Ann. Geophys., 26, 941–953, 2008
www.ann-geophys.net/26/941/2008/

Annales Geophysicae

Comparison of chaotic aspects of magnetosphere under various physical conditions using AE index time series

K. Unnikrishnan

School of Pure and Applied Physics, Mahatma Gandhi University, Priyadarshini Hills, Kottayam – 686 560, Kerala, India
also at: Department of Physics, N. S. S. Hindu College, Changanacherry – 686 102, Kerala, India

Received: 7 May 2007 – Revised: 6 March 2008 – Accepted: 10 March 2008 – Published: 13 May 2008

Abstract. The deterministic chaotic behaviour of magnetosphere was analyzed, using AE index time series. The significant chaotic quantifiers like, Lyapunov exponent, spatio-temporal entropy and nonlinear prediction error for AE index time series under various physical conditions were estimated and compared. During high solar activity (1991), the values of Lyapunov exponent for AE index time series representing quiet conditions (yearly mean = 0.5±0.1 min^{-1}) have no significant difference from those values for corresponding storm conditions (yearly mean = 0.5±0.17 min^{-1}). This implies that, for the cases considered here, geomagnetic storms may not be an additional source to increase or decrease the deterministic chaotic aspects of magnetosphere, especially during high solar activity. During solar minimum period (1994), the seasonal mean value of Lyapunov exponent for AE index time series belong to quiet periods in winter (0.7±0.11 min^{-1}) is higher compared to corresponding value of storm periods in winter (0.36±0.09 min^{-1}). This may be due to the fact that, stochastic part, which is D_{st} dependent could be more prominent during storms, thereby increasing fluctuations/stochasticity and reducing determinism in AE index time series during storms. It is observed that, during low solar active period (1994), the seasonal mean value of entropy for time series representing storm periods of equinox is greater than that for quiet periods. However, significant difference is not observed between storm and quiet time values of entropy during high solar activity (1991), which is also true for nonlinear prediction error for both low and high solar activities. In the case of both high and low solar activities, the higher standard deviations of yearly mean Lyapunov exponent values for AE index time series for storm periods compared to those for quiet periods might be due to the strong interplay between stochasticity and determinism during storms.

It is inferred that, the external driving forces, mainly due to solar wind, make the solar-magnetosphere-ionosphere coupling more complex, which generates many active degrees of freedom with various levels of coupling among them, under various physical conditions. Hence, the superposition of a large number of active degrees of freedom can modify the stability/instability conditions of magnetosphere.

Keywords. Magnetospheric physics (Magnetospheric configuration and dynamics; Solar wind-magnetosphere interactions; Storms and substorms)

1 Introduction

The Earth's magnetosphere is mainly affected by solar wind and interplanetary magnetic field, and it responds to external drivers in a highly organized and complex way (Klimas et al., 1996). This complex behaviour is due to a nonlinear dynamics related to the energy storage, transport and release in the geomagnetic tail regions. Moreover, as a consequence of the continuous solar wind driving, the coupled magnetosphere-ionosphere system is believed to be in an out-of-equilibrium configuration. In the recent past, studies have appeared on the role that chaos, turbulence and near-criticality dynamics might play in the magnetospheric dynamics (Baker et al., 1990; Roberts et al., 1991; Vassiliadis et al., 1990). In this framework, new research perspectives to the investigation of the magnetospheric dynamics were opened by the recent advances in the study of complexity and complex systems (Vassiliadis et al., 1990; Chang, 1999, 2001a, b; Consolini and Chang, 2001, 2002; Ukhorskiy et al., 2002, 2004; Balasis et al., 2006; Pulkkinen et al., 2006).

A common feature of systems displaying complexity is that all these systems are generally made of a huge number of interconnected and cross-coupled parts. The investigation of such systems allowed the introduction of several

Correspondence to: K. Unnikrishnan
(kaleekkal_unni@yahoo.com)

new concepts, dealing, for example, with the appearance of self-organization, criticality and scale-invariance in out of-equilibrium systems, and the role that disorder and fractal topologies might play in many natural systems.

The theory of nonlinear, deterministic dynamical systems provides a powerful theoretical tool to characterize geometrical and dynamical properties of the attractors of such systems (Hegger et al., 1999). Along with the theoretical understanding of these systems, many of the typical phenomena have been realized in laboratory experiments. Hence, nonlinear time series analysis is highly advantageous to reveal the underlying dynamics of a system. Besides the exponential divergence of trajectories, the most striking feature of chaotic dynamical systems is the irregular geometry of the sets in phase space visited by the system state point in the course of time.

In characterising chaos quantitatively, based on the dynamics, Lyapunov exponent is one of the best descriptors. In the case of dissipative systems, the effects of transcients associated with initial conditions fade away and the long-term behaviour is restricted to some attracting region or regions in state space. The reasons for quantifying chaotic behaviour are: the quantifiers may help to distinguish chaotic behaviour from noisy behaviour, enabling us to determine how many variables are needed to model the dynamics of the system, and may help us to sort systems into universality classes, and changes in quantifiers may be linked to important changes in the dynamical behavoiur of the system.

In recent years, nonlinear time series methods were employed to study the magnetospheric chaos, and these studies strongly support its existence (Vassiliadis et al., 1990; Roberts et al., 1991; Shan et al., 1991; Sharma et al., 1993; Pavlos et al.,1992). Pavlos et al. (1999a, b, c) have addressed the criticism over magnetospheric chaos by testing the null hypothesis, and their results further support the hypothesis of nonlinearity and chaotic behavior of the underlying dynamics of the magnetospheric system. Based on these studies, they observed that the hypothesis of low dimensional chaotic behavior of the magnetospheric dynamics is one of the possible and fruitful concepts, which must be developed further.

Bhattacharyya (1990) had studied the chaotic behavior of ionospheric density fluctuations, using amplitude and phase scintillation data, and found the existence of low-dimensional chaos. Also, Wernik and Yeh (1994) have studied chaotic behavior of ionospheric turbulence using scintillation data and numerical modeling of scintillation at high latitude. Kumar et al. (2004) reported the evidence of low dimensional chaos in a set of TEC data, obtained by Faraday rotation technique, measured at a high-latitude station, Goose Bay (47° N, 286° E), during the period, February to April 1976. Recently, Unnikrishnan et al. (2006a, b) analyzed the deterministic chaotic behavior of GPS TEC fluctuations at mid-latitude, by employing the nonlinear aspects like mutual information, fraction of false nearest neighbours, phase space reconstructions, and chaotic quantifiers. Also, they compared the possible chaotic behavior of ionosphere during geomagnetic storms, and quiet times, under different seasons, local times and latitudes using dynamical and topological invariants.

Recently, a tutorial review was made by Vassiliadis (2006) on systems theory and its applications to space plasma physics and, more broadly, on geophysics. It is known that, earth's magnetosphere is a spatially extended nonlinear system driven far from equilibrium by turbulent solar wind. Ukhorskiy et al. (2004) presented a data derived model of the solar wind-magnetosphere coupling that combines a nonlinear dynamical description of the global features with a statistical description of the multi-scale aspects. It is still unclear to what extent the fluctuations in the solar wind are reflected in the fluctuations of magnetosphere-ionosphere system. Numerous studies have tried to approach this problem by using both deterministic and stochastic paradigms (Tsurutani et al., 1990; Hnat et al., 2005; Cheng et al., 2005; Pulkkinen et al., 2006).

Although, studies on deterministic chaotic behavior of magnetosphere were conducted, the comparative studies based on the deterministic chaotic behavior of magnetosphere under various seasons, solar activities, and quiet/disturbed conditions have not been conducted so far. In the present study, the deterministic chaotic behaviour of magnetosphere under various physical conditions, using AE index time series, are analyzed. Particularly, the values of significant chaotic quantifiers, namely, Lyapunov exponent, the spatio-temporal entropy, and nonlinear prediction error of AE index time series during different seasons, solar activities, and quiet/disturbed conditions are compared. Based on the values of the above quantifiers, under various conditions, the features of chaotic behaviour of magnetosphere are briefly discussed.

2 Data analysis and methodology

The auroral electrojet index (AE), originally introduced by Davis and Sugiura (1966), is a measure of the global electrojet activity in the auroral zone. One of the main features of the AE-index is its intermittent character, which is evidence of a punctuated dynamics of the magnetospheric system in response to solar wind changes. In detail, the AE-index is characterised by periods of relative stasis punctuated by crises of different sizes (Consolini and De Michelis, 1998, 2002). The indices AU and AL provide instantaneous measures of eastward and westward electrojet currents, such that AE=AU−AL. Thus, AE indicates the total maximum amplitude of the two-current-system. More-over, the choice of the AE-index as an indicator of the global magnetospheric activity has been made because of the common point of view that AE-indices are able, in some sense, to sample the state space of the magnetospheric system (Hajkowicz, 1998). Also, AE index is a reliable indicator of the auroral activity and will

Table 1. List of time series of AE index considered for this study, representing quiet periods and storm periods with date, time of storm sudden commencement (SSC), and maximum value of D_{st} during storm.

Year	Season	Time series (quiet period)	Time series (storm period)	Date & UT of SSC (in brackets)	Maximum D_{st} value during storm
		5–11 Jan 1991	31 Jan–6 Feb 1991	1 Feb (1841)	−79 nT
	Winter	17–23 Jan 1991	7–13 Nov 1991	8 Nov (0647)	−354 nT
		15–21 Feb 1991	18–24 Nov 1991	19 Nov (0421)	−123 nT
1991		10–16 Apr 1991	23–29 Mar 1991	24 Mar (0341)	−298 nT
	Equinox	16–22 Sep 1991	8–14 Sep 1991	9 Sep (0516)	−96 nT
		11–17 Oct 1991	30 Sep–6 Oct 1991	1 Oct (1812)	−164 nT
		4–10 May 1991	3–9 Jun 1991	4 Jun (0337)	−223 nT
	Summer	25–31 Jul 1991	7–13 Jul 1991	8 Jul (1635)	−194 nT
			17–23Aug 1991	18 Aug (1833)	−170 nT
		4–10 Jan 1994	4–10 Feb 1994	5 Feb (0436)	−126 nT
	Winter	12–18 Nov 1994	20–26 Feb 1994	21 Feb (0900)	−144 nT
		17–23 Dec 1994	24–30 Nov 1994	25 Nov (1301)	−117 nT
1994		24–30 Apr 1994	6–12 Mar 1994	7 Mar (0357)	−109 nT
	Equinox	18–24 Sep 1994	1–7Apr 1994	2 Apr (0900)	−111 nT
			15–21 Apr 1994	16 Apr (2013)	−201 nT
		3–9 Aug 1994	2–8 May 1994	3 May (0112)	−79 nT
	Summer	17–23 Aug 1994	27 May–2 Jun 1994	28 May (1356)	−68 nT
			12–18 Jul 1994	13 Jul (1337)	−53 nT

reflect the irregularities associated with magnetospheric dynamics.

In the present study, AE index time series of one minute duration, representing disturbed and quiet conditions, falling under different seasons of high (1991 with yearly average of solar flux = 208) and low (1994 with yearly average of solar flux = 85.8) solar active years were analysed. The time series of length shorter than 7000 points will fail to give consistent estimate of Lyapunov exponent (Vassiliadis et al., 1991) and the time series used in the present study contain more than 10 000 points.

The number of geomagnetic storms considered is 9 each for high (1991) and low (1994) solar active years. However, it is very difficult to find 7 consecutive quiet days to form time series for quiet periods. To compare the nonlinear aspects of storm time with quiet time magnetosphere, the number of events (quiet periods) considered is 8 and 7, respectively during high (1991), and low (1994) solar activities (see Table 1). The source of AE index data is World Data Centre for Geomagnetism, Kyoto (http://swdcwww.kugi.kyoto-u.ac.jp).

The nonlinear time series analysis was performed for AE index (Hegger et al., 1999), for all the above periods, the respective chaotic quantifiers like, Lyapunov exponent, spatio-temporal entropy and nonlinear prediction error were estimated, and compared (For details of analysis see Unnikrishnan et al., 2006a, b).

3 Results

3.1 Time series and phase space

A time series, S_n is the sequence of scalar measurements of some quantity, which depends on the current state of a system, taken at multiples of a fixed sampling time (Δt), and Fig. 1a represents the typical time series of AE index.

According to embedding theorems, if we choose an appropriate delay based on the data, at most $2d+1$ delay coordinates are enough, where d is the fractal dimension of the attractor (Saucer et al., 1991). Mostly, the smallest integer greater than the correlation dimension is sufficient for the complete characterization of the attractor. The trajectories of the system may converge at a region in the phase space or the bounded subset of the phase space known as its attractor. To reveal the multidimensional aspects of the system, phase space reconstruction of the time series is required. For this, the proper choice of embedding dimension (m) and delay time (τ) are essential (Fraser and Swinney 1986; Saucer

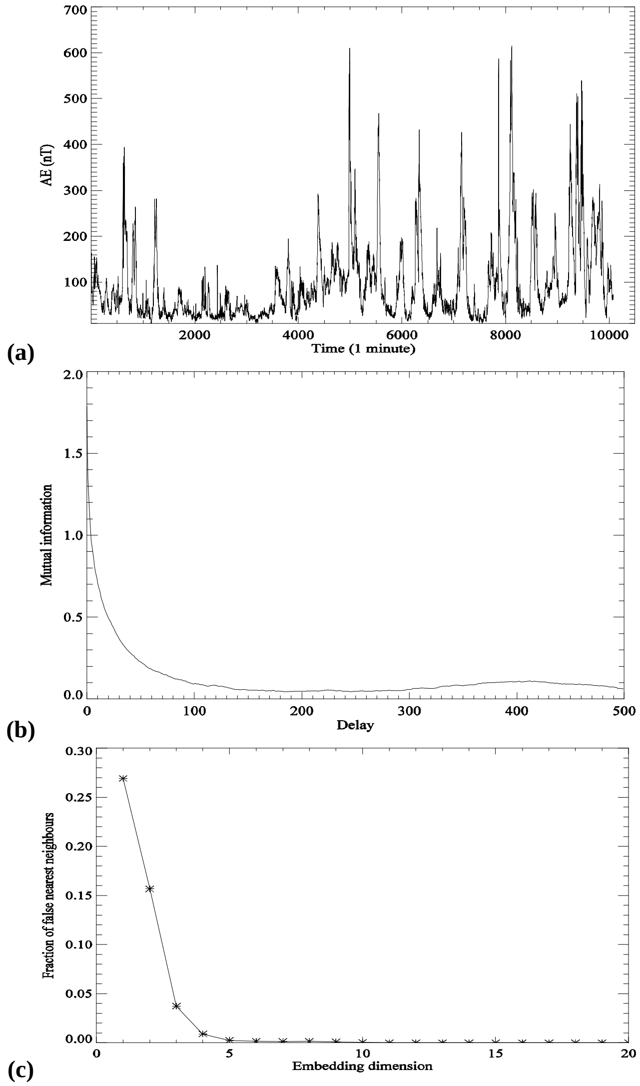

Fig. 1. **(a)** Time series of AE index, **(b)** mutual information of the time series as a function of delay, and **(c)** fraction of false nearest neighbours as a function of the embedding dimension m with τ=70, during the quiet period, 5–11 January 1991.

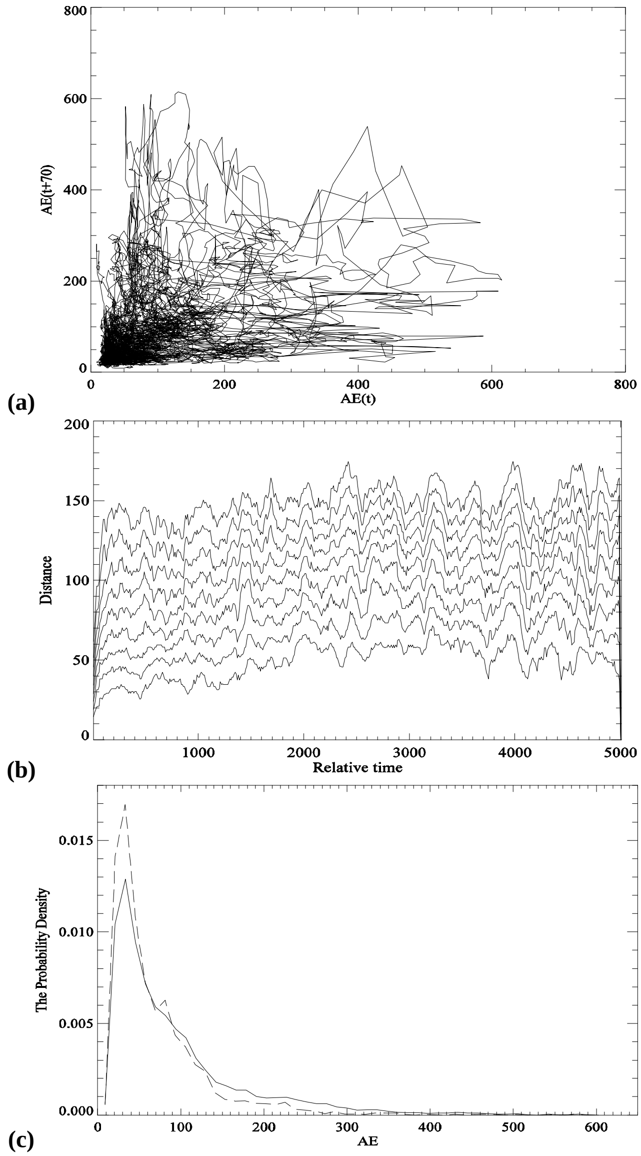

Fig. 2. **(a)** The plot of reconstructed phase space for the time series of AE index, using time delay embedding for τ=70 and m=5, **(b)** space time separation plot, and **(c)** the plot of probability density function based on the entire time series and on the first half of the series (continuous and dashed lines respectively), during the quiet period, 5–11 January 1991.

et al., 1991; Kennel et al., 1992). If the time delayed mutual information shows a marked minimum, that value can be considered as a reasonable time delay. Likewise, the minimal embedding dimension, which corresponds to the minimum number of false nearest neighbours can be found out, and is treated as the optimum value of m (Unnikrishnan et al., 2006a, b). With the help of time delayed mutual information (Fig. 1b) and the false nearest neighbour method (Fig. 1c), it is observed that any delay $\geq$60 and any dimension $\geq$4 are the suitable choices for τ and m, respectively. By repeating the similar analysis for all the time series considered in the present study, these choices for τ and m are found to be true. Hence, the choice of τ as 70 and m as 5 for further analysis of this study are reasonable. Based on embedding theorem, a multidimensional state space can be reconstructed as follows:

$$\boldsymbol{Y}_n = (S_{n-(m-1)\tau}, S_{n-(m-2)\tau}, \ldots, S_{n-\tau}, S_n) \quad (1)$$

where $\boldsymbol{Y}_n$ are the vectors. The reconstructed phase space for the time series of AE index (Fig. 2a), using time delay embedding for τ=70 and m=5, shows the convergence of the trajectories.

To identify the temporal correlation of the data, a stationarity test, called space-time separation plot, was conducted. The most common definition of a stationarity process is that, all conditional probabilities are constant in time. The interdependence existing in a system can be detected by plotting the number of pairs of points as a function of the two variables, namely, the time separation Δt, and the spatial distance, ϵ (Provenzale et al., 1992; Dasan et al., 2002). Figure 2b exhibits space time separation plots for the time series of AE index for τ=70 and m=5. From this figure, it is seen that the curves exhibit small scale oscillations and temporal correlation in the data is not prominent.

To check the stationarity, probability density of the time series of AE index was calculated, by dividing the range of values of AE index into short intervals, and then by counting the values of the series fall in each interval (George et al., 2002; Kumar et al., 2004; Unnikrishnan et al., 2006a, b). The probability density for the original time series of AE index (continuous line) and those of the first half of the series (dashed line) were calculated and shown in Fig. 2c. From this figure, it is evident that the trends of curves for the probability density of the original time series of AE index, and that of the first half of the time series of AE index are very similar, which supports stationarity of the data.

3.2 Comparison of chaotic quantifiers

It is known that, Lyapunov exponent is a measure of the rate of attraction to or repulsion from a fixed point in the state space. One of the most prominent evidences of chaotic behaviour of a dissipative deterministic system is the existence of positive Lyapunov exponent. A positive Lyapunov exponent indicates divergence of trajectories in one direction, or alternatively, expansion of an initial volume in this direction, and a negative Lyapunov exponent indicates convergence of trajectories or contraction of volume along another direction. For flows, there is always a zero Lyapunov exponent corresponding to the direction of the flow.

One of the drawbacks of the algorithm introduced by Wolf et al. (1985) to estimate Lyapunov exponent is its strong dependence on embedding dimension (Kantz, 1994; George et al., 2002; Kumar et al., 2004). The algorithm developed by Kantz (1994) to evaluate the maximal Lyapunov exponent was used here, as well as in previous studies (George et al., 2002; Kumar et al., 2004; Unnikrishnan et al., 2006a, b), and is given as

$$S(\Delta n)=\frac{1}{N}\sum_{n_0=1}^{N}\ln\left(\frac{1}{|U\left(y_{n_0}\right)|}\sum_{y_n\in U\left(y_{n_0}\right)} S_{n_0+\Delta n}-S_{n+\Delta n}\right) \quad (2)$$

We can compute $S(\Delta n)$ for a point y_{n_0} of the time series in the embedding space, where $U(y_{n_0})$ is the neighbourhood of y_{n_0} with diameter r. The computation of $S(\Delta n)$ for different values of the embedding dimension m and the diameter of the neighbourhood r is repeated. For an intermediate range of values of Δn, $S(\Delta n)$ increases with slope λ (Fig. 3a), which is known as maximal Lyapunov exponent. The value of Lyapunov exponent was calculated for various time delays by keeping embedding dimension a constant. Also, these calculations were conducted for various embedding dimensions, by keeping time delay a constant.

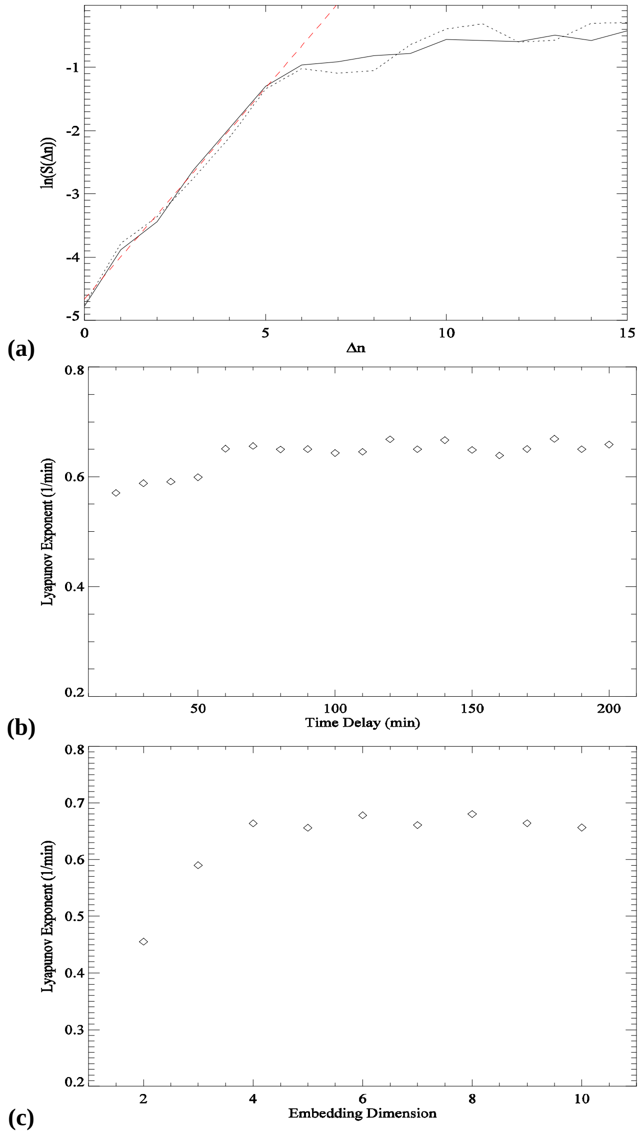

Fig. 3. Plots of **(a)** the curves of ln($S(\Delta n)$) for embedding dimensions m=5 and 6 (continuous and dotted lines respectively), with τ=70, Theiler window (ω)=380, whose slope will give the Lyapunov exponent, **(b)** the variation of Lyapunov exponent with time delay by keeping embedding dimension a constant, and **(c)** the variation of Lyapunov exponent with embedding dimensions by keeping time delay a constant, for the time series of AE index, during the quiet period, 5–11 January 1991.

A plot of Lyapunov exponent versus time delay, by keeping embedding dimension a constant (Fig. 3b) for a particular

Table 2. Values of Lyapunov exponent for time series of AE index during quiet periods.

Year	Season	Time Series	Lyapunov Exponent including error (unit is $\min^{-1}$)
		5–11 Jan 1991	0.6669±0.0156
	Winter	17–23 Jan 1991	0.3363±0.0065
		15–21 Feb 1991	0.4390±0.0412
1991		10–16 Apr 1991	0.4182±0.0141
	Equinox	16–22 Sep 1991	0.5401±0.0744
		11–17 Oct 1991	0.4536±0.0016
		4–10 May 1991	0.5292±0.0426
	Summer	25–31 Jul 1991	0.5633±0.0207
		4–10 Jan 1994	0.5722±0.0012
	Winter	12–18 Nov 1994	0.7254±0.0077
		17–23 Dec 1994	0.7844±0.1611
1994	Equinox	24–30 Apr 1994	0.6027±0.0114
		18–24 Sep 1994	0.5427±0.0178
		3–9 Aug 1994	0.5665±0.0289
	Summer	17–23 Aug 1994	0.5852±0.0239

time series shows that, its values initially increase up to a time delay 60, and thereafter remain almost constant. Figure 3c depicts the variation of Lyapunov exponent with embedding dimension, by keeping time delay a constant. It is seen that Lyapunov exponent initially increases upto m=4, and afterwards remains almost constant. Similar analyses were conducted for all the time series used for this study and it is observed that values of Lyapunov exponent are almost constant for time delay ≥60 and embedding dimension ≥4. This clearly indicates that values of Lyapunov exponent converge for optimal values of time delay (≥60) and embedding dimension (≥4). Hence, choice of time delay as 70, and embedding dimension as 5, are reasonable to estimate Lyapunov exponent and has been presented in Tables 2 and 3.

Figure 4 reveals the seasonal and solar activity dependence Lyapunov exponent and thereby the chaotic behaviour of magnetosphere, using the of AE index time series of quiet and disturbed conditions. During high solar activity (1991), the values of Lyapunov exponent for AE index time series belonging to quiet conditions have no significant difference from those corresponding storm conditions (Fig. 4a). The yearly average values of Lyapunov exponent during quiet and storm periods of high solar active period (1991) are $0.5\pm0.1\,\min^{-1}$ and $0.5\pm0.17\,\min^{-1}$, respectively.

During solar minimum period (1994), the seasonal mean value of Lyapunov exponent for AE index time series (Fig. 4b) representing quiet periods in winter ($0.7\pm0.11\,\min^{-1}$) is higher compared to that of storm periods ($0.36\pm0.09\,\min^{-1}$). The yearly average values of Lyapunov exponent during quiet and storm periods of low solar active period (1994) are $0.6\pm0.1\,\min^{-1}$ and $0.5\pm0.2\,\min^{-1}$, respectively. In the case of both high (1991) and low (1994) solar activities, standard deviations of yearly mean Lyapunov exponent values for AE index time series representing quiet periods are smaller than those for storm periods. Generally, the values of Lyapunov exponent of AE index obtained in the present study, are of the same order of magnitude as that obtained for AL index data ($0.11\pm0.05\,\min^{-1}$) computed by Vassiliadis et al. (1991).

Recurrence Plot (RP) first described by Eckmann, et al. (1987), and further modified by Zbilut and Webber (1992) is a powerful analytical tool for the study of nonlinear dynamical systems. With RP, one can graphically detect hidden patterns and structural changes in data or see similarities in patterns across the time series under study. If the time series is truly random and has no structure, the distribution of colors over the RP will be uniform, and has no identifiable patterns. On the other hand, if there is some determinism in the signal generator, it can be detected by some characteristic, distinct distribution of colors. The Shannon entropy of the probability distribution of the diagonal line lengths $p(l)$is given as:

$$\text{Entropy} = -\sum_{l=l_{\min}}^{N} p(l)\ln p(l) \tag{3}$$

where l is the lowest number of upward diagonal recurrent points required to define a deterministic line. Figure 5a presents the RP of the typical AE index time series which

Table 3. Values of Lyapunov exponent for time series of AE index during storm periods.

Year	Season	Time series	Lyapunov Exponent including error (unit is min^{-1})
1991	Winter	31 Jan–6 Feb 1991	0.3665±0.0026
		7–13 Nov 1991	0.4057±0.0033
		18–24 Nov 1991	0.6346±0.0309
	Equinox	23–29 Mar 1991	0.1596±0.0272
		8–14 Sep. 1991	0.6376±0.0746
		30 Sep–6 Oct 1991	0.4955±0.0003
	Summer	3–9 Jun 1991	0.6052±0.0666
		7–13 Jul 1991	0.4425±0.0587
		17–23 Aug 1991	0.7191±0.0938
1994	Winter	4–10 Feb 1994	0.4579±0.0282
		20–26 Feb 1994	0.3149±0.0263
		24–30 Nov 1994	0.2926±0.0069
	Equinox	6–12 Mar 1994	0.7881±0.0577
		1–7 Apr 1994	0.6194±0.0653
		15–21 Apr 1994	0.5134±0.0139
	Summer	2–8 May 1994	0.8616±0.2814
		27 May–2 Jun 1994	0.4060±0.0617
		12–18 Jul 1994	0.4184±0.0244

exhibit characteristic patterns. The RP prepared for all the time series studied here, and their entropies were estimated using Eq. (3).

Figure 5a and b depict the variation of seasonal mean values of entropy of AE index time series under quiet and disturbed conditions of different solar activities. During high solar activity (1991), significant difference is not observed between storm and quiet time values of entropy (Fig. 5b). However, It is seen that, during low solar active period (1994), the seasonal mean value of entropy for time series representing storm periods of equinox is greater than that for quiet periods (Fig. 5c). Also, it is observed that, seasonal mean values of nonlinear prediction error for time series representing storm periods have no considerable difference from those for quiet periods during both high (1991) and low (1994) solar activities (Fig. 6a and b, respectively).

4 Discussion

A common hallmark of out of equilibrium phenomena is their extraordinary complexity. Complex systems self-organize their internal structure and their dynamics, showing novel and surprising macroscopic properties, including coherent large-scale collective behaviours. A universal footprint seen in many complex systems near criticality is the self-affinity for energy release from the system that signals a fractal topology, namely a multi-scale process with no preferred spatial and temporal scales. Balasis et al. (2006) suggested that the development of an intense magnetic storm can be studied in terms of intermittent criticality, which is a more general character than the classical self-organized criticality phenomena, implying the predictability of the magnetosphere. Observations suggest that under the influence of the solar wind, the magnetosphere can be channeled into a globally non-equilibrium critical state (Chang, 1999; Consolini and Chang, 2001, 2002).

Ahn et al. (2000) have examined the seasonal and solar cycle variations of auroral electrojet indices for the past 20 years and found that the AU and AL indices maximize during summer and equinoctial months, respectively. Their study strongly suggests that the main modulator of the seasonal variation of the auroral electrojets is not the ionospheric conductivity but the electric field. It is also worth mentioning that the main reasons for the semi-annual variation of the AL index are the magnitude increase and the increased frequency of disturbed conditions during equinoctial season. It is a clear indication that the efficiency of the coupling between the solar wind and magnetosphere increases during equinoctial season.

It is known that the combination of the storm and substorm caused some unique and well-correlated phenomena in the magnetosphere and auroral-subauroral ionosphere (Huang et

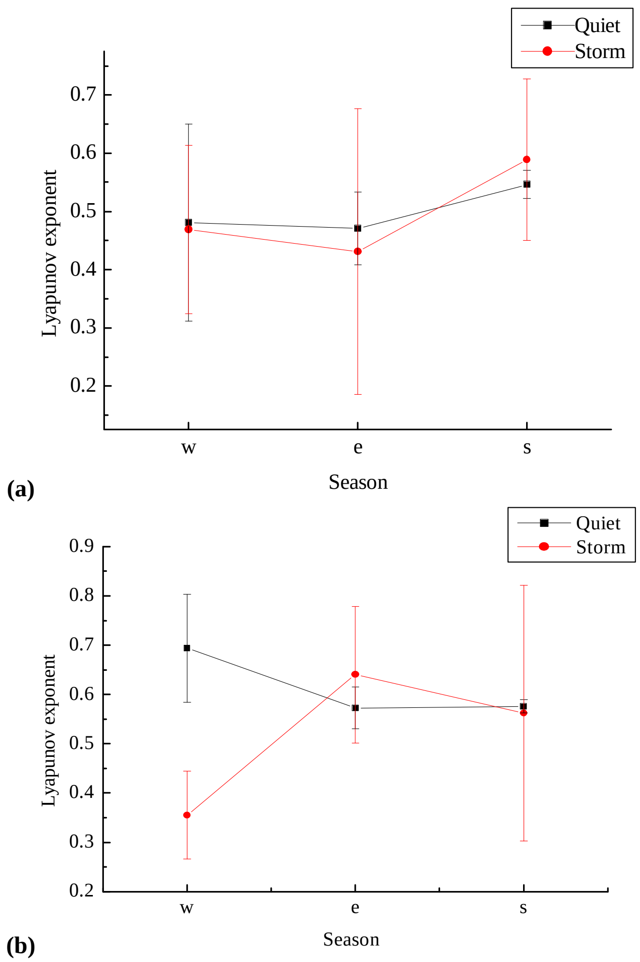

Fig. 4. Comparison of seasonal mean values of Lyapunov exponent of AE index time series for summer, equinox and winter (denoted by letters, "s", "e", and "w", respectively on X axis), during storm and quiet periods (red and black lines respectively) of **(a)** high (1991) and **(b)** low (1994) solar activity years. The vertical bars represent the standard deviation (red for storm and black for quiet period).

al., 2003). Moreover, other studies also observed that, magnetic substorm is the set of phenomena during which a reduction in topological complexity in the tail regions takes place (Chang, 2001a, b; Consolini and Chang, 2001, 2002). The role of the solar-wind driver would be to enhance the internal fluctuations that could induce a topological transition among metastable complex topologies. In such a case, the evolution of the magnetospheric system will be the result of the combined effects of local couplings of the magnetic and plasma structures, through the nonlinearities of the system. This point of view also supports the recent results of Sitnov et al. (2001) that the substorm activity resembles the non equilibrium (first and/or second order) phase transitions.

Various studies suggested that the ionospheric electron content perturbations were also caused by the penetration of magnetospheric electric fields, which were controlled or modulated by the oscillations in the IMF/solar wind pressure (Borovsky, et al., 1993; Fejer and Scherliess, 1998; Sobral et al., 1997; Huang et al., 2002; Unnikrishnan, et al., 2005). The energy and particle injection that takes place during magnetospheric disturbances produce multiple changes to the Earth's high latitude ionosphere-thermosphere system (Pincheira et al., 2002).

Since, magnetic storms are extreme forms of space weather, the external driving forces, mainly due to solar wind, make the solar-magnetosphere-ionosphere coupling more complex, and related disturbed electric field and wind patterns will develop. This in turn creates many active degrees of freedom with various levels of coupling among them. Hence, the superposition of a large number of active degrees of freedom can produce extremely complicated signals during a magnetic storm period. Thus a geomagnetic storm can modify the stability/instability conditions of magnetosphere, due to the superposition of various active degrees of freedom. A stochastic model for AE index was recently developed, by Pulkkinen et al. (2006) to investigate the role stochastic fluctuations in the magnetosphere-ionosphere system. It is shown that, the introduction of a stochastic component in their model could capture some essential features present in the measured AE index variations.

The dynamical framework presented by Vassiliadis (2006) provides a new approach alongside the traditional perturbative and statistical-mechanical methodologies and is directly relevant to the development of space weather applications. In this work, the relation between symmetries in the plasma system and modes in its structure and response is discussed. A framework of modeling methods is presented in order of increasing complexity: enumeration of the effective degrees of freedom, measurement of the linear dynamics and stability, and generalization to their nonlinear counterparts. Moreover, signal processing methods are presented, illustrated by examples, and their relative merits and limitations are discussed.

Infact, AE index fluctuations are a product of a number of different processes operating in the magnetosphere-ionosphere system, such as global ionospheric convection enhancements, geomagnetic pulsations, sudden geomagnetic commencements, substorm-related activity etc. Bursts in the AE index are a compound effect of numerous local events and because of the inherent randomness in the system (Pulkkinen et al., 2006). During high solar activity (1991), the values of Lyapunov exponent for AE index time series representing quiet conditions have no significant difference from those values for corresponding storm conditions. This implies that, for the cases considered here, geomagnetic storms may not be an additional source to increase or decrease the deterministic chaotic aspects of magnetosphere, especially during high solar activity.

Studies of Pavlos et al. (1992, 1999a, b, c) revealed that, the random character of the magnetospheric time series could

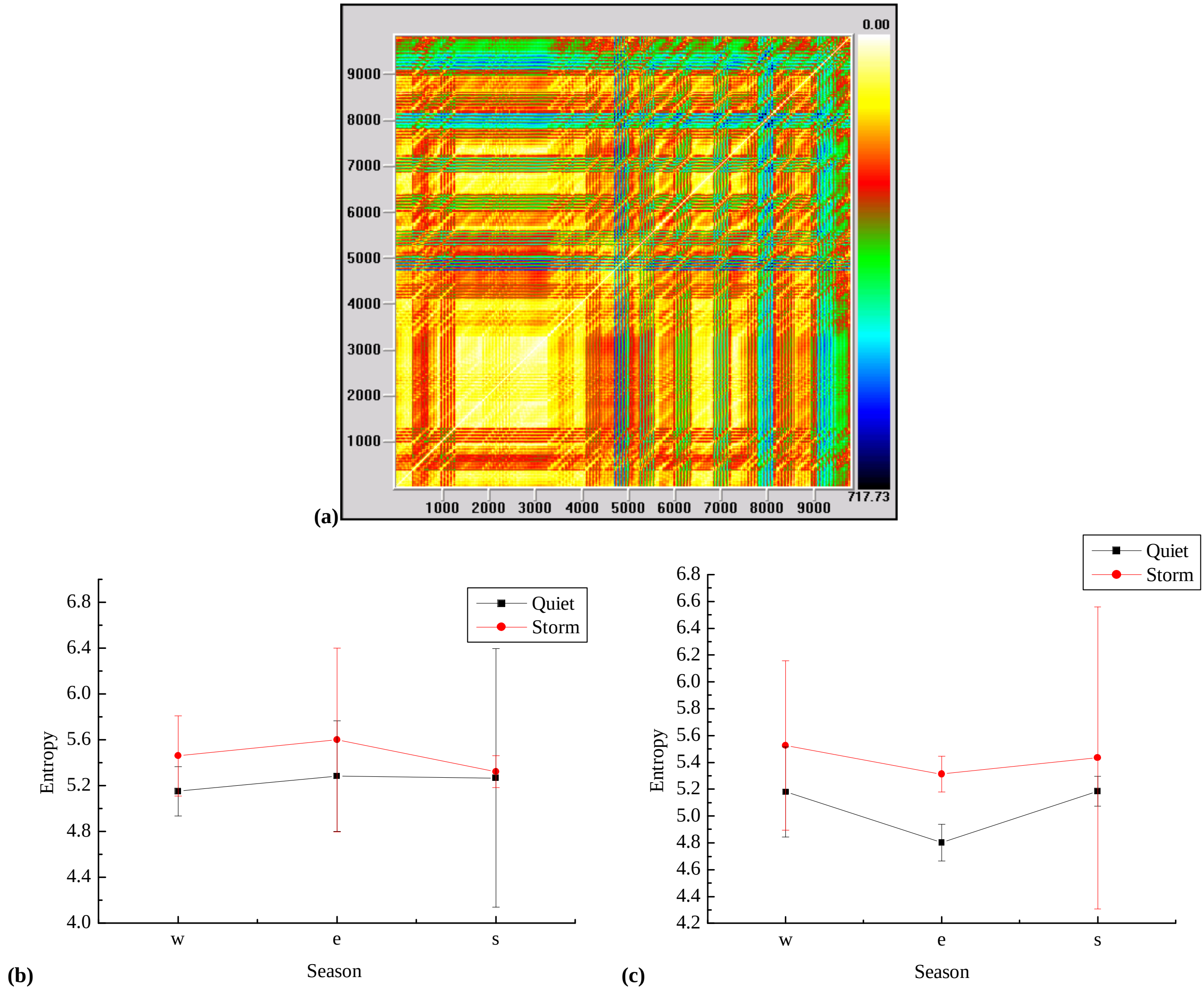

Fig. 5. **(a)** Recurrence plot (RP) of the AE index time series, during the quiet period, 5–11 January 1991, which exhibit characteristic patterns. Comparison of seasonal mean values of entropy of AE index time series for summer, equinox and winter (denoted by letters, “s”, “e”, and “w” respectively on x-axis) for quiet and storm periods (red and black lines respectively) during **(b)** high (1991), and **(c)** low (1994) solar activity years . The vertical bars represent the standard deviation (red for storm and black for quiet period).

be caused by the chaotic low-dimensional internal dynamics of the magnetospheric system, while this character only appears, when the solar wind input takes appropriate values. As the solar wind is changing continuously its state, the magnetospheric dynamics can live intermittently on a low dimensional chaotic attractor. Another study, using energetic ions’ signal also suggests the existence of two different physical processes related to the magnetospheric dynamics: the first process corresponds to a stochastic external component and the second process corresponds to a low dimensional chaotic component (Pavlos et al., 2003). These previous studies support the result of the present paper that, higher variabilities in values of Lyapunov exponent for time series representing storm periods, compared to quiet periods, could be due to the higher degree of coupling between solar wind and magnetosphere during storms.

Based on the model suggested by Pulkkinen et al. (2006), the nonlinear deterministic part of the fluctuation is made up of two components: a D_{st} dependent steady driving part and a background activity independent dissipative term representing the collective dissipative response of the system to driving. The dominating D_{st} dependent linear stochastic part is responsible for the complexity of the AE index fluctuations. The fundamental result of their study is that

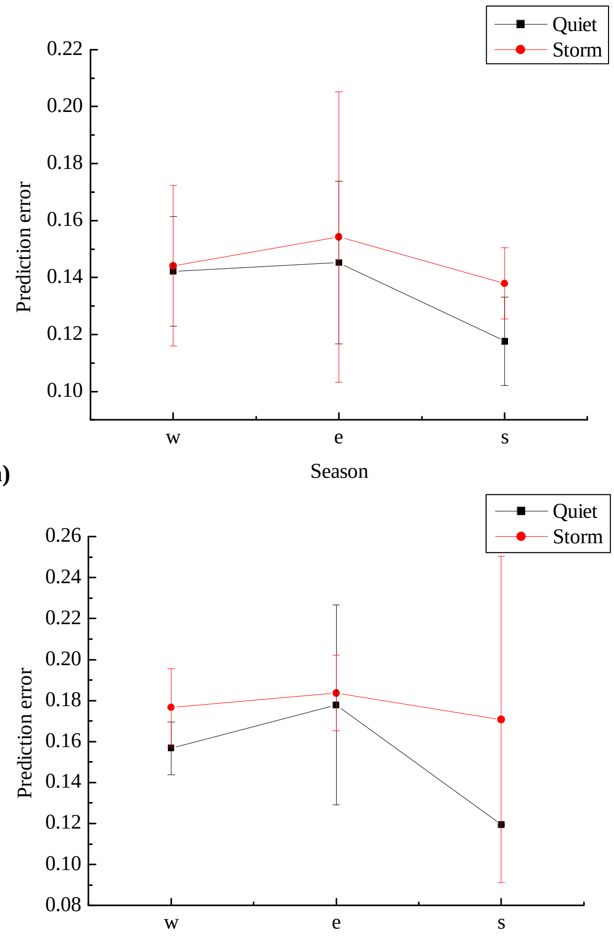

Fig. 6. Comparison of seasonal mean values of nonlinear prediction error for AE index time series for summer, equinox and winter (denoted by letters, "s", "e", and "w", respectively on x-axis), during storm and quiet periods (red and black lines respectively) of **(a)** high (1991) and **(b)** low (1994) solar activity years. The vertical bars represent the standard deviation (red for storm and black for quiet period).

the stochastic fluctuations play a central role in the evolution of the AE index and cannot be grossly neglected. It is reasonable to assume that the large-amplitude fluctuations are produced by internal bursts and the externally driven fluctuations contribute mostly to the small amplitude portion.

Takalo et al. (1993) have estimated the correlation dimension of AE index as 3.4 and reported that bicoloured noise shares many properties with AE index. It is believed that the stochastic nature of the AE index fluctuations is of internal, and not of external origin (Burlaga, 1995). The stochastic part, which is D_{st} dependent (Pulkkinen et al., 2006) may be more prominent during storms, thereby increasing fluctuations/stochasticity and reducing determinism in AE index time series during storms. This could be a reason for lower values of Lyapunov exponent observed during winter storms compared to corresponding values during quiet periods of low solar activity (1994).

Recent studies reveal that the magnetospheric dynamics is neither clearly low dimensional nor completely random, but exhibit combinations of these two aspects, which yields an improved and effective concept for forecasting space weather (Ukhorskiy et al., 2004). But basically, the fluctuations are of internal magnetospheric origin, though the bursts can be triggered by an external perturbation, and they are interplay of the deterministic and stochastic components of a stationary out of equilibrium system. In the case of both high and low solar activities, the higher standard deviations of yearly mean Lyapunov exponent values for AE index time series for storm periods compared to those for quiet periods might be due to the strong interplay between stochasticity and determinism during storms.

It was realized that randomness and disorder may introduce new and unexpected behaviours in physical systems. One of the most relevant characteristics of disordered systems is the occurrence of metastability as a consequence of the intrinsic space-time randomness. Due to the influence of solar activity, the earth's magnetic field becomes more and more stochastic. Also, it was reported that the magnitude of negative IMF Bz_{min} is larger in a high solar activity period than in a low activity period and the solar wind speed in an active period is faster than in a low activity period (Rawat et al., 2006). Hence, the internal instability of the magnetosphere system may be suppressed/modified and the system may transit more towards stochasticity rather than deterministic chaoticity. The complex behavior of magnetosphere is mainly due to the solar wind and the critical feature of persistency in the magnetosphere could be the result of a combined effect of solar wind and internal magnetospheric activity. Infact, entropy is the measure of disorder, and as it increases, the system transits more towards stochasticity. The higher values of seasonal mean of entropy for AE index time series for equinox storm periods compared to those of quiet periods of low solar activity may be explained as the dominance of stochasticity over determinism.

5 Conclusions

In the present study, the deterministic chaotic behaviour of magnetosphere under various physical conditions was analysed, using AE index time series. The significant chaotic quantifiers, namely, Lyapunov exponent, spatio-temporal entropy, and non linear prediction error for AE index time series during different seasons, solar activities, and quiet/disturbed conditions were estimated and compared. Based on the values of the above invariants, under various conditions, the features of chaotic behaviour of magnetosphere were briefly discussed.

With the help of time delayed mutual information and the false nearest neighbour method, it is observed that the choice of delay (τ) as 70 and embedding dimension (m) as 5 are reasonable for the present study. To identify the temporal correlation in the data, a stationarity test, called space-time separation plot, was conducted. To check the stationarity, probability density of the time series of AE index was calculated, by dividing the range of values of AE into short intervals, and then by counting the values of the series fall in each interval. It is evident that the trends of curves for the probability density of the original time series of AE index, and that of the first half of the time series of AE index are very similar, which supports stationarity of the data.

The value of Lyapunov exponent was calculated for various time delays by keeping embedding dimension a constant. Also, these calculations were conducted for various embedding dimensions, by keeping time delay a constant This clearly indicates that values of Lyapunov exponent converge for optimal values of time delay ($\geq$60) and embedding dimension ($\geq$4). Hence it is quiet reliable to estimate Lyapunov exponent by choosing time delay as 70 and embedding dimension as 5.

During high solar activity (1991), the values of Lyapunov exponent for AE index time series belong to quiet conditions have no significant difference from the corresponding storm conditions. The yearly average values of Lyapunov exponent during quiet and storm periods of high solar active period (1991) are $0.5\pm0.1\,\mathrm{min}^{-1}$and $0.5\pm0.17\,\mathrm{min}^{-1}$, respectively. This implies that geomagnetic storms may not be an additional source to increase or decrease the deterministic chaotic aspects of magnetosphere during high solar activity.

During solar minimum period (1994), the seasonal mean value of Lyapunov exponent for AE index time series belonging to quiet periods in winter ($0.7\pm0.11\,\mathrm{min}^{-1}$) is higher compared to corresponding values of storm periods in winter ($0.36\pm0.09\,\mathrm{min}^{-1}$). The yearly average values of Lyapunov exponent during quiet and storm periods of low solar active period (1994) are $0.6\pm0.1\,\mathrm{min}^{-1}$ and $0.5\pm0.2\,\mathrm{min}^{-1}$, respectively.

It is believed that the stochastic nature of the AE index fluctuations is of internal, not of external origin and the dominating D_{st} dependent linear stochastic part is responsible for the complexity of the AE index fluctuations. This stochastic part, which is D_{st} dependent may be more prominent during storms, thereby increasing fluctuations/stochasticity and reducing determinism in storm time AE index time series. This could be a reason for lower values of Lyapunov exponent observed during winter storms compared to corresponding values during quiet periods of low solar activity (1994).

In the case of both high (1991) and low (1994) solar activities, standard deviations of yearly mean values of Lyapunov exponent for AE index time series representing storm periods are higher than those corresponding to quiet periods. The higher variabilities in values of Lyapunov exponent for time series representing storm periods, compared to quiet periods, might be due to the strong interplay between stochasticity and determinism during storms.

Entropy is the measure of disorder, and as it increases, the system transits more towards stochasticity. The higher values of seasonal mean of entropy for AE index time series for equinox storm periods compared to those of quiet periods may be explained as the dominance of stochasticity over determinism. However, during high solar activity (1991), significant difference is not observed between storm and quiet time values of entropy and non linear prediction error.

The complex behavior of magnetosphere is mainly due to the combined effect of solar wind and inherent instabilities of magnetospheric dynamics. It is inferred that, the external driving forces, make the solar-magnetosphere-ionosphere coupling more complex, and the superposition of a large number of active degrees of freedom can modify the stability/instability conditions of magnetosphere.

Acknowledgements. The author expresses sincere thanks to World Data Centre for Geomagnetism, Kyoto for providing the AE index data. The author is grateful to both reviewers for giving valuable comments, which helped to improve this paper.

Topical Editor I. A. Daglis thanks two anonymous referees for their help in evaluating this paper.

References

Ahn, B.-H, Kroehl, H. W., Kamide, Y., and Kihn, E. A.: Seasonal and solar cycle variations of the auroral electrojet indices, J. Atmos. Sol. Terr. Phy., 62, 1301–1310, 2000.

Bhattacharyya, A.: Chaotic behaviour of ionospheric turbulence from scintillation measurements, J. Geophys. Res., 17, 733–738, 1990.

Borovsky, J. E., Nemzek, R. J., and Belian, R. D.: The occurrence rate of magnetospheric substorm onsets: random and periodic substorms, J. Geophys. Res., 98, 3807–3813, 1993.

Baker, D. N., Klimas, A. J., McPherron, R. I., and Büchner, J.: The evolution from weak to strong geomagnetic activity: and interpretation in terms of deterministic chaos, Geophys. Res. Lett., 17, 41–44, 1990.

Balasis, G., Daglis, I. A., Kapiris, P., Mandea, M., Vassiliadis, D., and Eftaxias, K.: From pre-storm activity to magnetic storms: a transition described in terms of fractal dynamics, Ann. Geophys., 24, 3557–3567, 2006, http://www.ann-geophys.net/24/3557/2006/.

Burlaga, L. F.: Interplanetary magnetohydrodynamics, Oxford Univ. Press, New York, 1995.

Chang, T.: Self-organized criticality, multi-fractal spectra, sporadic localized reconnections and intermittent turbulence in magnetotail, Phys. Plasmas, 6, 4137–4145, 1999.

Chang, T.: Colloid-like behavior and topological phase transitions in space plasmas: intermittent low frequency turbulence in the auroral zone, Phys. Scripta, T89, 80–83, 2001a.

Chang, T.: An example of resonances, coherent structures and topological phase transition – the origin of the low frequency broadband spectrum in the auroral zone, Nonlin. Processes Geophys., 8, 175–180, 2001b, http://www.nonlin-processes-geophys.net/8/175/2001/.

Cheng, C.-C., Russell, C. T., Reeves, G. D., Connors, M., and Moldwin, M. B.: On the relationship between double-onset substorm, pseudobreakup, and IMF variation: The 4 September 1999 event, J. Geophys. Res., 110, A07201, doi:10.1029/2004JA010778, 2005.

Consolini, G. and Chang, T.: Magnetic field topology and criticality in geotail dynamics relevance to substorm phenomena, Space Sci. Rev., 95, 309–321, 2001.

Consolini, G. and Chang, T.: Complexity, magnetic field topology, criticality, and metastability in magnetotail dynamics, J. Atmos. Sol. Terr. Phy., 64, 541–549, 2002.

Consolini, G. and De Michelis, P.: Non-Gaussian distribution function of AE index fluctuations: Evidence for time intermittency, Geophys. Res. Lett., 5(21), 4087–4090, 1998.

Consolini, G. and De Michelis, P.: Fractal time statistics of AE-index burst waiting times: evidence of metastability, Nonlin. Processes Geophys., 9, 419–423, 2002, http://www.nonlin-processes-geophys.net/9/419/2002/.

Dasan, J., Ramamohan, T. R., Singh, A., and Nott, P. R.: Stress fluctuations in sheared stokesian suspensions, Phys. Rev. E, 66, 21 409–21 422, 2002.

Davis, T. N. and Sugiura, M.: Auroral electrojet activity index AE and its universal time variations, J. Geophys. Res., 71, 785–801, 1966.

Eckmann, J. P., Oliffson Kamphorst, S., and Ruelle, D.: Recurrence plots of dynamical systems, Europhys. Lett., 4, 973–977, 1987.

Fejer, B. G. and Scherliess, L.: Mid- and low-latitude prompt-penetration ionospheric zonal plasma drifts, Geophys. Res. Lett., 25, 3071–3075, 1998.

Fraser, A. M. and Swinney. H. L.: Independent coordinates for strange attractors from mutual information, Phys. Rev. A, 33, 1134–1141, 1986.

George, B., Renuka, G., Satheesh Kumar, K., Anil Kumar, C. P., and Venugopal, C.: Nonlinear time series analysis of the fluctuations of the geomagnetic horizontal field, Ann. Geophys., 20, 175–183, 2001, http://www.ann-geophys.net/20/175/2001/.

Hajkowicz, L. A.: Longitudinal (UT) effect in the onset of auroral disturbances over two solar cycles as deduced from the AE-index, Ann. Geophys., 16, 1573–1579, 1998, http://www.ann-geophys.net/16/1573/1998/.

Hegger, R., Kantz, H., and Schreber, T.: Practical implementation of nonlinear time series method, The TISEAN Package, Chaos, 9, 413–430, 1999.

Hnat, B., Chapman, S. C., and Rowlands, G.: Scaling and a Fokker-Planck model for fluctuations in the geomagnetic indices and comparison with solar wind ϵ as seen by Wind and ACE, J. Geophys. Res., 110, A08206, doi:10.1029/2004JA010824, 2005.

Huang, C. S., Foster, J. C., and Erickson, P. J.: Effects of solar wind variations on the midlatitude ionosphere, J. Geophys. Res., 107(A8), 1192, doi:1029/2001JA00905, 2002.

Huang, C. S., Foster, J. C., and Goncharenko, L. P., Sofko, G. J., Borovsky, J. E., and Rich, F. J.: Mid latitude ionospheric disturbances during magnetic storms and substorms, J. Geophys. Res., 108(A6), 1244, doi:10.1029/2002JA009608, 2003.

Kantz, H.: A robust method to estimate the maximal Lyapunov exponent of a time series, Phys. Lett. A, 185, 77–87, 1994.

Kennel, M. B., Brown, R., and Abarbanel, H. D. I .: Determining minimum embedding dimension using a geometrical construction, Phys. Rev. A., 45, 3403–3411, 1992.

Klimas, A. J., Vassiliadis, D., Baker, D. N., and Roberts, D. A.: The organised nonlinear dynamics of the magnetosphere, J. Geophys. Res., 101, 13 089–13 113, 1996.

Kumar, K. S., Kumar, C. V. A., George, B., Renuka, G., and Venugopal, C.: Analysis of the fluctuations of the total electron content, measured at Goose Bay using tools of nonlinear methods, J. Geophys. Res., 10, A02308, doi:10.1029/2002JA009768, 2004.

Pavlos, G. P., Kyriakov, G. A., Rigas, A. G., Liatsis, P. I., Trochoutsos, P. C., and Tsonis, A. A.: Evidence for strange attractor structures in space plasma, Ann. Geophys., 10, 309–315, 1992, http://www.ann-geophys.net/10/309/1992/.

Pavlos, G. P., Athanasiu, M. A., Kugiumtzis, D., Hantzigeorgiu, N., Rigas, A. G., and Sarris, E. T.: Nonlinear analysis of magnetospheric data Part I. Geometric characteristics of the AE index time series and comparison with nonlinear surrogate data, Nonlin. Processes Geophys., 6, 51–65, 1999a, http://www.nonlin-processes-geophys.net/6/51/1999/.

Pavlos, G. P., Kugiumtzis, D., Athanasiu, M. A., Hantzigeorgiu, N., Diamantidis, D., and Sarris, E. T.: Nonlinear analysis of magnetospheric data Part II. Dynamical characteristics of the AE index time series and comparison with nonlinear surrogate data, Nonlin. Processes Geophys., 6, 79–98, 1999b, http://www.nonlin-processes-geophys.net/6/79/1999/.

Pavlos, G. P., Athanasiu, M. A., Diamantidis, D., Rigas, A. G., and Sarris, E. T.: Comments and new results about the magnetospheric chaos hypothesis, Nonlin. Processes Geophys., 6, 99–127, 1999c, http://www.nonlin-processes-geophys.net/6/99/1999/.

Pincheira, X. T., Abdu, M. A., Batista, I. S., and Richards, P. G.: An investigation of ionospheric responses, and disturbance thermospheric winds, during magnetic storms over South American sector, J. Geophys. Res., 107(A11), 1379, doi:10.1029/2001JA000263, 2002.

Provenzale, A., Smith, L. A., Vio, R., and Murante, G.: Distinguishing between low-dimensional dynamics and randomness in measured time series, Physica D, 58, 31– 35, 1992.

Pulkkinen, A., Klimas, A., Vassiliadis, A., and Uritsky, V.: Role of stochastic fluctuations in the magnetosphere-ionosphere system: A stochastic model for the AE index variations, J. Geophys Res., 111, A10218, doi:10.1029/2006JA011661, 2006.

Rawat, R., Alex, S., and Lakhina, G. S.: Low-latitude geomagnetic signatures during major solar energetic particle events of solar cycle-23, Ann. Geophys., 24, 3569–3583, 2006, http://www.ann-geophys.net/24/3569/2006/.

Roberts, D. A, Baker, D. N., Klimas, A. J., and Bargatze, L. F.: Indications of low dimensionality in magnetosphere dynamics, Geophys. Res. Lett., 18, 151–154, 1991.

Saucer, T., Yorke, J. A., and Casdagli, M.: Embedology, J. Stat. Phys., 65, 579–585, 1991.

Shan, H, Hansen, P., Goertz, C. K., and Smith, K. A.: Chaotic appearance of the ae index, J. Geophys Res., 18, 147, 1991.

Sharma, A. S., Vassiliadis, D., and Papadopoulos, K.: Reconstruction of low-dimensional magnetospheric dynamics by singular spectrum analysis, Geophys. Res. Lett., 20(5), 335–338, 1993.

Sitnov, M. I., Sharma, A. S., Papadopoulos, K., and Vassiliadis, D.: Modeling substorm dynamics of the magnetosphere: from self-organization and self-organized criticality to nonequilibrium phase transitions, Phys. Rev. E., 65, 16 116–16 126, 2001.

Sobral, J. H. A., Abdu, M. A., Gonzalez, W. D., Tsurutani, B. T., Batista, I. S., and Gonzalez, A. L. C.: Effects of intense storms and substorms on the equatorial ionosphere/thermosphere system in the American sector from ground-based and satellite data, J. Geophys. Res., 102, 14 305–14 314, 1997.

Takalo, J., Timonen, J., and Koskinen, H.: Correlation dimension and affinity of AE data and bicolored noise , Geophys. Res. Lett., 20, 1527–1530, 1993.

Tsurutani, B. T., Sugiura, M., Iyemori, T., Goldstein, B. E., Gonzales, W. D., Akasofu, S. I., and Smith, E. J.: The nonlinear response of AE to the IMF Bs driver: A spectral break at 5 hours, Geophys. Res. Lett., 17, 279–282, 1990.

Ukhorskiy, A. Y., Sitnov, M. I., Sharma, A. S., and Papadopoulos, K.: Global and multiscale aspects of the magnetospheric dynamics in local-linear filters, J. Geophys. Res., 107(A11), 1369, doi:10.1029/2001JA009160, 2002.

Ukhorskiy, A. Y., Sitnov, M. I., Sharma, A. S., and Papadopoulos, K.: Global and multi-scale features of solar wind-magnetosphere coupling: From modeling to forecasting, Geophys. Res. Lett., 31, L08802, doi:10.1029/2003GL018932, 2004.

Unnikrishnan, K., Saito, A., Otsuka, Y., Yamamoto. M., and Fukao, S.: Transition region of TEC enhancement phenomena during geomagnetically disturbed periods at mid-latitudes, Ann. Geophys., 23, 1–12, 2005, http://www.ann-geophys.net/23/1/2005/.

Unnikrishnan, K., Saito, A., and Fukao, S.: Differences in magnetic storm and quiet ionospheric deterministic chaotic behavior: GPS TEC Analyses, J. Geophys Res., 111, A06304, doi:10.1029/2005JA011311, 2006a.

Unnikrishnan, K., Saito, A., and Fukao, S.: Differences in day and night time ionospheric deterministic chaotic behavior: GPS TEC analyses, J. Geophys. Res., 111, A07310, doi:10.1029/2005JA011313, 2006b.

Vassiliadis, D.: Systems theory for geospace plasma dynamics, Rev. Geophys., 44, RG2002, doi:10.1029/2004RG000161, 2006.

Vassiliadis, D. V., Sharma, A. S., and Papadopoulos, K.: Lyapunov exponent of magnetospheric activity from AL time series, Geophys. Res. Lett., 18, 1643–1646, 1991.

Vassiliadis, D. V., Sharma, A. S., Eastman, T. E., and Papadopoulos, K.: Low-dimensional chaos in magnetospheric activity from AE time series, Geophys. Res. Lett., 17, 1841–1844, 1990.

Wernik, A. W. and Yeh, K. C.: Chaotic behavior of ionospheric scintillation-modeling and observations, Radio Sci., 29, 135–139, 1994.

Wolf, A., Swift, J. B., Swinney, H. L., and Vastano, J. A.: Determining Lyapunov exponents from a time series, Physica D, 16, 285–317, 1985.

Zbilut, J. P. and Webber Jr., C. L.: Embeddings and delays as derived from quantification of recurrence plots, Phys. Lett., 171, 199–203, 1992.