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Magnetic guide field generation in collisionless current sheets

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Abstract. In thin ($\Delta <$ few λ_i) collisionless current sheets in a space plasma like the magnetospheric tail or magnetopause current layer, magnetic fields can grow from thermal fluctuation level by the action of the non-magnetic Weibel instability (Weibel, 1959). The instability is driven by the counter-streaming electron inflow from the "ion diffusion" (ion inertial Hall) region into the inner current (electron inertial) region after thermalisation by the two-stream instability. Under magnetospheric tail conditions it takes ∼50 e-folding times (∼100 s) for the Weibel field to reach observable amplitudes $|\mathbf{b}_\mathrm{W}| \sim 1$ nT. In counter-streaming inflows these fields are of guide field type.

Keywords. Space plasma physics (Magnetic reconnection)

1 Introduction

In this communication we investigate the self-consistent generation of a so-called magnetic guide field in a thin collisionless current layer that initially lacks the presence of any guide field. Guide fields are believed – and have been shown by numerical simulations (see e.g. Drake et al., 2006; Pritchett, 2005; Cassak et al., 2007, and others) – to be of prime importance in collisionless reconnection. The reason for their presence in a thin collisionless current sheet is not evident. For this we render responsible the Weibel ("current filamentation") instability (Weibel, 1959). It may be capable of generating a guide field that could become a non-negligible fraction of the undisturbed external field B_0 being of strength $B_g/B_0 < 1$ and directed along the current layer.

Magnetic guide fields have two implications which, in a thin current sheet with unmagnetised ions, affect basically only the electrons:

– that the centre of the current sheet is not free of magnetic fields, and

– that the sheet current $\mathbf{J}$ attains a guide-field-aligned component $J_\parallel$ which becomes important in reconnection scenarios.

2 Weibel scenario

The Weibel instability (Weibel, 1959) produces stationary magnetic fields under conditions when the plasma exhibits certain anisotropies in flow and/or temperature. It is driven either by electrons or ions with the ion instability being much weaker than the electron instability.

The original proposal by Weibel (1959) referred to a temperature anisotropy in the unmagnetised electron distribution providing the free energy for a stationary (very low frequency $\omega \sim 0$) magnetic instability. Anisotropies in flow refer to streaming or beams. So far application was mostly intended in either laser (inertial) plasma fusion or violent conditions present in astrophysical systems.

Figure 1 sketches the "ion-diffusion region" (conventionally called so even though there is no diffusion) of a thin collisionless plane current sheet. The ions become non-magnetic here. The electrons continue their inward $\boldsymbol{E} \times \boldsymbol{B}$-drift motion transporting the magnetic field to the centre of the thin current sheet. Close to the centre in a region of size of few λ_e the electrons become demagnetised while maintaining their inward velocity $\mathbf{V} = \pm V_b \hat{\mathbf{z}}$ (with $V_b \lesssim V_n$) on both sides of the current sheet. The two flows pass across each other without (direct) interaction thereby realising a counter-streaming electron-beam configuration which according to Weibel (1959) and Fried (1959) may become electromagnetically unstable.

The Weibel instability generates a non-oscillating ($\omega \sim 0$) transverse magnetic field $\mathbf{b}$ with $\mathbf{k}$-vector about

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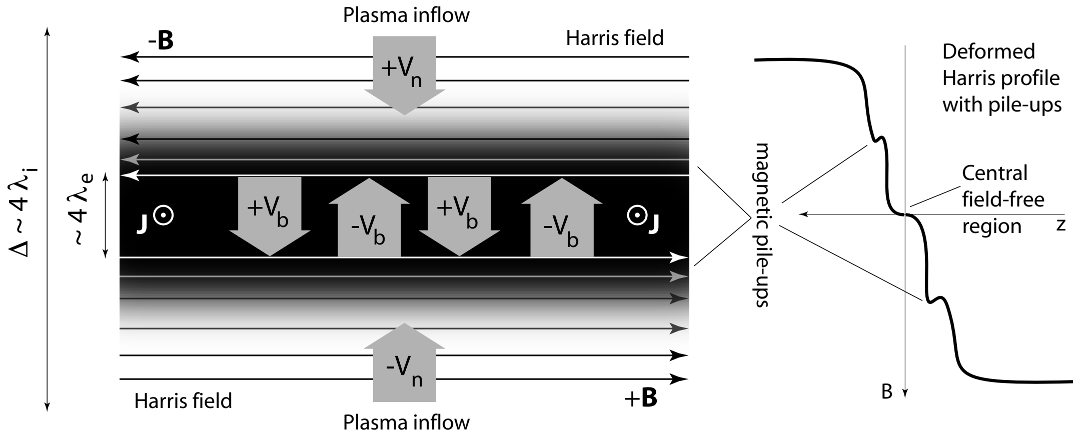

Fig. 1. Sketch of a homogeneous collisionless current sheet with plasma inflow from both sides and central electron inertial region. The central sheet current **J** is crossed by two (symmetric counter-streaming) electron flows which are unstable against the termal-anisotropic Weibel mode. On the right the magnetically deformed Harris profile is shown.

perpendicular to the electron beams ($k_\perp \gg k_\|$, subscripts refer to the direction of **V**) or temperature anisotropy $A = T_\|/T_\perp - 1 > 0$. In space plasma this instability is non-relativistic and weak. Under certain conditions its effect may be not negligible. Investigating beam instability we work in the fluid approximation of cold ($T_b < T_e$) symmetric beams of density $N_b/2$ (the cold beam approximation being justified because the lobe electron inflow is indeed substantially colder than the background plasma), also for simplicity assuming that the plasma is cold as well, even though the centre of the reconnecting current layer contains a denser $N > N_b$ thermal plasma of temperature $T = T_e + T_i$ (to which we will return when estimating the thermal fluctuation level of the instability). At very low frequencies the electromagnetic dispersion relation factorises (Yoon and Davidson, 1987; Achterberg and Wiersma, 2007, and others).

In the slab geometry of Fig. 1, the factor describing plane electromagnetic fluctuations of frequency $\omega \approx 0$ becomes

$$\mathsf{D}_{xx}\mathsf{D}_{zz} - |\mathsf{D}_{xz}|^2 = 0 \tag{1}$$

where D_{ij} are the components of the dispersion tensor $\mathsf{D}(\omega, \mathbf{k}) = k^2c^2\mathsf{I} - \epsilon(\omega, \mathbf{k})$, with plasma dielectric tensor function $\epsilon(\omega, \mathbf{k})$. Under the assumed symmetric conditions $\mathsf{D}_{xz} \equiv 0$, and the dispersion relation simplifies to

$$\mathsf{D}_{zz} = n^2 - 1 + \sum_s {}^s\chi_{zz} = 0, \qquad n^2 = k^2c^2/\omega^2 \tag{2}$$

where n is the refraction index, **k** wave number, ω wave frequency, and ${}^s\chi_{ij}$ the susceptibility tensor of species s the only surviving component which, in a symmetric electron/electron-beam plasma configuration, is given by

$$\chi_{zz} = \frac{k^2V_b^2}{\omega^2}\frac{\omega_b^2}{\omega^2} + \frac{\omega_e^2}{\omega^2}\left(1 + \frac{m_e}{m_i}\right)^{-1} \tag{3}$$

Subscripts e and b indicate background and beam parameters, respectively, ω_b is the beam plasma frequency for symmetric beam density $N_\pm = N_b/2$, and in the last term the (negligibly small) neutralising background-ion contribution to the plasma frequency is taken into account for correctness in the electron-to-ion mass ratio term m_e/m_i. When the background plasma is at rest, the "wave" becomes non-oscillating with $\omega = \pm i\gamma_{\mathrm{W}}$ (otherwise when the plasma moves at velocity $\mathbf{V}_0$ the wave frequency will be Doppler shifted by the amount $\mathbf{k}\cdot\mathbf{V}_0$, a case that may be realised under magnetopause conditions, there with $\mathbf{V}_0$ being the magnetosheath flow velocity tangential to the magnetopause).

Solving for $\gamma_{\mathrm{W}} > 0$ yields the non-evanescent growth rate

$$\frac{\gamma_{\mathrm{W}}}{\omega_b} = \frac{V_b}{c}\left\{1 + \frac{\omega_b^2}{k^2c^2}\left[1 + \frac{2N}{N_b}\left(1 + \frac{m_e}{m_i}\right)^{-1}\right]\right\}^{-\frac{1}{2}} \tag{4}$$

It maximises when the second term in the braced expression becomes small

$$k\lambda_{eb} \gg [1 + (2N/N_b)]^{\frac{1}{2}} > 1 \tag{5}$$

which is the case at relatively short wavelengths and yields

$$\gamma_{\mathrm{W,max}} \approx \omega_b V_b/c \sim 0.6 \quad \mathrm{s}^{-1} \tag{6}$$

when we use $V_b \sim 10$ km/s, and $N_b \sim 0.1N \sim 10^5\,\mathrm{m}^{-3}$.

The Weibel instability thus leads to comparably small-scale magnetic structures populating the beam-electron inertial range of size $\lambda \sim 2\pi\lambda_{eb} = 2\pi c/\omega_b$. Since the beam density is substantially less than the density of the ambient plasma in the centre of the current sheet, one has $\lambda_{eb} > \lambda_e$, the condition that $k^{-1} \lesssim \lambda_e$ is thus not in contradiction with the condition of maximum growth.

Because $kc/\omega > 1$, the unstably generated magnetic structures are of mixed polarisation, with longitudinal electric "wave" field component $\mathbf{e}_\ell || \mathbf{k}$ which in the symmetric beam case is small. The transverse electric field component $\mathbf{e}_{\mathrm{T}}$ is along the electron beam direction. Since the "wave"

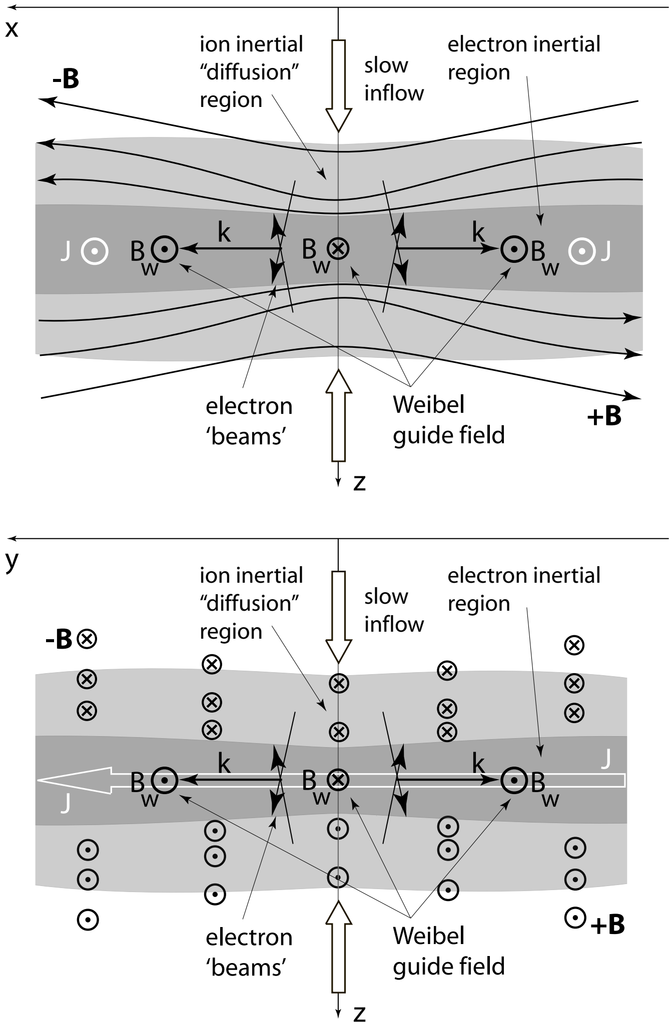

Fig. 2. Conditions inside a local electron inertial region. Inside the inertial region the electron components form two oblique beams which become smeared out by the two-stream instability. The resulting thermal-anisotropic Weibel instability generates a magnetic vortex of field $\mathbf{b}_\mathrm{W}$ with wavenumber $\mathbf{k}\perp\mathbf{V}_b$ perpendicular to the beam configuration. These vortices close in the current plane (x,y). The sense of the magnetic vortices alternates along the current direction. Shown is the part of the lowest order vortex where the field is along the sheet current $\mathbf{J}$. At a phase of $\pi/2$ along the current the Weibel fields point along the ambient magnetic field causing wobbling of the current layer.

magnetic field $\mathbf{b}$ satisfies the solenoidal condition $\mathbf{b}\cdot\mathbf{k}=0$, $\mathbf{k}\perp\mathbf{V}_b$, and $\omega\mathbf{b}=\mathbf{k}\times\mathbf{e}_\mathrm{T}$, the Weibel magnetic field lies in the plane perpendicular to the inflow into the current sheet (cf. Fig. 2).

The beam-excited Weibel-mode model might not be realistic. The two counter-streaming electron flows should readily thermalise due to the action of the high-frequency electrostatic two-stream instability which has large maximum growth rate

$$\frac{\gamma_{ts}}{\omega_e}\simeq\sqrt{3}\left(\frac{N_b}{2N}\right)^{\frac{1}{3}}\approx 0.6 \tag{7}$$

under conditions of $N_b\sim 0.1N$ with $k\sim\omega_e/V_b\sim 6\gg k_\mathrm{W}\sim\omega_e/c$. This growth rate is much faster than the above estimated Weibel growth rate $\gamma_{\mathrm{W,max}}$. Hence within one e-folding time of the Weibel field it smears out the beams into a flat-top distribution equivalent to heating the electrons and producing a weak temperature anisotropy $T_\parallel > T_\perp$.

After this happens, the original thermal-anisotropic Weibel mode (Weibel, 1959) takes over. It grows for $k\lambda_e < k_0\lambda_e=\sqrt{A_e}$ at maximum growth rate

$$\frac{\gamma_w}{\omega_e}\simeq A_e^{\frac{5}{2}}\sqrt{\frac{8}{27\pi}\frac{T_{e\perp}}{m_ec^2}}\approx 4\times 10^{-6} \tag{8}$$

which holds for a weak anisotropy $A_e\sim 0.1$ and $T_{e\perp}\sim 10\,\mathrm{eV}$. With plasma density of $N\sim 1\,\mathrm{cm}^{-3}$ this yields slow growth $\gamma_w\sim 0.2\,\mathrm{s}^{-1}$ of the Weibel field at maximum growing wave number $k_m=\lambda_e^{-1}\sqrt{A_e/3}\sim 3.5\times 10^{-5}\,\mathrm{m}^{-1}$ which corresponds to wavelengths $\lambda_w\sim 200\,\mathrm{km}$.

The Weibel magnetic field is directed either parallel or anti-parallel to the sheet current $\mathbf{J}=J\hat{\mathbf{y}}$ while being confined to the electron inertial zone. Such a field has the required properties of a guide field. Since it cannot be stronger than the magnetic field B_0 in the external inflow region, the Weibel instability generates weak guide fields only satisfying $b_\mathrm{W}/B_0<1$.

The above smallness condition on its wavelength implies that the Weibel guide field forms a comparably short-scale wavy magnetic structure along the sheet current, thereby structuring the current sheet magnetically in the direction of the current flow. Its transverse electric field $\mathbf{e}_\mathrm{T}||\mathbf{V}_b$ component along the electron beam inflow direction is confined to the electron inertial region, while the longitudinal electric field is in the direction $\mathbf{e}_\ell\perp\mathbf{V}_b$.

The condition that the magnetic field be free of divergence forces the Weibel magnetic field to form closed magnetic vortices in the (x,y)-plane. Along the sheet current they periodically amplify and weaken the external magnetic field on one side of (above or below) the current layer producing a spatial oscillation around the symmetry plane of the current layer of wavelength of the Weibel magnetic vortices.

3 Thermal Weibel level

So far we have been dealing with linear growth of the Weibel instability. In order to obtain its saturation level one needs to investigate the nonlinear evolution of the Weibel instability. Here we ask for how long it takes the Weibel instability to grow from thermal fluctuation level until reaching any measurable magnetic field strength.

The magnetic spectral energy density $\langle b_i b_j \rangle_{k\omega}$ of thermal fluctuations in an isothermal plasma is determined from Eq. (2.52) in Sitenko (1967) as

$$\langle b_i b_j \rangle_{k\omega} = \frac{\mu_0 T n^2}{\omega}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\frac{\mathrm{Im}\epsilon_\perp(\omega,\mathbf{k})}{|\epsilon_\perp - n^2|^2} \tag{9}$$

In a thermally anisotropic plasma the temperature T is replaced by the effective temperature $T_\perp T_\parallel/(T_\perp + T_\parallel)$ in this expression.The transverse dielectric response function $\epsilon_\perp(\omega,\mathbf{k})$ in the anisotropic case reads

$$\epsilon_\perp = 1 - \frac{\omega_e^2}{\omega^2}\left\{1 - \frac{T_\parallel}{T_\perp}\left[1 - \Phi(z) + i\pi^{\frac{1}{2}} z \mathrm{e}^{-z^2}\right]\right\} - \frac{\omega_i^2}{\omega^2} \tag{10}$$

Here $z = \omega/\sqrt{2}kv_{e\perp}$ is a variable that vanishes with $\omega \to 0$, $n^2 \equiv (kc/\omega)^2 = \epsilon_\perp$ is the refraction index of transverse fluctuations, and the real function $\Phi(z) \approx 2z^2$ for $z \ll 1$.

With the help of these expressions the spectral energy density $\langle |\mathbf{b}|^2 \rangle_{k\omega}$ can be brought into the form

$$\langle |\mathbf{b}|^2 \rangle_{k\omega} = \frac{\mu_0}{\omega_e}\frac{\sqrt{2\pi}(c/v_{e\parallel})T_\perp[\theta/(1+1/\theta)]\kappa \mathrm{e}^{-z^2}}{[\kappa^2 - \tilde{\omega}^2 + 1 - \theta(1-\Phi)]^2 + \pi\theta^2 z^2 \mathrm{e}^{-2z^2}} \tag{11}$$

where $\kappa \equiv k\lambda_e, \tilde{\omega} \equiv \omega/\omega_e, \theta \equiv T_\parallel/T_\perp = A_e + 1$, and $\lambda_e = c/\omega_e$ is the background plasma electron inertial scale. This holds for $\omega \simeq 0$.

The spectral energy density of the Weibel field is obtained in the limit $z = \omega = 0$, which yields

$$\langle |\mathbf{b}|^2 \rangle_{k0} = \frac{\mu_0}{\omega_e}\sqrt{\frac{\pi T_\perp}{m_e c^2}}\frac{m_e c^2 (A_e+1)^2 \kappa}{(A_e+2)[\kappa^2 - A_e - \mu]^2} \tag{12}$$

Here the ion contribution has been retained in the term $\mu \equiv m_e/m_i$. It prevents the fluctuations to diverge at $\kappa \to 0$ in the isotropic case $\theta = 1$ which scales as k^{-3}. (The pole at $\kappa^2 = A_e + \mu$ disappears if full ion dynamics is included.)

In a current sheet of density $N \sim 10^6\,\mathrm{m}^{-3}$, temperature $T \sim 0.1\,\mathrm{keV}$ which implies electron inertial and Debye lengths $\lambda_e \approx 6\,\mathrm{km}$ and $\lambda_D \approx 7.5\,\mathrm{m}$, respectively, presumably corresponding to conditions in the near-Earth magnetotail current sheet, the Weibel spectral energy density at maximum growing wave number k_m becomes

$$\langle |\mathbf{b}|^2 \rangle_{k_m 0} \approx 1.6 \times 10^{-27} A_e^{\frac{3}{2}} \frac{(A_e+1)^2}{A_e+2}\sqrt{T_{[\mathrm{eV}]}}\ \frac{\mathrm{V}^2\mathrm{s}^3}{\mathrm{m}} \tag{13}$$

where $T_{[\mathrm{eV}]}$ is the background plasma electron temperature in eV.

4 Growth time in the magnetotail current layer

We are interested in the time required for the Weibel instability to grow to measurable guide magnetic field values under conditions in the magnetotail reconnection region. The maximum growth rate of the thermal-anisotropic Weibel instability was found to

$$\gamma_{\mathrm{W,max}} \sim 0.2\,\mathrm{s}^{-1} \tag{14}$$

Instability growth implies for the time evolution of the spectral energy density of the magnetic field

$$\langle |\mathbf{b}(t,\mathbf{k},0)|^2 \rangle \approx \left\langle |\mathbf{b}^2(\mathbf{k},0)| \right\rangle_{\mathrm{W,th}} \exp(2\gamma_{\mathrm{W}} t) \tag{15}$$

where on the left is the linearly growing spectral density at time t, and on the right the thermal level from where the instability starts growing. (No nonlinear saturation effects are included at this time.) An observable magnetic field in the magnetotail should be roughly of order $b \sim 1\,\mathrm{nT}$, a value which one may use to find the growth time τ_{W} required to reach this field strength

$$\tau_{\mathrm{W}} \approx \frac{1}{2\gamma_w}\ln\frac{\langle |\mathbf{b}^2(\mathbf{k},\tau_w)| \rangle}{\langle |\mathbf{b}^2(\mathbf{k},0)| \rangle_{\mathrm{W,th}}} \tag{16}$$

Inserting for the thermal energy density $\left\langle |\mathbf{b}^2(\mathbf{k},0)| \right\rangle_{\mathrm{W,th}}$ from $\langle |\mathbf{b}|^2 \rangle_{k_{\mathrm{m}}0}$ and using the spectral energy density of a $b \sim 1\,\mathrm{nT}$ magnetic field, $\langle b^2_{\mathrm{1nT}} \rangle_{k0} \approx 4.3 \times 10^{-12}\,\mathrm{V}^2\mathrm{s}^3/\mathrm{m}$, the typical growth time to reach this magnetic field level at a weak anisotropy of just $A_e \sim 0.1$ becomes

$$\tau_{\mathrm{W,max}} \sim 92\ \mathrm{s} \tag{17}$$

corresponding to ~ 50 e-folding times. This time of roughly $\tau \sim 1.5\,\mathrm{min}$ is not unreasonable for the processes in the magnetotail current sheet.

5 Conclusions

The result of this investigation is that inside the electron inertial region (of transverse size of a few λ_e) in the current sheet, the inflow of electrons into the sheet from its two sides (in the geomagnetic tail from the lobes) may well be capable of self-consistently generating a weak magnetic guide field via the thermal-anisotropic Weibel instability. Given sufficient time of, say, 50 e-foldings weak guide fields will evolve on wavelengths the order of a few 100 km.

The Weibel-guide field B_g is limited to be weaker than the ambient external magnetic field B_0. In the symmetric magnetospheric tail current sheet it may reach up to $\lesssim 10\%$ of the ambient field unless nonlinear effects set on earlier to saturate the field on a lower level. Such nonlinear processes have not been investigated here. They can in principle be treated by assuming a stationary final state and calculating the thermal saturation level of the Weibel instability.

Guide fields are important in the dynamics of the current sheet and in particular for reconnection. They have been observed in the geomagnetic tail in various cases (see e.g., Runov et al., 2003; Nakamura et al., 2006, 2008). They remagnetise the electron plasma in the central current region. Pointing along the electric field that drives the current, they cause particle acceleration, which amplifies the current, generates energetic particles, and cause a number of secondary effects that affect the stability of the plasma in the current sheet. The role of guide fields in collisionless magnetic reconnection and the various effects it may cause have in the past decade been thoroughly investigated in numerical simulations both for guide fields perpendicular (e.g., Ricci et al., 2003) and parallel (e.g., Pritchett, 2005; Cassak et al., 2007; Egedal et al., 2008; Le et al., 2009) to the sheet current.

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