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Field-aligned structure of poloidal Alfvén waves in a finite pressure plasma

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Abstract. This paper is devoted to analytical and numerical studies of the parallel structure of poloidal Alfvén waves in a finite pressure plasma. The effects of finite-beta plasma are especially essential near the magnetospheric equator, where an opaque (non-transparent) region for Alfvén waves can be formed. This region is bounded by two turning points which restrain penetration of the wave energy far from the ionosphere, and an Alfvén resonator appears on a part of the field line adjacent to the ionosphere. Due to this effect the ULF pulsations in the Northern and Southern Hemispheres can be non-conjugated. Another result is a peculiar field-aligned structure of the wave magnetic field: its fundamental harmonic must have three nodes, rather than one node as is the case in cold plasma.

Keywords. Magnetospheric physics (MHD waves and instabilities) – Space plasma physics (Kinetic and MHD theory; Waves and instabilities)

1 Introduction

The Alfvén waves in the magnetosphere, observed by satellite and ground magnetometers as Pc3-5 pulsations, are usually classified according to the predominant polarization in the plane transverse to geomagnetic field **B**: azimuthally polarized (toroidal), where the magnetic field vector oscillates in an azimuthal direction, and radially polarized (poloidal), where the magnetic field vector oscillates in a radial direction (Radoski, 1967). This difference in polarization reflects the difference of the wave transverse spatial structure: toroidal waves are more strongly localized in radial direction (that is their azimuthal wave number $m \simeq 1$), whereas poloidal waves are mostly localized in the azimuthal direction ($m \gg 1$). The poloidal Pc5 waves are a persistent element of a magnetic storm observed at its recovery phase. In earlier studies they were even named storm-time Pc5 pulsations (Barfield and McPherron, 1978). The poloidal Alfvén waves are strongly screened by the ionosphere from the ground magnetometers, so their transverse spatial structure was first studied using combined observations at the ionospheric STARE radar facility and geosynchronous satellite GOES-2 (Allan and Poulter, 1984; Walker et al., 1982). More recently these waves were observed with multi-satellite CLUSTER mission (Eriksson et al., 2005; Sinha et al., 2005; Schäfer et al., 2007, 2008) and SuperDARN radars (Fenrich et al., 2006; Yeoman et al., 2006).

Theoretical studies on the poloidal Alfvén modes predicted that this mode is very sensitive to the finite plasma pressure effect and magnetic field geometry. In a curved magnetic field with finite-β the usual Alfvén dispersion relation $\omega^2 = k_\parallel^2 A^2$ is replaced for the poloidal mode by $\omega^2 = k_\parallel^2 A^2 + H$ (e.g. Ohtani and Tamao, 2002), where the additional term H is named the ballooning term. In (Safargaleev and Maltsev, 1986; Golovchanskaya and Maltsev, 2005) this term was designated as ω_g^2, by analogy with the gravity mode in an inhomogeneous atmosphere. Its sign depends on the radial gradient of the plasma pressure P: for the inward gradient ∇P, $H<0$ and the system may be unstable due to ballooning (or convective) instability under a sufficiently steep plasma gradient (Liu, 1997): plasma confined inside the system tends to expand outward. At the outward gradient ∇P (e.g., the inner edge of the plasma sheet) $H>0$.

Many recent theoretical efforts were also devoted to the poloidal wave structure across magnetic shells. It was found that it corresponds to the traveling wave localized between two magnetic surfaces (Leonovich and Mazur, 1993). This effect is substantially strengthened in the presence of the azimuthal inhomogeneity of plasma (Klimushkin et al., 1995) and finite plasma pressure (Mager and Klimushkin, 2002; Klimushkin et al., 2004).

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However, the field aligned structure of the poloidal waves in the magnetosphere was never examined in full detail. Implicitly, it is commonly assumed that this structure is more or less similar to that of the shear toroidal waves and has a typical scale of about the field line length between the conjugate ionospheres. However, at high latitudes where the magnetic field lines are essentially non-dipole, a field aligned structure can be different. As Pilipenko et al. (2005) showed, a steep "bending" of field lines in some regions of the magnetosphere can cause a partial reflection of poloidal Alfvén waves, thus enabling the formation of Alfvén quasi-resonator on a part of a field line. However, the model developed in Pilipenko et al. (2005) was not fully self-consistent since a drastic change of the magnetic field geometry necessarily should involve electric currents and finite plasma pressure to maintain the MHD equilibrium, which were neglected.

In this paper we examine a possible field-aligned structure of Alfvén waves taking into account the plasma inhomogeneity across magnetic shells and in the direction along an external magnetic field, the field line curvature, and finite plasma pressure $\beta \equiv 8\pi P/B^2$.

2 Model and principal equations

Here we present the basic equations describing poloidal Alfvén waves in a finite pressure plasma immersed into a curved magnetic field. Let us introduce an orthogonal curvilinear coordinate system $\{x^1, x^2, x^3\}$, in which the field lines play the role of coordinate lines x^3, the stream lines are coordinate lines x^2, and the surfaces of constant pressure (magnetic shells) are coordinate surfaces x^1=const. The coordinates x^1 and x^2 represent the radial and azimuthal coordinates, and we use the parameter L and azimuthal angle φ, respectively, to represent them. The physical length along a field line is expressed in terms of an increment of the corresponding coordinate as $dl_\parallel = \sqrt{g_3}dx^3$, where g_3 is the component of the metric tensor, and $\sqrt{g_3}$ is the Lamé coefficient. Similarly, for the transverse direction one has $dl_1 = \sqrt{g_1}dx^1$, and $dl_2 = \sqrt{g_2}dx^2$.

The electric field of Alfvén waves can be represented in the form

$$\mathbf{E} = -\nabla_\perp \Phi,$$

where $\nabla_\perp = \mathbf{e}_x \partial/\partial x^1 + \mathbf{e}_y \partial/\partial x^2$ is the transverse operator, and Φ is a scalar function ("potential"). Time-space variations of Φ in an axisymmetric model of the magnetosphere can be specified in the form $\propto \exp(-i\omega t + im\varphi)$, where m is the azimuthal wave number.

In the poloidally polarized Alfvén waves the physical components of the wave electric field has predominant azimuthal component and the magnetic field has predominant radial component as follows

$$E \propto \frac{1}{\sqrt{g_2}}\Phi, \qquad b \propto \frac{1}{\sqrt{g_2}}\frac{\partial}{\partial l_\parallel}\Phi. \tag{1}$$

In an inhomogeneous plasma, the Alfvén waves are coupled with the fast and slow compressional MHD modes. The coupling with the fast mode can be neglected because a high azimuthal wave number m makes it ineffective (e.g. Allan and Poulter, 1984). In a finite-β plasma, the coupling with the slow mode due to plasma pressure and field line curvature must be considered (Southwood and Saunders, 1985; Walker, 1987; Woch et al., 1988). The coupling with a slow mode influences both radial structure of the Alfvén wave and its eigenfrequencies (Klimushkin, 1998). However, even for equatorial region of the magnetosphere, where β can be as high as about unity, the eigenfrequenices of Alfvén and slow modes differ for more than two orders of magnitude (Cheng et al., 1993). The reason seems to lie in the fact that at high latitudes β decreases rapidly because of the crowding of field lines, so its field-line integrated value is small. This huge difference between eigenfrequencies of Alfvén and slow modes enables one to treat the Alfvén mode separately. In this case, the coupling results only in emergence of a compressional component attached to the Alfvén wave field (Mager and Klimushkin, 2002), also indirectly influencing its eigenfrequency (Klimushkin, 1998). Furthermore, if the poloidal Alfvén mode is considered, the problems of finding the parallel and transverse structure of the wave field can be treated separately, as described by Leonovich and Mazur (1993), Vetoulis and Chen (1996) and Klimushkin et al. (2004).

An equation describing the field-aligned structure of Alfvén waves in a finite pressure plasma may be written as follows (Denton, 1998; Denton and Vetoulis, 1998; Mager and Klimushkin, 2002; Klimushkin et al., 2004)

$$\sqrt{\frac{g_1}{g_2}}\frac{\partial}{\partial l_\parallel}\sqrt{\frac{g_1}{g_2}}\frac{\partial}{\partial l_\parallel}\Phi + \frac{g_1}{g_2}\left(\frac{\omega^2}{A^2} + \eta\right)\Phi = 0. \tag{2}$$

Here A is the Alfvén speed, ω is the wave frequency, and η is the contribution of a hot plasma

$$\eta = -2K\left(\frac{4\pi}{c}\frac{j_\perp}{B} + K\gamma\beta\right),$$

where K is the local curvature of a field line, and γ is the adiabatic index. The equilibrium transverse current $j_\perp$ is related to the hot plasma pressure gradient: $j_\perp \propto \partial P/\partial x^1$. A part of the η term is caused by the coupling with the slow mode (Klimushkin, 1998).

Let us change the field-aligned variable $l_\parallel$ to ξ in Eq. (2) as follows

$$dl_\parallel = \sqrt{\frac{g_1}{g_2}}d\xi.$$

As a result, Eq. (2) can be presented in the form of the Schrödinger equation:

$$\frac{\partial^2}{\partial \xi^2}\Phi + \frac{g_1}{g_2}\frac{\omega^2 - H(\xi)}{A^2}\Phi = 0, \tag{3}$$

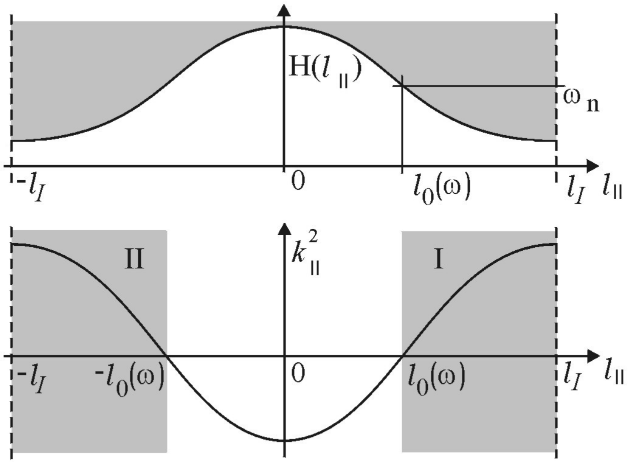

Fig. 1. Sketch demonstrating the dependence of H (top) and $k_\parallel^2$ (bottom) on $l_\parallel$ and showing probable locations of the transparent and opaque regions. Here $\pm l_0$ are the turning points for a harmonic with eigenfrequency ω_n, $\pm l_I$ are the intersection points of a field line with the ionosphere. The shaded areas I and II correspond to the transparent regions.

where

$$H(\xi) = -A^2\eta = A^2 2K\left(\frac{4\pi}{c}\frac{j_\perp}{B} + K\gamma\beta\right). \quad (4)$$

For the ideally conducting ionosphere, the boundary conditions for the potential are written as

$$\Phi|_{\pm l_I} = 0, \quad (5)$$

where $\pm l_I$ denotes the points of the intersection of a field line with the ionosphere.

3 Analysis of the field-aligned structure in the WKB approximation

To give some insight into the physics of the Alfvén wave propagation in a finite-β plasma Eq. (3) will be solved first in the WKB approximation. Strictly speaking, this approximation cannot be applied for fundamental wave harmonics, but it usually gives correct qualitative results for them, so it can be used for heuristic purposes. The resulting field-aligned component of the wave vector is determined from the expression

$$k_\parallel^2(\xi) = \frac{\omega^2 - H(\xi)}{A^2}. \quad (6)$$

In a cold plasma, when H=0, an Alfvén wave has no turning point, that is $k_\parallel^2$>0 everywhere. Thus the entire field line between the conjugate ionospheres is transparent for the Alfvén wave and the oscillation has a familiar sinusoidal structure. Let us denote the harmonic wave number N as a number of nodes plus 1 (that is, the fundamental harmonic has N=1). Then, the eigenfrequency of the poloidal mode Ω_{PN} is approximately proportional to this number: $\Omega_{PN} \propto N$ (e.g. Leonovich and Mazur, 1993).

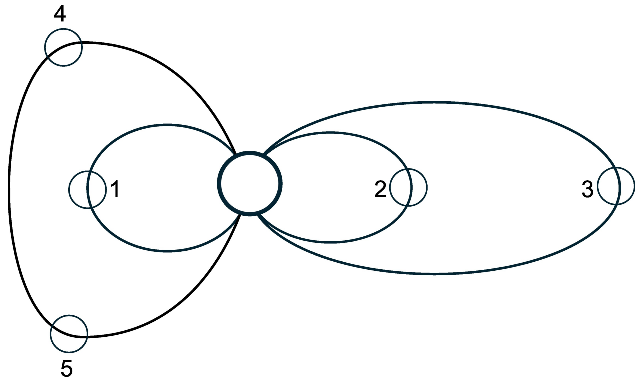

Fig. 2. Cartoon showing possible locations of opaque regions for poloidal Alfvén waves in the magnetosphere.

In a finite pressure plasma, the sign and magnitude of the ballooning term in Eq. (3) determines the field-aligned structure of the wave disturbance. An interesting effect occurs under the outward gradient of pressure, $\partial P/\partial x^1$>0. Let us find the region along the field line where the function $H(\xi)$ has its maximum. Usually, the second term in Eq. (4) dominates, thus H varies along the parallel coordinate as $H \propto K^2\rho^{-1}$, where ρ is the plasma density. The plasma pressure is to be constant along the field line, whereas the field line curvature K and inverse density ρ^{-1} both peak near the equator. Therefore, the function $H(\xi)$ has a maximum at the equator (Fig. 1) and H>0.

In this case Eq. (6) predicts that the function $k_\parallel^2(\xi)$ can change its sign along the field line. The point l_o, where $k_\parallel^2(l_0)$=0, is the turning point for a poloidal Alfvén wave. The region where $k_\parallel^2$<0, is an opaque region, where the wave becomes evanescent, and the region where $k_\parallel^2$>0 is transparent for waves.

When H has a maximum at the magnetospheric equator, which is usually the case, the opaque region (marked as spots 1–3 in Fig. 2) is located in the vicinity of the top of a field line. Thus, two sub-resonators (regions I and II in Fig. 1) are formed, bounded by the ionosphere and the turning point near the equator, $\pm l_0$.

Let us introduce auxiliary functions $F_n^{(\pm)}$ serving as eigenfunctions of Eq. (3). Each of them has the boundary conditions at the conjugated ionospheres

$$F_n^{(\pm)}\Big|_{\pm l_I} = 0,$$

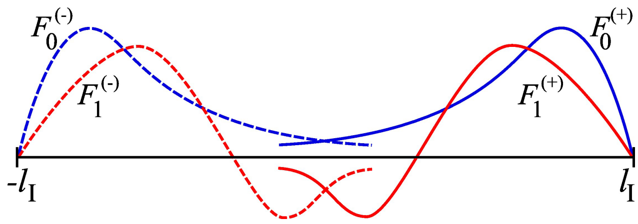

Fig. 3. Two principal harmonics of the functions $F_n^{(\pm)}$.

and decays beyond its transparent region. For example, the function $F_n^{(+)}$ has the form (Fig. 3)

$$F_n^{(+)} = \frac{C}{|k_\parallel^2|^{1/4}} \begin{cases} \frac{1}{2}\exp(-\int\limits_{l_\parallel}^{l_0} \sqrt{-k_\parallel^2 dl_\parallel}), & 0 < l_\parallel < l_0; \\ \sin(\int\limits_{l_0}^{l_\parallel} \sqrt{k_\parallel^2} dl_\parallel + \pi/4), & l_0 < l_\parallel < l_I. \end{cases}$$

Here $k_\parallel = k_\parallel(\omega_n)$, and $l_0 = l_0(\omega_n)$ is the turning point obtained from the condition $k_\parallel(\omega_n)=0$. The n-th harmonic eigenfrequency ω_n is obtained from Bohr–Sommerfeld quantization condition

$$\int\limits_{l_0(\omega_n)}^{l_I} k_\parallel(\omega_n) dl_\parallel = \pi \left(n + \frac{3}{4} \right). \qquad (7)$$

If the opaque region is wide enough, the resonators in the Northern and Southern Hemispheres are practically not coupled. The function $F_n^{(-)}$ is determined simply as $F_n^{(-)}(l_\parallel) = F_n^{(+)}(-l_\parallel)$, and both functions have the same set of eigenfrequencies.

The functions $F_n^{(\pm)}$ are the building blocks for the construction of the function Φ_N, the complete solution of Eq. (3) with boundary conditions (5). These wave functions Φ_N are composed from the symmetric and antisymmetric combinations of $F_n^{(-)}$ and $F_n^{(+)}$. For example (Fig. 4), the fundamental harmonic (N=1) is

$$\Phi_1 = \frac{1}{\sqrt{2}}[F_0^{(+)} + F_0^{(-)}],$$

the second harmonic (N=2) is

$$\Phi_2 = \frac{1}{\sqrt{2}}[F_0^{(+)} - F_0^{(-)}], \text{ etc.}$$

The eigenfrequency is determined by the condition (7). The eigenfrequencies for the functions $F_n^{(-)}$ ans $F_n^{(+)}$ are the same, thus in the presence of an opaque region like in Fig. 1 the frequencies of the lowest two harmonics of the poloidal eigenfunctions Φ_N have almost the same frequencies: $\Omega_{P1} \approx \Omega_{P2}$.

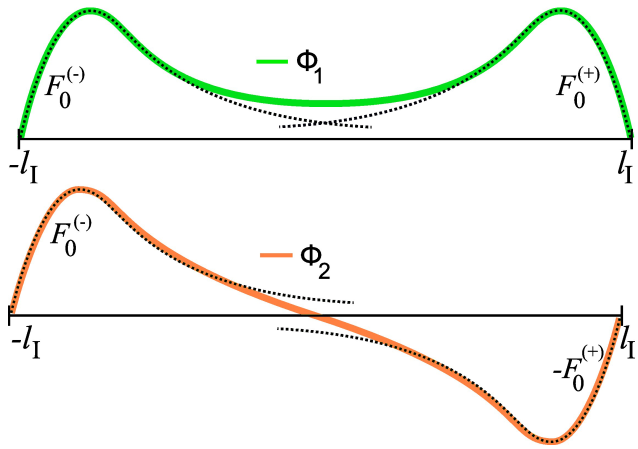

Fig. 4. Two principal harmonics of the wave potential Φ.

In a case of coupled oscillations in resonators I and II, one should take into account their coupling caused by the barrier tunneling. Due to the tunneling, each eigenfrequency ω_n splits into ω_{n+} and ω_{n-}:

$$\omega_{n\pm}^2 = \omega_n^2 \pm \Delta\omega_n^2, \qquad (8)$$

where

$$\Delta\omega_n^2 \approx \exp(-\int\limits_{-l_0}^{l_0} \sqrt{-k_\parallel^2 dl_\parallel}) \cdot [\int\limits_{l_0}^{l_I} A^{-2} k_\parallel^{-1} dl_\parallel]^{-1}.$$

In Eq. (8) the lower (upper) sign corresponds to the symmetric (antisymmetric) modes. Thus, the frequencies of the fundamental mode Φ_1 and second harmonic Φ_2 must be somewhat different:

$$\Omega_{P1}^2 = \omega_0^2 - \Delta\omega_0^2, \qquad \Omega_{P2}^2 = \omega_0^2 + \Delta\omega_0^2 \qquad (9)$$

(note that index "0" here denotes the principal n-harmonic, i.e. n=0).

The width of the opaque region decreases with the harmonic number N. The difference in the frequencies of the pair of the neighboring harmonics (N=1 and 2, N=3 and 4, etc.) grows with the decrease of the opaque region width. So, the lowest harmonics must have the smallest difference between the frequencies.

The structure of the wave potential Φ_1 for the fundamental mode must be most deformed in comparison with the cold plasma case, since the opaque region is largest and "deepest" for it. Furthermore, in the magnetosphere the opaque region may not exist for higher harmonics. In a cold plasma, the amplitude of the electric field of the fundamental harmonic (N=1) has a maximum at the equator. In a finite pressure plasma with an opaque region near the top of a field line, it has a minimum at the equator and two maxima in both hemispheres, that is a total of three extremes. The wave transverse

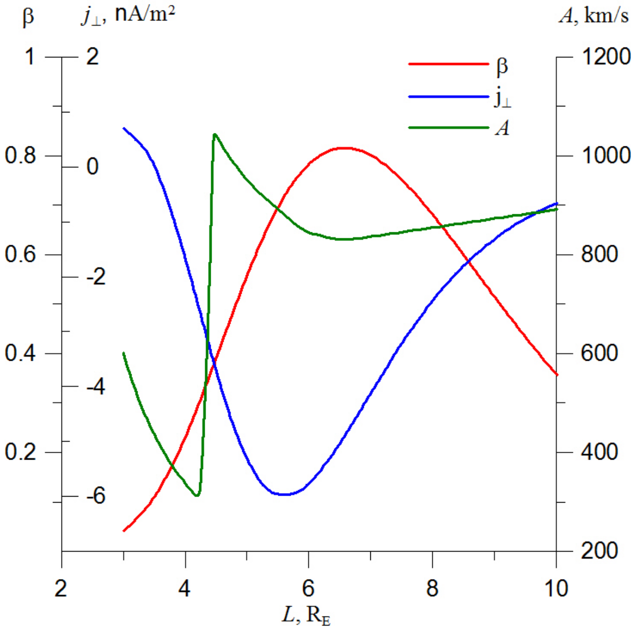

Fig. 5. The radial profiles of Alfvén speed $A(L)$, $\beta(L)$ and transverse current $j_\perp(L)$ in the equatorial plane. Here $L=r/R_E$ is the magnetic shell parameter.

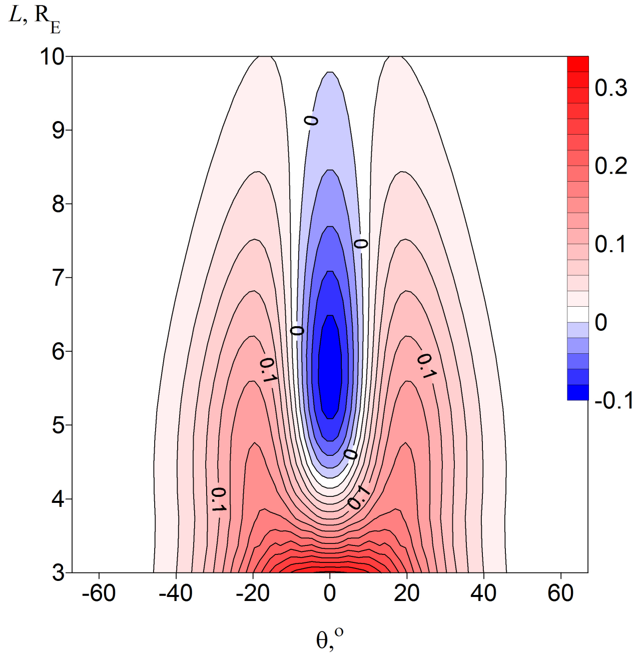

Fig. 6. Dependence of $k_\parallel^2$ (in R_E^{-2}) on geomagnetic latitude θ and parameter $L=r/R_E$ for the fundamental harmonic.

magnetic field b is determined by the field-aligned derivative of potential Φ (see Eq. 1), therefore, the magnetic field of the fundamental harmonic must have three nodes, rather than one node as in a cold plasma. The magnetic field of the second harmonic must have only two nodes, just like in the $\beta=0$ case.

4 Numerical solution for a dipole magnetosphere with finite plasma pressure

Since the WKB approximation cannot give the quantitative results for the fundamental modes, to find the wave structure and eigenfrequencies we need to perform a numerical solution of the wave Eq. (2).

Numerical studies by Cheng (1992) found that even for equatorial plasma β as high as unity, at geosynchronous L-shells the field lines almost coincide with the dipole ones, although the field value changes more significantly. Another study (Chan et al., 1994) showed that finite pressure shifts the top of field line at most for half R_E. Thus, the dipole model of the magnetosphere can be used as a reasonable approximation even for a finite-β plasma. In this model, the variation of the curvature along the field line is expressed as

$$K=\frac{3}{L\cos\theta}\frac{1+\sin^2\theta}{(1+3\sin^2\theta)^{3/2}},$$

where θ is the geomagnetic latitude, which can be used instead of the longitudinal coordinate. The physical length along a field line is $dl_\parallel=h_\theta d\theta$, where $h_\theta=L\cos(\theta)(1+3\sin^2\theta)^{1/2}$, thus the wave magnetic field is

$$b\propto\frac{1}{h_\theta\sqrt{g_2}}\frac{\partial}{\partial\theta}\Phi.$$

The longitudinal profile of the Alfvén speed is given by the formula

$$A(\theta)\propto\frac{B}{B_{eq}}\cos^3\theta,$$

where B_{eq} is the magnetic field in the equator plane. Note that the longitudinal variation of the Alfvén speed is the main reason of the peak of the function H near the equator due to low plasma density in that region. We set the radial profiles of Alfvén speed A (we model the plasmapause by a sharp change of Alfvén speed near $L=4.3$) and plasma pressure, which gives us the radial profiles of $\beta(L)$ and $j_\perp(L)$. The plasma pressure and current profiles are related as $\nabla P=\boldsymbol{j}_\perp\times\boldsymbol{B}$ (SI units). The resulted plots are presented in Fig. 5. They are in a reasonable agreement with the observational data (see, e.g. Lui et al., 1987).

The function $k_\parallel^2(\theta,L)$ calculated for the fundamental harmonic ($N=1$) is shown in Fig. 6. From this figure we notice that there is an opaque region ($k_\parallel^2<0$) near the equator, in accordance with the analysis made in the previous section. This region is extended along the radial coordinate. The field-aligned structure of the fundamental harmonic is shown in Fig. 7 for an L-shell with the "deepest" opaque region (for

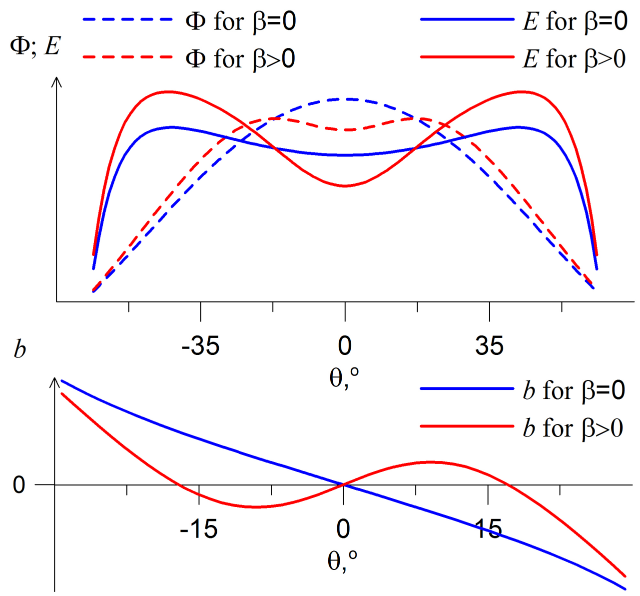

Fig. 7. The field-aligned structure of the fundamental harmonic at L=6 shell in plasma with β=0 and β>0. Here Φ is the "potential", E and b are the wave electric and magnetic fields, accordingly, and θ is the geomagnetic latitude.

comparison, the wave parallel structure is also shown for the cold plasma case, η=0, for the same L-shell). In accordance with the results of the WKB analysis, there are a minimum of the potential Φ near the equator and three nodes of the wave magnetic field (compare Figs. 4 and 7). Note that the wave electric field E (unlike the potential Φ) has a minimum at the equator even in the β=0 case due to Lamé coefficient behaviour as $\sqrt{g_2}=L\cos^3\theta$ in Eq. (1) (Cummings et al., 1969). It is a representation of the effect described by Pilipenko et al. (2005), but the figure shows that in the finite pressure case the minimum is much deeper.

5 Discussion: analysis of assumptions and possible observational consequences

In the analysis above we neglected the coupling of the Alfvén wave with the slow mode because of huge difference of their frequencies. Nevertheless, it is worth studying whether this coupling can significantly change our results. For this purpose, let us write a dispersion equation for the coupled poloidal Alfvén and slow modes (e.g. Klimushkin and Mager, 2008):

$$(\omega^2 - k_\parallel^2 A^2) - \frac{2P'K}{\rho} = \frac{\omega^2 - k_\parallel^2 v_s^2}{4\omega^2 v_s^2 K^2}. \qquad (10)$$

Here the prime means differentiating with respect to the radial coordinate, $V_s=SA/\sqrt{S^2+A^2}$ is slow magnetosonic speed, $S=\sqrt{\gamma P/\rho}$ is the sound speed.

Since the parallel reflection points are determined as the points where the condition $k_\parallel$=0 is satisfied, it is easily can be seen that the reflection still takes place where ω^2=H, with the only difference that the sound speed is replaced by the slow magnetosonic speed, $S\to V_s$. Therefore, we may conclude that the coupling with the slow mode cannot drastically change the results: it does not significantly shift the parallel reflection point found in this study, nor it introduces additional reflection points.

The account of the finite-pressure effect has shown that regions with strongly bent field lines may not just partially reflect poloidal Alfvén waves, as was predicted in Pilipenko et al. (2005), but even impose a stop-band for them. We have to indicate that the effect of partial or even total Alfvén wave reflection from this region may occur for an azimuthally-small scale (poloidal) mode. The Alfvénic disturbances of this type are to accompany localized bursty or non-steady processes in the nighttime magnetosphere. A possibility of this effect for an azimuthally large-scale (toroidal) mode needs a special consideration, but most probably it is to be much less pronounced. But it can be observed also in the compressional component of the magnetic field $b_\parallel$ which is coupled with electric field in a finite-pressure plasma as

$$b_\parallel = \frac{c}{\omega}\frac{1}{\sqrt{g_1 g_2}}\frac{\eta}{2K}m\Phi$$

(e.g. Klimushkin et al., 2004). This component is often measured with satellites.

The presence of an opaque region for Alfvén waves in the near-equatorial region may practically decouple the field line oscillation in the Northern and Southern magnetospheric hemispheres. As a result, non-conjugate long-period (Pc5–6, Pi2–3) pulsations can be observed at high latitudes in the night time magnetosphere. In fact, the wave process will not involve the entire field line, but only that part of it, which is terminated by the high-latitude ionosphere and reflection point above/below the equatorial current sheet, in the Northern or Southern Hemispheres, depending on the location of the driver. This effect will deteriorate the conjugacy of all wave-related processes: Alfvénic aurora, periodic modulation of particle fluxes and precipitation, etc. In contrast with the traditional notions of the field line resonance theory, a standing Alfvénic structure can be formed not between the conjugated ionospheres, but along a part of a field line far extended into the magnetotail, e.g. between the ionosphere and the current sheet.

Let us briefly mention the possibility of the field-aligned Alfvénic resonator occurrence in other parts of the magnetosphere. In the day-side high-latitude entry regions, a steep bending of field lines and local increase of curvature K may occur (see spots 4 and 5 in Fig. 2). In these regions, the function H takes large values due to large field line curvature K, and, consequently, value of $k_\parallel^2$ can be negative. Thus, a resonator can be formed between these two regions.

At high latitudes in the nighttime magnetosphere the magnetic field topology is essentially different from a dipole: the field lines are strongly stretched into the magnetotail under the influence of the magnetotail currents and are sharply bent in the equatorial plane (see Figs. 9 and 14 from Wanliss et al., 2002). The possibility of such a resonator forming in these regions was mentioned earlier by Safargaleev and Maltsev (1986). This resonator can exist also in night-side regions (in the near tail). To consider this possibility, let us rewrite the ballooning term (see Eq. 4) in another form:

$$H = \frac{2\gamma P}{\rho}\left(\frac{1}{2\gamma}\frac{K}{\sqrt{g_1}}\frac{\partial \ln P}{\partial x^1} + K^2\right) \tag{11}$$

Near the top of the tail field line, the field line curvature is rather high, but the pressure gradient is negative and can also be rather high. Thus, the first and second terms in brackets can be almost in balance, or the first one can even overweight. This can result in the local minimum of the ballooning term near the equator, which leads to the positive value $k_\parallel^2 \equiv (\omega^2 - H)/A^2 > 0$ near the equator. This possibility, however, will be studied in more detail elsewhere.

6 Conclusions

The differential equation describing the field aligned structure of poloidal Alfvén waves in a finite pressure plasma contains an additional ballooning term. Under realistic conditions, this term results in the occurrence of the opaque region in the vicinity of the equatorial plane of the magnetosphere. Consequently, the wave is composed of two weakly related parts adjacent to the ionosphere with a dip of an amplitude in the equatorial region.

The predicted effect is to be most evident for field lines extended deep into the plasmasheet. Due to this effect non-conjugate long-period (Pc5–6 and Pi2–3) pulsations may be observed, or oscillations with a peculiar field-aligned structure: the wave magnetic field of the fundamental harmonic must have three nodes, rather than one node as is the in cold plasma.

The ability of the multi-systems (such as Cluster) to separate the spatial structure from the temporal evolution in the spacecraft reference system can be used to verify theoretical predictions of the spatial structure of the Pc5 waves in the magnetosphere.

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