales Geophysicae Discussions

On the ion-inertial range density power spectra in solar wind turbulence

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Abstract.– De-magnetised ions in the ion inertial range of quasi-neutral plasmas respond to Kolmogorov inertial range velocity turbulence power spectra via the spectrum of the velocity turbulence related random mean square induction electric field. Maintenance of electrical quasi-neutrality in this field causes deformation in the power spectral density of the density turbulence. Experimentally confirmed Kolmogorov inertial range spectra in solar wind velocity turbulence and observations of density power spectra suggest that the observed unexplained scale-limited occasional "bumps" in the density power spectrum may be caused by this electric ion response. Magnetic power spectra react passively to the density spectrum by warranting pressure balance. This effect still neglects contribution of Hall currents and is restricted to the ion inertial range scale. It occurs under certain conditions only. While both density and magnetic turbulence spectra in the range of ion inertial scales deviate from Kolmogorov, the velocity turbulence preserves its Kolmogorov inertial range shape in this process to which spectral advection turns out to be secondary.

1 Introduction

The solar wind is a turbulent flow of origin in the solar corona. Power spectral densities of this turbulence have been measured for several decades already. They include spectra of the magnetic field (cf. e.g., Goldstein et al., 1995; Tu & Marsch, 1995; Zhou et al., 2004; Podesta, 2011, for reviews among others), but with improved instrumentation also of the fluid velocity (Podesta et al., 2007; Podesta, 2009; Šafránková et al., 2013), occasionally the electric field (Chen et al., 2011, 2012), temperature (Šafránková et al., 2016) and (starting with Celnikier et al., 1983, who already reported its main properties) also of the (quasi-neutral) solar wind density (Chen et al., 2012; Šafránková et al., 2013, 2015, 2016).

The information content of density fluctuations has so far remained obscured by the lack of their relation to developed solar wind turbulence. It is of course clear that density fluctuations are inherent to pressure fluctuations δP. Their nature is compressive. Under ideal gas conditions like those assumed to hold in the solar wind, at least on the larger MHD scales and for high solar wind speeds, they are just secondary to turbulence. They can be determined from total pressure balance. Their information content is thus to some extent trivial as they signal the presence of a population of compressive (magnetoacousticlike) fluctuations in the solar wind in addition to the usually assumed about Alfvénic turbulence, the dominant fluid-magnetic fluctuation family dealing with the related velocity and magnetic fields. The latter is subject to the famous Kolmogorov (Kolmogorov, 1941a, b, 1962) or Iroshnikov-Kraichnan (Iroshnikov, 1964; Kraichnan, 1965, 1966, 1967) spectra respectively their anisotropic generalisation (Goldreich & Sridhar, 1995). Presence of the Kolmogorov inertial range spectra in the solar wind reaching down into the dissipation range have been confirmed by a wealth of observations (cf., e.g., Goldstein et al., 1995; Tu & Marsch, 1995; Zhou et al., 2004; Alexandrova et al., 2009; Boldyrev et al., 2011; Matthaeus et al., 2016; Lugones et al., 2016; Podesta, 2011; Podesta et al., 2006, 2007; Sahraoui et al., 2009, and others for reviews, original observations, including MHD simulations). Since the mean fields B_0, T_0, N_0, U_0 already obey pressure balance one simply has for the turbulent fluctuations

$$\frac{\langle|\delta \boldsymbol{B}|^2\rangle}{B_0^2} = \frac{\sqrt{\langle|\delta N|^2\rangle}}{N_0} + \frac{\sqrt{\langle|\delta T|^2\rangle}}{T_0} \tag{1}$$

Alfvénic fluctuations compensate themselves separately due to their magnetic and velocity fluctuations being related; they do not contribute to compression. In order to infer the contribution of density fluctuations one may thus compare their spectral densities with those of the temperature or magnetic field. This requires normalisation to the means. Solar wind densities at 1 AU are of the order of $N_0 \sim 10$ cm^{-3}, while ion thermal speeds are of the order of $v_i \sim 30$ km s^{-1}. Since only fluctuations of the latter have been measured (Šafránková et al., 2013, 2015, 2016) comparison can be done between the two.

An example is shown in Fig. 1 based on solar wind measurements on July 6, 2012 (Šafránková et al., 2016). There is not much freedom left in choosing the mean densities and temperatures in Fig. 1. Densities at 1 AU barely exceed 10 cm^{-3}. Decreasing or increasing the mean ion thermal speed makes as well not much sense. Of course, electron temperatures and their fluctuations are much larger, but we are bound here to the observations of only thermal ion fluctuations. Moreover, electron temperatures are insensitive to those low frequency density fluctuations. High mobility makes their reaction isothermal.

The data in Figure 1 show the relative dominance of the turbulent density fluctuations over the ion temperature fluctuations at all frequencies larger than the lowest accessible MHD fluctuations in view of their contribution to pressure balance. This is not very surprising because one would not expect a large temperature effect. Ion thermalisation is a slow process which does not react to any fast fluctuations in the pressure cause by the density or magnetic field. It just shows that the turbulent thermal pressure is mainly due to density fluctuations over most of the frequency range. In addition, of course, the kinetic pressure of turbulent eddies must be considered, when not talking about relative but total pressure equilibrium.

2 Response of nonmagnetic ions

The question arises of what is the cause of those ion density fluctuations which cause the observed scale-limited deviation of the turbulent power spectra in density from the expected monotonic power law decay towards higher wavenumbers respectively shorter scales. At low frequencies in the MHD range the above mentioned magnetosonic waves including their sidebands as well as some unknown strongly damped "virtual" evanescent modes will contribute to both density and temperature. However, in the higher frequency range their existence needs to be explained otherwise.

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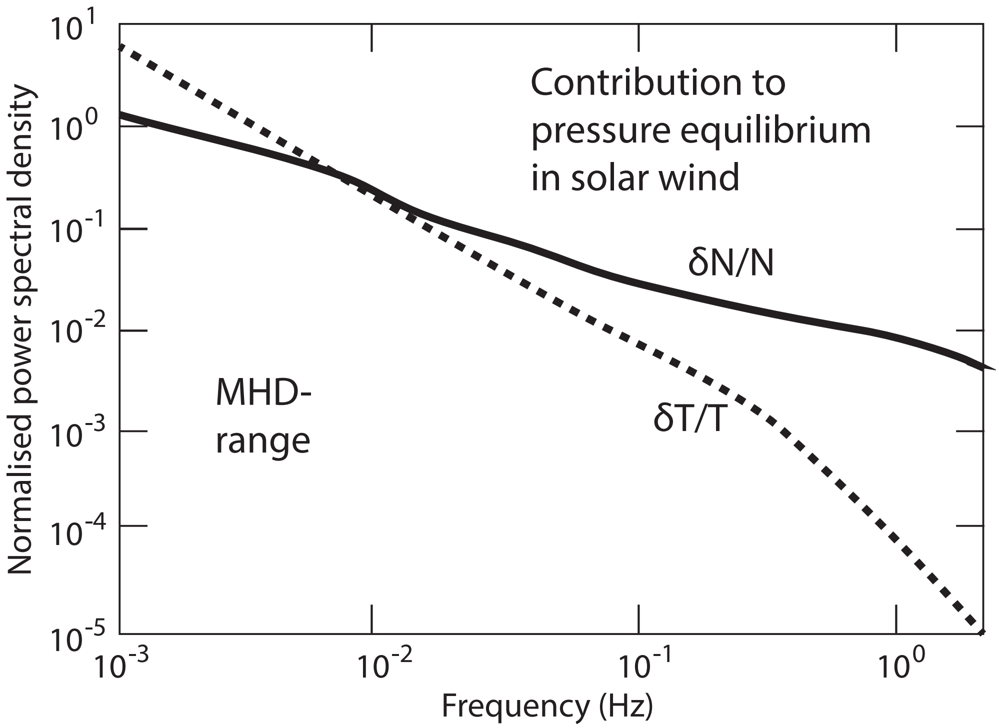

Figure 1. Normalised solar wind power spectra of turbulent temperature and density fluctuations. The curves are based on data from Šafránková et al. (2016) obtained on July 6, 2012. The data have been rescaled and normalised to the main density N_0 and temperature T_0 in order to show their relative contributions to an assumed solar wind pressure balance. The interesting result is that in the lowest MHD frequency range density fluctuations are irrelevant with respect to pressure balance. At higher frequencies, however, the density fluctuations dominate the temperature fluctuations.

The steep decay of the normalised fluctuations in ion temperature above frequencies $> 10^{-1}$ Hz is certainly due to the drop in ion dynamics at those frequencies exceeding the ion cyclotron frequency. In this range we enter the (dissipationless) ion inertial or Hall (electron-MHD) domain (cf., e.g., Huba, 2003) where ions become nonmagnetic, currents are carried by the magnetised electrons, and both species decouple magnetically and contribute through the Hall current. At frequencies far below the electron plasma frequency they couple mainly through the condition of quasi-neutrality, i.e. via the turbulent induction electric field which in the ion inertial domain becomes

$$\begin{aligned}\delta\boldsymbol{E} = \; & - \;\; \delta\boldsymbol{V}_\perp \times \boldsymbol{B}_0 - \boldsymbol{U}_0 \times \delta\boldsymbol{B} - \\ & - \;\; \frac{1}{eN_0}\boldsymbol{B}_0 \times \delta\boldsymbol{J} + \\ & + \;\; \left\langle \delta\boldsymbol{V} \times \delta\boldsymbol{B} + \frac{1}{eN_0}\delta\boldsymbol{J} \times \left(\frac{\delta N}{N_0}\boldsymbol{B}_0 - \delta\boldsymbol{B}\right)\right\rangle\end{aligned} \qquad (2)$$

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The inert ions are bound to react to this field in order to maintain electrical quasi-neutrality. The last three averaged nonlinear terms on the right within the angular brackets $\langle \ldots \rangle$ are the contributions of the electromotive force to the electric field and may be assumed to have been absorbed already in the mean quantities where they contribute to mean field processes like convection, dynamo action, and turbulent diffusion. They do not vary on the fluctuation scale and, in considering the effect of the electric field on turbulence, can be dropped. The remaining three terms distinguish between directions parallel and perpendicular to the main field $\boldsymbol{B}_0$, with the third term being the genuine Hall contribution perpendicular to it. If the advection $\boldsymbol{U}_0 \| \boldsymbol{B}_0$ is parallel, and from Ampere's law with $\mu_0 \delta \boldsymbol{J} = \nabla \times \delta \boldsymbol{B}$, we have

$$\begin{aligned} \delta \boldsymbol{E}_\perp &= \boldsymbol{B}_0 \times \left[\delta \boldsymbol{V}_\perp - \frac{U_{0\|}}{B_0} \delta \boldsymbol{B}_\perp - \right. \\ & \left. - \frac{1}{e\mu_0 N_0} \left(\nabla \times \delta \boldsymbol{B} \right)_\perp \right] - \boldsymbol{U}_{0\perp} \times \delta \boldsymbol{B}_\| \qquad (3) \\ \delta \boldsymbol{E}_\| &= -\boldsymbol{U}_{0\perp} \times \delta \boldsymbol{B}_\perp \Longrightarrow 0 \end{aligned}$$

The second of these equations is of no interest because the low frequency parallel electric field it produces is readily compensated by electron displacements along $\boldsymbol{B}_0$, which leaves us with the fluctuating perpendicular induction field in the first Eq. (3). Here, parallel advection $U_{0\|}$ attributes to the perpendicular velocity fluctuations from perpendicular magnetic fluctuations $\delta \boldsymbol{B}_\perp$. On the other hand, any present parallel compressive magnetic fluctuations $\delta \boldsymbol{B}_\| = \boldsymbol{B}_0(\delta B_\| / B_0)$ add through perpendicular advection. In their absence the last term disappears.

The complete Hall contribution to the electric field, viz. the last term in the brackets, can be written

$$\delta \boldsymbol{E}_\perp^H = -\frac{B_0}{e\mu_0 N_0} \left(\boldsymbol{\nabla}_\perp \delta B_\| - \nabla_\| \delta \boldsymbol{B}_\perp \right) \qquad (4)$$

Even for $U_{0\|} = 0$ it contributes through the turbulent fluctuations in the magnetic field. As both these contributions depend only on $\delta \boldsymbol{B}$ we can isolate them for separate consideration. One observes, however, that in the absence of any compressive magnetic components $\delta B_\|$ and homogeneity along the mean field $\nabla_\| = 0$ there is no contribution of the turbulent Hall term to the electric induction field. In that case only velocity turbulence contributes. Below we consider this case.

3 Effect of velocity turbulence

Let us assume that advection by large-scale energy carrying eddies is perpendicular $\boldsymbol{U}_0 = \boldsymbol{U}_{0\perp}$, and there are no compressive magnetic fluctuations $\delta \boldsymbol{B}_\| = 0$. This reduces to considering in Eq. (2) only the first term containing the velocity fluctuations. We ask for its effect on the density fluctuations in the ion-inertial domain on inertial scales where the ions demagnetise.

The inert non-magnetic ions in the Hall domain (with scales shorter than their thermal gyroradius $\rho_i = v_i/\omega_{ci}$ or inertial length $\lambda_i = c/\omega_i$, depending on the direction to the mean magnetic field $\boldsymbol{B}_0$ and the value of plasma beta $\beta = 2\mu_0 N_0 T_0 / B_0^2$, with $\omega_{ci} = eB_0/m_i$ ion cyclotron and $\omega_i = e\sqrt{N_0/\epsilon_0 m_i}$ ion plasma frequency, respectively) experience the induction field caused by the spectrum of velocity fluctuations as an external potential field. Quasi-neutrality then (for an electron-proton plasma) implies a density fluctuation $\delta N_i = \delta N_e \equiv \delta N$ which is obtained from Poisson's equation

$$\nabla \cdot \delta \boldsymbol{E} = \frac{e}{\epsilon_0} \delta N \qquad \Longrightarrow \qquad i\boldsymbol{k} \cdot \delta \boldsymbol{E}_{\boldsymbol{k}} = \frac{e}{\epsilon_0} \delta N_{\boldsymbol{k}} \qquad (5)$$

It contains only the ions, because only they feel the electric fluctuations. Since power spectral densities are measured, one switches to their Fourier representations on the right. For completeness we note that the Hall contribution to the Poisson equation in Fourier space reads

$$i\boldsymbol{k}_\perp\cdot\delta\boldsymbol{E}^H_{\perp\boldsymbol{k}} = \frac{B_0}{e\mu_0 N_0}\left(k_\perp^2\delta B_{\|\boldsymbol{k}} - k_\|\boldsymbol{k}_\perp\cdot\delta\boldsymbol{B}_{\perp\boldsymbol{k}}\right) = \frac{e}{\epsilon_0}\delta N^H_{\boldsymbol{k}} \tag{6}$$

Again it becomes obvious that absence of parallel (compressive) magnetic turbulence eliminates the first term in this expression while purely perpendicular propagation eliminates the second term. Alfvénic turbulence, for instance, with $\delta B_\| = 0$ and $k_\perp = 0$ has no effect on the modulation of the density spectrum, a fact which is of course reasonably known. On the other hand, for perpendicular wavenumbers $k = k_\perp$ only the compressive magnetic turbulence contributes.

4 Advected Kolmogorov spectrum of turbulent velocity

We will be interested in the spectral behaviour of the power spectral density of the turbulent density while, based on the above assumptions, not taking into account the Hall term.

Multiplication of the remaining first term in the electric induction field Eq. (3) with the turbulent wavenumber $\boldsymbol{k}$ selects wavenumbers $\boldsymbol{k}_\perp$ perpendicular to $\boldsymbol{B}_0$. Combining Eq. (2) and the Poisson equation yields the expression for the power spectrum of turbulent density fluctuations

$$\langle|\delta N|^2\rangle_{k_\perp} = \left(\frac{\epsilon_0 B_0}{e}\right)^2 k_\perp^2\langle|\delta\boldsymbol{V}|^2\rangle_{k_\perp} \tag{7}$$

which in this case is determined by the power spectrum of the turbulent velocity. This can also be written as

$$\begin{aligned}\frac{\langle|\delta N|^2\rangle_{k_\perp}}{N_0^2} &= \left(\frac{V_A}{c}\right)^2\left(\frac{k_\perp}{\omega_i}\right)^2\langle|\delta\boldsymbol{V}|^2\rangle_{k_\perp}\\ &= \frac{\langle|\delta\boldsymbol{V}|^2\rangle_{k_\perp}}{c^2}\left(\frac{V_A}{c}\right)^2(k_\perp\lambda_i)^2\end{aligned} \tag{8}$$

with $V_A^2 = B_0^2/\mu_0 m_i N_0$ the Alfvén speed squared, and $\omega_i^2 = e^2N_0/\epsilon_0 m_i$ the squared proton plasma frequency. This form is still non-normalised. It however suggests that the contribution to density fluctuations requires perpendicular scales $\lambda_\perp \ll \lambda_i$ smaller than the ion inertial length $\lambda_i = c/\omega_i$. This is the case when $\lambda_\perp < \rho_i$ is smaller than the ion gyroradius $\rho_i = v_i/\omega_{ci}$, the condition for ions becoming de-magnetised. Then the condition on the scale simply reduces to a condition on the ion thermal speed $v_i \ll V_A$ which in the solar wind is almost ever satisfied. Thus, knowing the power spectral density of velocity turbulence, the power spectrum of the density will also be known. This requires that a spectrum of velocity turbulence is to be given a priori. In application to the solar wind we make, in the following, use of the celebrated Kolmogorov spectrum.

The velocity spectrum of the turbulent eddies gives rise to a broad spectrum in $\boldsymbol{k}_\perp$. In order to infer its effect on the power spectrum of density we refer to the Kolmogorov inertial range spectra (Kolmogorov, 1941a, b, 1962; Obukhov, 1941) as these are most appropriate for the mechanical turbulence. Their inertial range power spectral density in wavenumber space is

$$\left\langle|\delta\boldsymbol{V}|^2\right\rangle_k \equiv \mathcal{E}_K(k) = C_K\epsilon^{\frac{2}{3}}k^{-\frac{5}{3}} \qquad \text{for} \qquad k_{in} < k < k_d \tag{9}$$

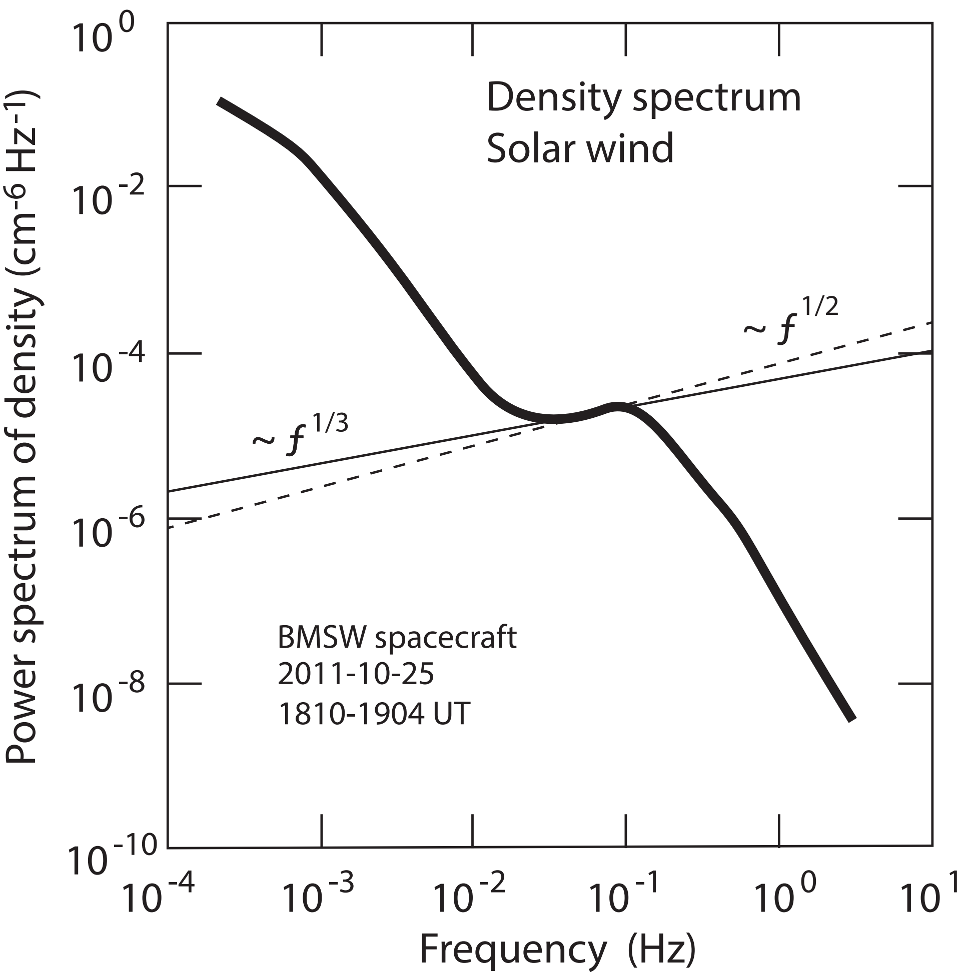

Figure 2. Solar wind power spectra of turbulent density fluctuations (based on BMSW spacecraft data from Šafránková et al., 2013, obtained on Oct 10, 2011). The power spectrum exhibits a so-called bump at intermediate frequencies of positive slope $\sim \omega^{\frac{1}{3}}$. This is in agreement with it being caused by the response of the nonmagnetic ions to the electric induction field of the turbulent mechanical fluctuations in the solar wind velocity in Kolmogorov (K) inertial range turbulence (straight line). The dashed line corresponds to an Iroshnikov-Kraichnan (IK) spectrum. Uncertainty in the data inhibits distinguishing between K and IK inertial range velocity turbulence.

This applies to advected (mechanical) velocity turbulence (Fung et al., 1992; Kaneda, 1993) in the inertial range between energy injection rate ϵ at k_{in} and dissipation at k_d, with $C_K \approx 1.65$ Kolmogorov's constant of proportionality (as determined by Gotoh & Fukayama, 2001, using numerical simulations). Advection by the large eddy flow at velocity U_0 just causes a weak spectral modification (cf., e.g., Fung et al., 1992; Kaneda, 1993, and others) or Doppler broadening at given fixed k

$$\mathcal{E}^{ad}_{k\omega_k} = \frac{1}{2}\frac{\mathcal{E}_K(k)}{\sqrt{2\pi}kU_0}\sum_{\pm}\exp\Big[-\frac{1}{2}\frac{\omega_\pm^2}{(kU_0)^2}\Big], \qquad \omega_\pm = \omega_k \pm \lambda k^{\frac{2}{3}} \tag{10}$$

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which is due to the decorrelation of the small and large eddies caused by the advective transport. $\lambda \sim O(1)$ is some constant. The $k^{\frac{2}{3}}$ dependence in the frequency results from advection (Doppler shift) $k\delta V$ by neighbouring eddies in the inertial range at a velocity of $\delta V \propto k^{-\frac{1}{3}}$ (Fung et al., 1992). The frequency ω_k stands for the internal dependence of the turbulent frequency on the turbulent wavenumber $\boldsymbol{k}$, i.e. it can be understood as a "turbulent dispersion relation", which by no means is the solution of a linear wave eigenmode equation. Ignoring ω_k, the advected power spectrum at large k is power law and decays $\propto k^{-\frac{8}{3}}$ which, when integrated with respect to k yields the Kolmogorov law in frequency space:

$$\int\limits_{k_{in}}^{k_d} dk\, \mathcal{E}^{ad}_{k\omega_k} \sim \mathcal{E}^{ad}_K(\omega) \,\propto\, \omega^{-\frac{5}{3}} \tag{11}$$

an expression which clearly demonstrates that the purely mechanical Kolmogorov velocity fluctuation spectrum can be mapped from k-space into ω-space.

On the other hand, in the long wavelength range the exponential dependence $\exp(-\lambda^2/U_0^2k^{\frac{2}{3}})$ suppresses the spectrum a bit which fakes a flattening of the spectrum towards the small wavenumber end k_{in} of the inertial range by bending the spectrum. This effect depends on the value of U_0. It is partially compensated by large advection speeds. (The spectral break point where the power maximises is found at $k_{min} = \lambda^3/16U_0^3\sqrt{2}$. Note that below k_{in} Kolmogorov theory does not apply. Hence one may just observe a flattening but will not encounter the point $k_{min} < k_{in}$ in the k spectrum for large U_0.)

It is however not known whether ω_k can indeed be ignored. In the large Reynolds number case of fast advection $\omega_\pm \approx kU_0$, the two exponentials reduce to numbers, and the dispersive modification of the frequency ω_k can indeed be neglected. Integration with respect to k then transforms the velocity fluctuation spectrum in k space into the above Kolmogorov spectrum in frequency space over the entire inertial domain. This is the case corresponding to Taylor's hypothesis in application to the advected mechanical power spectrum $\langle|\delta\boldsymbol{V}|^2\rangle$. For magnetic turbulence $\langle|\delta\boldsymbol{B}|^2\rangle$ a similar conclusion is, however, valid under further severe restrictions only (Treumann et al., 2018). This makes the inclusion of the turbulent Hall electric field Eq. (4) in the general case difficult (not speaking about the additional effect introduced by the Hall term).

5 Ion inertial range density power spectrum

Use of the advected power spectral density Eq. (10) of the velocity field in the transformed Poisson equation with $k \to k_\perp$ yields for the non-convected but advected turbulent ion-inertial range density-power spectrum in the stationary large eddy turbulence frame

$$\begin{aligned}\langle|\delta N|^2\rangle^{ad}_{\omega_k k_\perp} &= \frac{\epsilon_0^2 B_0^2}{e^2} k_\perp^2 \langle|\delta\boldsymbol{V}|^2\rangle_{\omega_k k_\perp} \\ &= \frac{\epsilon_0^2 B_0^2}{e^2} k_\perp^2 \mathcal{E}^{ad}_{k_\perp\omega_k} \\ &\propto k_\perp^{-\frac{2}{3}} \sum_\pm \exp\Big[-\frac{1}{2}\frac{\omega_\pm^2}{(kU_0)^2}\Big]\end{aligned} \tag{12}$$

This wavenumber space spectrum exhibits the advected broadening by the internal turbulence. Integrating it with respect to $k_\perp$ under the above assumption on $\omega_\pm \approx k_\perp U_0$ yields for the Eulerian (Fung et al., 1992) density power spectrum in frequency space $\omega_\ell < \omega < \omega_u$ in the ion-inertial domain of the turbulent inertial range :

$$\langle|\delta N|^2\rangle^{ad}_\omega \sim \omega^{\frac{1}{3}}, \qquad k_{ir}^{\frac{2}{3}}\epsilon^{\frac{1}{3}} = \omega_\ell < \omega < \omega_u \tag{13}$$

This is the proper local turbulent density spectrum *in the flowing turbulence frame* as function of frequency, which holds when the electric induction field is determined solely by turbulent velocity fluctuations. Here $k_{ir} \approx 2\pi\omega_i/c$ (or $2\pi v_i/\omega_{ci}$) is the wavenumber corresponding to the lower end of the ion inertial range. The upper bound on the frequency ω_u remains undetermined. It may be caused by processes which reduce the extension of validity of the above equation to a wavenumber range narrower than the full Kolmogorov inertial range. One assumption would be that ω_u is at the lower hybrid frequency which is intermediate to the ion and electron cyclotron frequencies. At this frequency electrons become capable of discharging the electric induction field thus breaking the spectrum to return to its Kolmogorov spectral decay at further increasing frequency. In contrast to the Kolmogorov law this advected density power spectrum Eq. (15) *increases* with frequency. Clearly, this increase is limited supposing that the Kolmogorov dissipation range has not been entered yet.

In an MHD flow like the fast streaming solar wind this density power spectrum is convected past the stationary observer (spacecraft). It is thus subject to a Doppler shift in frequency. The frequency in the spacecraft frame is then given by

$$\omega_s = \omega \pm \boldsymbol{k}\cdot\boldsymbol{V}_0 \approx k(k\epsilon \pm V_0\cos\alpha) \tag{14}$$

where α is the angle between $\boldsymbol{k}$ and $\boldsymbol{V}_0$. Assuming that around k_{ir} perpendicular wavenumbers $k_\perp\epsilon \ll V_0$ have a nonvanishing component parallel to the streaming speed, and that V_0 is large, the wavenumber spectrum near $k \gtrsim k_{ir}$ maps directly to the frequency spectrum with $\omega_s \gtrsim \omega_\ell$. In the above frequency spectrum we thus may replace $\omega \to \omega_s$, which yields

$$\langle|\delta N|^2\rangle_{\omega_s} \sim \omega_s^{\frac{1}{3}}, \qquad k_{ir}^{\frac{2}{3}}\epsilon^{\frac{1}{3}} = \omega_\ell < \omega_s < \omega_u \tag{15}$$

This assumption corresponds to the complete neglect of the turbulent dispersion relation and thus also neglect of any advection effects. It thus corresponds to the straightforward use of the Kolmogorov spectrum in Poisson's equation

$$\langle|\delta N|^2\rangle_{k_\perp} = \left(\frac{\epsilon_0 B_0}{e}\right)^2 k_\perp^2\mathcal{E}_K \propto \epsilon^{\frac{2}{3}}k_\perp^{\frac{1}{3}} \tag{16}$$

with $k_\perp \propto \omega_s/V_0$. At larger $k_\perp$ and lower convection speeds V_0 as well as nonaligned wavenumbers and flow velocity the density spectrum will exhibit deviations. The required alignment of turbulent wavenumbers and flow velocity also excludes any non-aligned eddies in the velocity spectrum from contributing to the density turbulence, causing deviations in the spectral slope of the power spectrum as function of frequency.

This behaviour of the density spectrum is seen in Fig. 2 as a spectral increase with spacecraft frequency $\propto f^{\frac{1}{3}}$. The data used in this figure were taken from published data (Šafránková et al., 2013) where the change in slope to positive values is referred to as a “bump” in the spectrum believed to indicate either the presence of new waves, which the authors suspected to propagate in the kinetic Alfvén mode, or attributed to dissipation. Though this is neither impossible nor can it be excluded here, the rather more convincing conclusion is that we are dealing with de-magnetised ions in the ion-inertial domain which respond to the turbulent induction electric field and become swept over the spacecraft by the fast solar wind flow. The scatter

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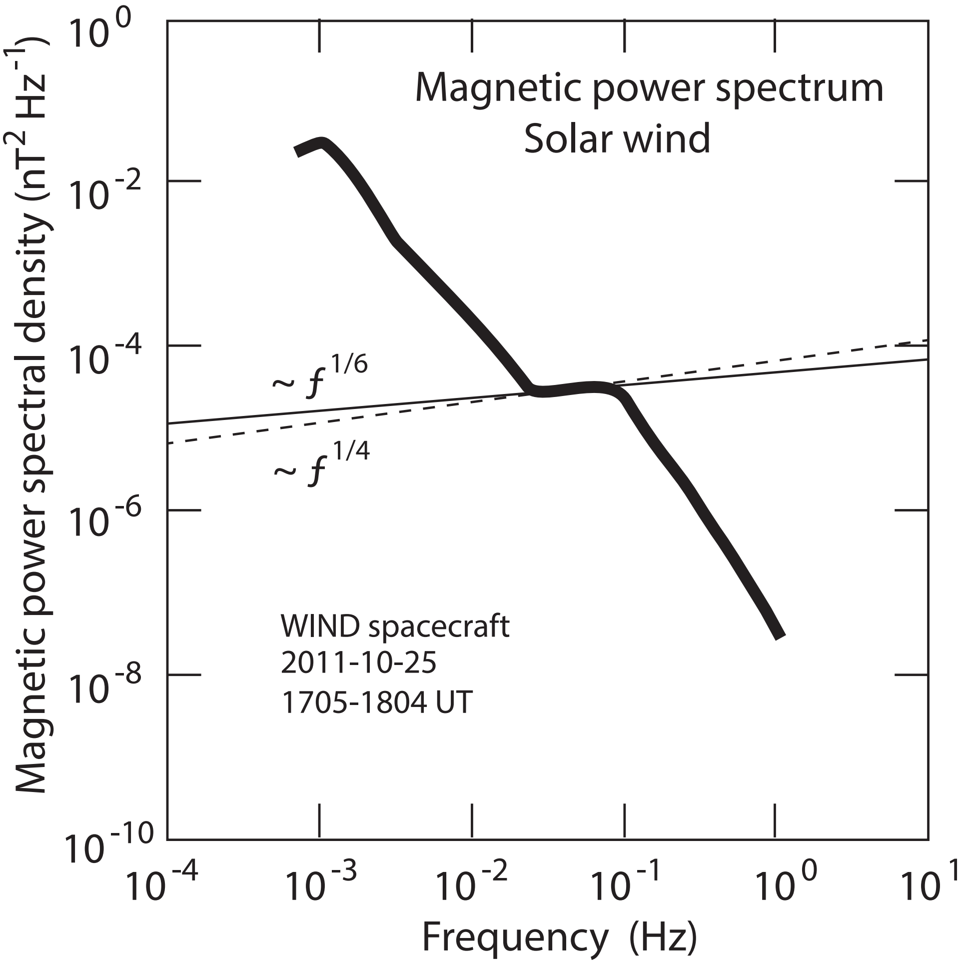

Figure 3. Solar wind power spectra of the turbulent magnetic field for the same time interval as in Figure 2 measured by the WIND spacecraft (data from Šafránková et al., 2013) which was located in the Lagrange point L1. The magnetic turbulence spectrum exhibits a deformation similar to that in the density power spectrum. The positive slope $\sim \omega^{\frac{1}{6}}$ in the deformation confirms its maintenance by pressure balance. The straight (dashed) line corresponds to the K (IK) spectrum. Scatter of these data was substantial inhibiting distinction between the two cases. The data curve shown is obtained by smearing out the scattered data.

of the originally measured data was substantial (Šafránková et al., 2013) allowing for different positive slopes as indicated by the solid and dashed straight lines with the latter being of slope $\propto f^{\frac{1}{2}}$, which would correspond to Iroshnikov-Kraichnan scaling. Both slopes cannot be distinguished in the data. However, the agreement of the average positive slope with our theory is presumably not coincidental. It also shows that the density fluctuations in this domain of nonmagnetic ions are intimately related to the turbulent velocity fluctuations whose spectrum under weak conditions (Treumann et al., 2018) directly transforms into the spacecraft frequency frame.

Table 1. Advected ion inertial range power spectral indices $k^{-a}, \omega_s^b, \mathcal{E}_{Bs} \sim \omega_s^c$

$\langle\lvert\delta V\rvert^2\rangle$	a	$a-2$	b	c
$\mathcal{E}_K$	$\frac{5}{3}$	$+\frac{1}{3}$	$\frac{4}{3}$	$\frac{2}{3}$
$\mathcal{E}_K^{ad}$	$\frac{8}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
$\mathcal{E}_{IK}$	$\frac{3}{2}$	$+\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{4}$
$\mathcal{E}_{IK}^{ad}$	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$

We might check whether the increased density in the limited frequency range agrees with the required pressure balance. Šafránková et al. (2013) referred to turbulent magnetic power spectra from the WIND spacecraft. WIND was located in the L1 Lagrange point where it measured magnetic field fluctuations related in time to the BMSW observations by the solar wind flow. In spite of their large scatter, the data were sufficiently stationary for comparison to the density measurements.

Figure 3 shows the spectral shape of the WIND magnetic field turbulence. We have smeared out the scatter in the original data (Šafránková et al., 2013, Fig. 3) such that the spectrum becomes a broad continuous line. This results in a "bump" respectively knee shape of the spectrum in the same frequency interval as the density spectrum. Its slope is positive being very close to the root of the density spectrum $\langle|\delta B|^2\rangle_{\omega_s} \sim {\omega_s}^{\frac{1}{6}}$ which corresponds to Kolmogorov (K) spectra. Similarly, for the steeper slope of $\propto f^{\frac{1}{2}}$ in the density spectrum, pressure balance is as well warranted at magnetic slope $\propto f^{\frac{1}{4}}$, slopes that correspond to Iroshnikov-Kraichnan (IK) inertial range spectra. Observations of density and magnetic power spectra are thus related via pressure balance Eq. (1) in the frequency interval in question. Given the scatter in the original data, the turbulent density and magnetic field are convincingly in pressure balance. Fluctuations in temperature do, within the experimental uncertainty, not to play any major role, both being expected physically. Comparing absolute powers makes no sense because of the different instrumentations, techniques, and locations of the spacecraft which inhibit any further analysis.

Pressure balance just checks that the effect of the density fluctuations is put into reasonable terms with equilibrium physics. The obvious conclusion would be that the density spectrum obeys the $\sim \omega^{\frac{1}{3}}$ shape, which in the stationary frame of turbulence is the straight consequence of an advected turbulent velocity K spectrum (or IK spectrum). On the other hand, observation of this same spectral shape in the spacecraft frame located in the fast solar wind stream suggests rather that advection plays only a marginal role in shaping the turbulent ion-inertial range density spectrum. Straightforwardly mapping the Poisson-modified turbulent velocity K-spectrum into the spacecraft frequency frame does reproduce the required slope. Such a mapping is permitted because it applies directly to the spectrum of turbulent velocity fluctuations $\langle|\delta \boldsymbol{V}|^2\rangle$. Considering the large scatter in the data within the narrow frequency interval provided by the observations, this might still be coincidental. Other reasons could as well be imagined for the production of spectral "bumps". Their rather occasional presence might, for instance, be caused by the passage of interplanetary discontinuities or shock waves which for the time of their passages just interrupt the turbulent spectrum over the distance of their effective widths. Pressure balance could not distinguish between this and the above spectral K (or IK) cases.

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Table 2. Convected SW density power spectral indices $k^{-a}, \omega_s^b, \mathcal{E}_{Bs} \sim \omega_s^c$

Spectrum	a	$a-2$	b	c
$\mathcal{E}_K$	$\frac{5}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{6}$
$\mathcal{E}_K^{ad}$	$\frac{8}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$
$\mathcal{E}_{IK}$	$\frac{3}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{4}$
$\mathcal{E}_{IK}^{ad}$	$\frac{5}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$

6 Discussion

Above we considered the effect of ordinary and advected K (IK) velocity turbulence on the turbulent density in the inertial range for scales when the ions de-magnetise, the so-called ion-inertial (or Hall) range. This is not the only option for choosing a turbulent velocity spectrum as reason for the deformation of the turbulent density spectrum. Other possible options are ordinary and advected IK spectra $\mathcal{E}_{IK}, \mathcal{E}_{IK}^{ad}$. Each of these spectra yields a different ion inertial scale range power spectrum in k-space and, consequently, also a different power law spectrum in ω-space. These power laws including K (IK) spectra are summarised in Table 1 for pure advection and absence of convective transport of turbulence by a fast solar wind stream.

The frequency spectra in Table 1 are the result of the response of ions to the turbulent induction electric field in the ion inertial range caused by the velocity turbulence. (We may note at this place, that concerning IK spectra it seems improbable that they would be realised in the scale range of the ion-inertial domain.) They can be considered to result from the direct mapping of the turbulent velocity K spectrum into the spacecraft frame because the non-relativistic velocity turbulence, to rather good approximation, translates its wavenumber spectrum into a frequency spectrum if only neglecting any internal turbulent dispersion relation (Treumann et al., 2018). The latter is in most cases well justified. This does, however, not hold for the spectrum of magnetic turbulence which depends in a more complicated way on the velocity turbulence.

The advected density K-spectrum in Table 1 in the frame of the stationary turbulence acquires a positive slope in a scale range restricted to the ion inertial range. However, observations of this slope in the spacecraft frame undermine this conclusion, suggesting that it is the ordinary velocity turbulence K (or if anisotropy is taken into account, the Kolmogorov-Goldreich-Sridhar KGS) spectrum in the ion-inertial range which causes the deformation of the power spectral density of density turbulence. Table 2 shows the case of fast streaming on the turbulent density spectral slopes. Clearly, the slopes in Table 2 when compared to the observations in Fig. 2 support the ordinary K or KGS (IK) spectra as only these models of turbulence reproduce the observed positive spectral slopes of the ion-inertial-region spectral domain under question in the power spectra of the density turbulence.

This observation diminishes the importance of advection of small-scale turbulence by large-scale eddies in the ion-inertial domain of the K- inertial range spectra. We should, however, account for the large uncertainty introduced by the scatter in the original data. The determination of the slope in the narrow frequency interval remains sufficiently imprecise to allow also for IK turbulence in Table 2 and Fig. 2.

It is important to note that the entire effect of spectral deformation is a cause of the undeformed K or IK inertial range velocity turbulence. It applies to density turbulence to which the magnetic turbulence responds via pressure balance. The occasional presence of a bump in the turbulent density power spectra suggests that there are conditions in the solar wind when the ion-inertial domain uncovers its existence by turning the turbulent power spectra of density fluctuations over from negative to positive slopes in the electric induction field related to the mechanical turbulence. Pressure equilibrium requirements then force the magnetic power spectrum to follow. The question remaining is why this distortion of the normal spectral power law happens only occasionally. Its answer seems to be related to the streaming conditions of advection of small scale eddies by the energy carrying large scale eddies. One possibility is that the solar wind ion temperature may become high. When this happens and the ion thermal speed $v_i > V_A$ exceeds the Alfvén velocity in the region of observation, then the charge neutralising effect on the density is diminished because the ion inertial range is shifted to scales shorter than observed.

Another reason for the relative rarity of the occurrence of those inertial range spectral deformations causing in the power spectra of density and magnetic inertial range turbulence might be found in our neglect of parallel transport of the turbulence in the first term of Eq. (3). The large eddies in relatively weak magnetic fields carry large amounts of kinetic energy and will thus be insensitive to magnetic effects. Similarly, large convective flows dominate the magnetic field. Therefore transport in most cases will be oblique and for large $U_{0\|}$ will dominate the first term in Eq. (3). Being determined by the magnetic component only under those conditions, one does not expect that the turbulence contributes to Poisson's equation. This leaves the inertial range power spectrum of turbulence in the density undeformed. Observationally this seems to be the general more frequently realised case.

We conclude with a remark on the Hall term Eq. (6) which has so far been neglected. This term results from taking into account the difference in electron and ion dynamics in the ion inertial domain leading to Hall currents, i.e. the reduced electron momentum conservation equation in two fluid theory of the plasma. As discussed above the Hall term can be neglected for $\delta B_\| = 0$ and perpendicular propagation. Violation of these restrictions would force its inclusion. This requires a separate investigation.

Acknowledgement. This work was part of a Visiting Scientist Programme at the International Space Science Institute Bern. We acknowledge the reserved interest of the ISSI directorate as well as the generous hospitality of the ISSI staff.

Author contribution. All authors contributed equally to this paper.

Data availability. No data sets were used in this article.

Competing interests. The authors declare that they have no conflict of interest.

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